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EXPERIMENT AND ANALYSIS TO ASSESS A PREFERENCE ORDERING SCHEME - THE ANALYTICAL HIERARCHY PROCESS

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ORAE PROJECT REPORT No. PR 232

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R. BIJOOR

OTTAWA, CANADA NOVEMBER 1983

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R. BIJOOR

Report Prepared under
ORAE Project 96513-10

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ABSTRACT

An experiment was conducted by the Directorate of Logistics Analysis to gain some insight into both the practical and theoretical aspects of ranking systems with many attributes. The method used to analyze the experiment is what is popularly known as the Analytical Hierarchy Process (AHP).

On the basis of the experiment it was concluded that the AHP is a simple ranking method well attuned to the decision making process as envisaged by the Project Manager Low Level Air Defence (PM LLAD)

RÉSUMÉ

La Direction de l'analyse logistique a effectué une expérience afin de mieux comprendre les aspects théoriques et pratiques des systèmes de rangement hiérarchique à attributs multiples. La méthode d'analyse employée est celle de la "hiérarchie analytique".

A partir de cette expérience, nous pouvons conclure que la "hiérarchie analytique" est une méthode simple d'évaluation, tout à fait appropriée au processus de décision envisagé par le "Project Manager Low Level Air Defense" (PM LLAD).

FOREWORD

This report discusses a method of ranking a group of weapon systems taking into account not only the engineering aspects of the systems but such other factors as socio-economic gains, Canadian content, logistics etc. as well.

For illustrative purposes let us take engineering aspects, socio-economic implications and logistics to be the total number of facets to be considered. It follows then that a decision made as to the relative importance of each of these factors is a decision made at a certain level. For convenience let us call this a level 1 decision and the factors as level 1 factors. Now each of these level 1 factors can be split into smaller components, (for example the engineering factor could be split into acquistion range, single shot kill probability etc.) with a level 2 decision concerning level 2 factors being made.

Finally, each pair of weapon systems is compared in the light of each of the sub-components and the result of the comparison noted, this then is the level 3 decision on level 3 factors. Combining all these decisions at the various levels we arrive at what Saaty (Reference 3) calls the "focus" - the final ranking of the weapon systems. The entire ranking process is termed as an "Analytical Hierarchy Process" or AHP.

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EXPERIMENT AND ANALYSIS TO ASSESS A PREFERENCE ORDERING SCHEME - THE ANALYTICAL HIERARCHY PROCESS

INTRODUCTION

1. This report is written in response to a request for assistance (Reference 1) by the staff members of the Low Level Air Defence (LLAD) project office. The very essence of the Evaluation Plan for the Low Level Air Defence System issued by the LLAD project office (Reference 2) is a complex scheme for ranking various air defence systems taking into account not only the purely technical and logistic aspects of the contending systems, but factors such as industrial benefits and Canadian content, as well.

2. After investigating various ranking methods where the objects to be ranked have multiple attributes it was decided to use Saaty's methods (Reference 3). The hierarchial method has two very appealing aspects, firstly, it is mathematically simple and, secondly, it fits in very closely with the LLAD evaluation plan.

3. The preferred method of gaining an insight into the Analytical Hierarchy Process (AHP) is to conduct an experiment and apply AHP techniques to the experimental data. Consequently at a meeting between D Log A and PM LLAD Staffs, it was decided to design and carry out just such a trial.

DESCRIPTION OF EXPERIMENT

4. Consider for example the air defence of an airfield. It is assumed that there are five different ground to air missile systems which are to be evaluated. Each of the missiles has five attributes which are:

a. Acquisition range;

b. Identification, Friend or Foe (IFF);

c. Reaction time;

d. Single shot kill probability; and

e. Missile velocity.

Table I gives us the values of each of these characteristics for each of the missiles. Except for the IFF feature, which a missile system possesses or does not possess, all characteristics have numerical values.

5. Using Saaty's terminology we can consider the selection of a missile system as the "focus"; the missile characteristics and their relative importance are the second level of the hierarchy and, finally, the actual comparison of the missiles is the third level.

