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PROJECT REPORT 678

AN ALGORITHM FOR HOWITZER MUZZLE VELOCITY PREDICTION

by

R.M.H. Burton

September 1994

OTTAWA, CANADA

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Department of National Defence
Canada

Operational Research and Analysis

Directorate of Mathematics and Statistics

Project Report 678

An Algorithm for Howitzer Muzzle Velocity Prediction

by

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Ottawa, Canada September 1994

Abstract

This project report describes a number of ways in which muzzle velocity can be forecasted for howitzers such as the M109 155mm, C1 105mm or the L5 105mm. A simple time series model is selected and proposed for use with the Canadian Forces Artillery Calibration System (ACS) project. The proposed model is called the double exponentially weighted moving average (DEWMA) model.

Résumé

Le rapport décrit différentes façons de prédire la vitesse initiale d'une bouche d'obusier tel que le M109 155mm, le C1 105mm ou le L5 105 mm. Un modèle à série chronologique simple fut choisi et suggéré pour utilization pour le project des systèmes de régimage. Le modèle proposé est appelé "modèle à moyenne mobile à double exponentielle".