6. The experiment was conducted by D Log A with the help of three members of PM LLAD who will be identified as evaluators A, B and C. Two sets of forms were distributed to the evaluators (Figures 1 and 2). Of the evaluators, only one was briefed by the author and he in turn briefed the others. At this point the first principle to be observed in the conduct of the experiment was flouted in that all three evaluators should have been briefed by the author.

7. At this time it is worth digressing a bit to explain the mathematics underlying the AHP analysis. We assume that the evaluators are comfortable with the semantic variables Equally, Slightly, Greatly, Very greatly and Decisively and, furthermore, they are quite at ease in making pairwise comparisons between missile characteristics. We can then

write a matrix of preferences with elements a_{ij}, which is a measure of the intensity with which characteristic i is preferred to characteristic j. We further specify that $a_{ij} = 1/a_{ji}$ and that $a_{ii} = 1$. Suppose now that the characteristics we are discussing are capable of being measured and let W_i be the outcome of the measurement; we can then calculate the preference intensity as the ratio W_i/W_j. As a_{ij} is an estimate of W_{ij} we can write:

$$a_{ij} \frac{W_j}{W_i} = 1 \text{ - - - - - - - - - (1)}$$

if our estimate a_{ij} is perfect. It follows then that when n characteristics are considered equation 1 becomes:

$$\sum_{j=1}^{n} a_{ij} W_j = n W_i \quad (i=1,....n) \text{ - - (2)}$$

which in matrix notation is:

$$[A][W] = n[W] \text{ - - - - - - (3)}$$

where A is a square matrix, W is a column vector and n is a scalar. In the light of matrix theory W is an eigen vector of A and n is its eigen value. Under the ideal conditions stipulated so far the eigen value n would be the only non-zero eigen value.

8. In real life however, the preference intensities can not be accurately estimated and equation (3) does not hold, but small perturbations in the elements of A result in small perturbations in the value of the eigen value. It follows then that the measure of "goodness" or "consistency", as Saaty calls it, is how close the value of the eigen value which we calculate is to the number n. Consistency is a measure of the transitivity; for example, if A is preferred threefold to B and B is preferred

threefold to C, then A should be preferred ninefold to C. In actual practice A might only be preferred fivefold to C. In some cases A might be preferred to B and B to C but C might be preferred to A hence intransitivity of choice. These are normal facets of human judgement and must be accepted as such in any mathematical analysis of ranking.

9. Equation 3 can now be expressed in a form which is commonly used in texts on Matrix methods:

$$[A - \lambda I][W] = 0 \text{ - - - - -(4)}$$

where again λ is the eigen value and I is a unit matrix (a matrix with the principal diagonal of unity and zero elsewhere). It follows, that for equation (4) to have non-trivial solutions the determinant (det)

$$\det|A - \lambda I| = 0 \text{ - - - - - (5)}$$

The mathematical details and the computer programs necessary to evaluate the eigen value, eigen vector and the intransitivity of choice are given in Reference 6.

DISCUSSION OF THE DATA COLLECTION FORM

10. The data collection form was designed to utilize a preference scale extending from 1 to 9; this is in accord with Saaty's recommendation that scales with 7 + 2 points is more than adequate to distinguish between stimuli. With reference to Figure 1, we assign the value 9 to the phrase "Decisively More Important". It follows, therefore, that according to Figure 1 the preference intensity of acquisition range to reaction time is 3 and conversely the preference intensity of reaction time to acquisition range is 1/3. Table II is an example of a preference intensity

matrix generated by the data forms filled out by evaluator A. Figure 2 shows the form where a pair of missiles are compared with respect to one of the characteristics and Table II shows the relevent matrix of preference intensities.

11. Consistency Ratio As noted in paragraph 13 consistency is a measure of goodness, the consistency index (CI) is a measure of how far our sample deviates from the ideal, hence:

$$CI = (\max - n)/(n - 1) \quad (6)$$

If a matrix is constructed such that the preference intensities are generated in a random fashion, the principal diagonal constrained to be one and the appropriate reciprocal preference intensities noted, then a distribution of CIs called Random Index (RI) can be calculated. Saaty suggests that the ratio of CI to RI (called the Consistency Ratio) be considered acceptable if it is 0.10 or less.

12. The consistency ratio for a hierarchy is obtained by multiplying the first level priority row vector by the second level column vector of CIs and adding it to the first level CI. Similarly, the RI is computed for the hierarchy, the usual ratio calculated and the resulting Consistency Ratio for the Hierarchy (CRH) arrived at. The CRH is to be considered tolerable if it is 0.10 or less.

13. Multiple Judges For the present, "Consensus" amongst judges or evaluators is arrived at by using the geometric mean of their responses. As the LLAD study progresses other means might be considered.

14. Final Ranking The final ranking is arrived at by pre-multiplying the column vector of normalized eigen vectors of the priority matrix by a matrix whose columns are the normalized eigen vectors of the preference matrices conditioned on each of the priorities.

15. As we have many judges it is imperative to ascertain the degree of concordance amongst them; to this end we compute a statistic called the coefficient of concordance. The coefficient of concordance has a value of 1 when there is complete agreement between judges and a value of 0 when there is no accord. The formula to compute this coefficient is given in Annex A.

DISCUSSION OF RESULTS

16. Figures 3 and 4 are the computer output of a single evaluator's (evaluator A) assessment. Reference 6 describes how the input matrices are constructed and the data inserted into the computer. Figure 3 indicates the intransitivity in A's priority matrix and this intransitivity is ignored. Normally A would be approached and asked to resolve the anomaly; as this was merely an excercise, the author resolved the intransitivity and the differences in the final ranking are clear.

17. Table III lists the consistency ratio of the hierarchy as realised by each of the evaluators. We now need to specifiy a "Consensus Maker" who must synthesize the deliberations of each of the evaluators; following the practice of Reference 5 (vidè pages 515, 516) the consensus maker will be referred to as "she".* The consensus maker

---

* As the entire process is one of decision making at various levels, it is easier to label the consensus maker as "She". If one feels that women's suffrage is insufferable, then the final decision maker can be referred to as "He".

has several courses of action open to her to arrive at a consensus. She can:

a. Disregard C's analysis because of an unacceptable CRH, this is not remarkable in view of C being insufficiently briefed;

b. Noting that the geometric mean of the responses of all evaluators is very heavily influenced by C's strong preference for missile system C (Table V) which causes the rank ordering to change (Table IV) thus automatically affording C a greater voice in the ordering than A or B, she can on this basis disregard C's analysis; and

c. Noting that the coefficient of concordance amongst the three evaluators is .96 and is statistically significant at both the 5% and 1% level, she can decide to use the simple rank totals and accept the ranking thus generated. In this case she would have used all the information available to her to arrive at a consensus.

LESSONS LEARNED

18. Based on the results of this experiment the following lessons were learned concerning the application of the AHP technique:

a. The questionnaire (Figures 1 and 2) in its present form is inadequate on two counts which are:

(i) A single paired comparison is made on each sheet of paper, this leads to a formidable sheaf of papers when even a moderate number of systems are to be compared.* This can be confusing and perhaps aggravating to both the person filling in the form and the person entering the data from the form into a matrix.

(ii) The word "more" between the adverb and the adjective should be modified to accomodate the word "less" as well. In keeping with good English usage the adverb "greatly" should be replaced by "much". The phrase would then be written as "Much more/less important" rather than the present "Greatly more important".

b. All participants in the experiment must be thoroughly informed on the precise meaning of each attribute being examined.

c. Informal conversations with PM LLAD staff suggest using a scale of 1 to 5 rather than the present 1 to 9 scale for denoting the priority matrix preference intensities.

d. No hard and fast rules for arriving at a consensus can be established; each case must be examined on its merit.

---

* In this study each evaluator had to fill in 60 pages of forms.

CONCLUSIONS

19. The conclusions arrived at from the conduct of the experiment are:

a. The AHP method is a simple, flexible ranking method which readily lends itself to the decision making process as envisaged by PM LLAD.

b. No hard and fast methods of arriving at a consensus can be predicated; however, each case requiring a consensus will have to be judged on its own merits.

RECOMMENDATIONS

20. Experience gained from the conduct of this experiment suggests the following:

a. The AHP method should be used to rank the competing weapon systems.

b. The present form (Figure 1) should be changed. The suggested revised form is shown in Figure 5.

c. When trials are conducted at units the trials team must have a carefully prepared briefing kit so that differences in briefings at the various units can be minimized.

Evaluator: A

Remarks

Reaction Time is the time between target indentification and opening fire.

☐ Equally important

☐

☒ Slightly more important 3

☐

☐ Greatly more important

☐

☐ Very greatly more important

☐

☐ Decisively more important

Acquisition Range

When compared to:

Reaction Time

If you can't decide between a pair of the above statements check the box in the middle.

FIGURE 1: Attributes Questionnaire

Evaluator: A

Preferences

Comparison of: Acquisition Ranges.

System: Missile A - 10.5 km

System: Missile B - 16.0 km

Comparing Missile A with Missile B My preference can be described as:

☐ Equal preference

☐

☐ Slightly prefer to

☐

☐ Greatly prefer to

☐

☐ Very greatly prefer to

☐

☒ Absolutely prefer B to A

If you can't decide between a pair of the above statements check the box in the middle.

FIGURE 2: Preference Questionnaire

```
RANKING
INPUT PRIORITY ENTRIES
⎕:
      7 3 1 3 .2 .2 .33333 3 1 3

YOU ARE INTRANSITIVE...
ACQR =SSKP <REACT<ACQR  => ACQR <ACQR
YOU ARE INTRANSITIVE...
REACT=VELOC<SSKP <REACT => REACT<REACT
DO YOU WANT TO MAKE CHANGES TO PRIORITY MATRIX ?(YES/NO)
NO

EIGENVECTOR: 0.3599 0.04138 0.2482 0.2255 0.125
MAX EIGENVALUE 5.543
CONSISTENCY INDEX: 0.1357
CONSISTENCY RATIO: 0.1212

INPUT ENTRIES CORRESPONDING TO PRIORITY 1
⎕:
      .11111 .11111 .11111 .11111 .25 .16667 .25 .25 1 4

EIGENVECTOR: 0.02385 0.0869 0.1936 0.5021 0.1936
MAX EIGENVALUE 5.551
CONSISTENCY INDEX: 0.1378
CONSISTENCY RATIO: 0.123

INPUT ENTRIES CORRESPONDING TO PRIORITY 2
⎕:
      1 1 1 9 1 1 9 1 9 9

EIGENVECTOR: 0.2432 0.2432 0.2432 0.2432 0.02703
MAX EIGENVALUE 5
CONSISTENCY INDEX: ¯1.665E¯16
CONSISTENCY RATIO:  ¯1.487E¯16

INPUT ENTRIES CORRESPONDING TO PRIORITY 3
⎕:
      5 .33333 1 5 .2 .25 4 3 5 4

EIGENVECTOR: 0.2269 0.08395 0.4399 0.2028 0.04649
MAX EIGENVALUE 5.413
CONSISTENCY INDEX: 0.1034
CONSISTENCY RATIO: 0.09229

INPUT ENTRIES CORRESPONDING TO PRIORITY 4
⎕:
      4 .16667 .25 6 .2 .25 4 4 5 4

EIGENVECTOR: 0.1437 0.07508 0.4982 0.2411 0.04189
MAX EIGENVALUE 5.84
CONSISTENCY INDEX: 0.2099
CONSISTENCY RATIO: 0.1874

INPUT ENTRIES CORRESPONDING TO PRIORITY 5
⎕:
      1 1 .142857 3 1 .142857 4 .142857 4 9

EIGENVECTOR: 0.1037 0.1112 0.1112 0.6351 0.03877
MAX EIGENVALUE 5.144
CONSISTENCY INDEX: 0.03601
CONSISTENCY RATIO: 0.03215
FINAL RANKING
SYSTD   0.3749
SYSTC   0.3152
SYSTA   0.1205
SYSTE   0.09663
SYSTB   0.09301
```

FIGURE 3: Ranking (Intransitive)

```
      RANKING
INPUT PRIORITY ENTRIES
⎕:
      7 3 1 3 .2 .2 .33333 3 1 3

YOU ARE INTRANSITIVE...
ACQR =SSKP <REACT<ACQR  => ACQR <ACQR
YOU ARE INTRANSITIVE...
REACT=VELOC<SSKP <REACT  => REACT<REACT
DO YOU WANT TO MAKE CHANGES TO PRIORITY MATRIX ?(YES/NO)
YES

INPUT PRIORITY ENTRIES
⎕:
      7 3 1 3 .2 .2 .33333 1 1 1

YOU ARE INTRANSITIVE...
ACQR =SSKP =REACT<ACQR  => ACQR <ACQR
DO YOU WANT TO MAKE CHANGES TO PRIORITY MATRIX ?(YES/NO)
YES

INPUT PRIORITY ENTRIES
⎕:
      7 1 1 3 .2 .2 .33333 1 1 1

EIGENVECTOR: 0.3172 0.04729 0.2317 0.2317 0.1721
MAX EIGENVALUE 5.132
CONSISTENCY INDEX: 0.03301
CONSISTENCY RATIO: 0.02947

INPUT ENTRIES CORRESPONDING TO PRIORITY 1
⎕:
      .11111 .11111 .11111 .11111 .25 .16667 .25 .25 1 4

EIGENVECTOR: 0.02385 0.0869 0.1936 0.5021 0.1936
MAX EIGENVALUE 5.551
CONSISTENCY INDEX: 0.1378
CONSISTENCY RATIO: 0.123

INPUT ENTRIES CORRESPONDING TO PRIORITY 2
⎕:
      1 1 1 9 1 1 9 1 9 9

EIGENVECTOR: 0.2432 0.2432 0.2432 0.2432 0.02703
MAX EIGENVALUE 5
CONSISTENCY INDEX: ¯1.665E¯16
CONSISTENCY RATIO:  ¯1.487E¯16

INPUT ENTRIES CORRESPONDING TO PRIORITY 3
⎕:
      5 .33333 1 5 .2 .25 4 3 5 4

EIGENVECTOR: 0.2269 0.08395 0.4399 0.2028 0.04649
MAX EIGENVALUE 5.413
CONSISTENCY INDEX: 0.1034
CONSISTENCY RATIO: 0.09229

INPUT ENTRIES CORRESPONDING TO PRIORITY 4
⎕:
      4 .16667 .25 6 .2 .25 4 4 5 4

EIGENVECTOR: 0.1437 0.07508 0.4982 0.2411 0.04189
MAX EIGENVALUE 5.84
CONSISTENCY INDEX: 0.2099
CONSISTENCY RATIO: 0.1874

INPUT ENTRIES CORRESPONDING TO PRIORITY 5
⎕:
      1 1 .142857 3 1 .142857 4 .142857 4 9

EIGENVECTOR: 0.1037 0.1112 0.1112 0.6351 0.03877
MAX EIGENVALUE 5.144
CONSISTENCY INDEX: 0.03601
CONSISTENCY RATIO: 0.03215
FINAL RANKING
SYSTD  0.3829
SYSTC  0.3094
SYSTA  0.1228
SYSTB  0.09505
SYSTE  0.08983
```

FIGURE 4: Ranking (Transitive)

	The characteristic in the left margin compared with that in the right hand margin is:									
Characteristic	Absolutely Un- Important	Very Much Less Important	Much Less Important	Slightly Less Important	Equally Important	Slightly More Important	Much More Important	Very Much More Important	Decisively More Important	Characteristic
Acquisition Range	☐	☐	☐	☐	☐	☐	☐	☐	☐	Reaction Time
"										"
"										"
"										"
etc.										etc.

FIGURE 5: Attribute Questionnaire (Proposed)

TABLE I

MISSILE CHARACTERISTICS

		Systems				
Serial	Parameter	A	B	C	D	E
1	Acq Range	10.5km	16km	18km	20km	18km
2	Ident IFF	Y	Y	Y	Y	N
3	Reaction Time	7 Secs	9 Secs	6 Secs	7 Secs	10 Secs
4	SSKP	.70	.65	.80	.75	.55
5	Msl Velocity	500m/s	500m/s	500m/s	1000m/s	425m/s

TABLE II

PREFERENCE INTENSITY MATRIX

Reaction Time						
System 1	A	B	C	D	E	Eigen Vector*
A	1.0	5.0	0.33	1.0	5.0	0.227
B	0.2	1.0	0.2	0.25	4.0	0.084
C	3.0	5.0	1.0	3.0	5.0	0.440
D	1.0	4.0	0.33	1.0	4.0	0.203
E	0.2	0.25	0.2	0.25	1.0	0.046

Eigen Value	5.412
Consistency Ratio	0.092
Consistency Index	0.103

* Normalized Eigen Vector

TABLE III

CONSISTENCY RATIO OF THE HIERARCHY (C.R.H.)

	C.R.H.
Evaluator A	.07
Evaluator B	.04
Evaluator C	.22
Geometric Mean	.06

TABLE IV

FINAL RANKING OF MISSILE SYSTEMS

	System A	System B	System C	System D	System E
Evaluator A	3	4	2	1	5
Evaluator B	3	4	2	1	5
Evaluator C	3	4	1	2	5
Rank Totals	9	12	5	4	15
Ranking(1)	3	4	2	1	5
Geometric Mean Method Ranking(2)	3	4	1	2	5

Coefficient of Concordance = 0.96 (significant at 5% and 1% level).

(1) Using Rank Sums.
(2) Using Geometric Mean of Responses.

TABLE V

NORMALIZED EIGEN VECTOR COMPONENTS OF FIRST THREE SYSTEMS

	Evaluator A	Evaluator B	Evaluator C	Geometric Mean
System D	.383	.337	.270	.328
System C	.309	.289	.447	.359
System A	.122	.175	.148	.148

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D Log A Staff Note 83/11 F. Audet (in preparation).

ANNEX A
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COEFFICIENT OF CONCORDANCE

1. The following is taken from Moroney's Fact from Figures. (Ref. 4)If n options are to be ranked by m judges then the total sum of ranks would be:

$$m\ n(n + 1)/2 \quad \text{- - - - - - -} \quad (1)$$

where

$$n(n + 1)/2 \quad \text{- - - - - - -} \quad (2)$$

is the sum of the first n natural numbers. If the judges were incapable of any genuine ability to evaluate the options presented the expected rank totals of each option would be:

$$m(n + 1)/2 \quad \text{- - - - - - -} \quad (3)$$

If the judges were in complete accord the rank totals would form a series:

$$m,\ 2m,\ 3m.....nm \quad \text{- - - - - -} \quad (4)$$

2. If we take the null hypothesis H_0 to be the statement that, "there is absolutely no accord between the judges", the maximum possible sum of squares of the difference between the "no accord" and "complete accord" is given by:

$$S_{max} = m^2(n^3 - n)/12 \quad \text{- - - -} \quad (5)$$

The coefficient of concordance W is then:

$$W = S/S_{max}$$

where S is the sum of the squares of the differences between the rank totals arrived at by experiement and equation (3).

3. A continuity correction is applied to the estimate of W by subtracting 1 from S and adding 2 to S_{max}. The significance of W is tested by the F statistic where:

$$F = (m-1)W/(1 - W) - - - - (7)$$

with the numerator degrees of freedom given by:

$$(n-1) - (2/m) - - - - (8)$$

and denominator degrees of freedom as:

$$(m-1)\left[(n-1) - (2/m)\right] - - - - (9)$$

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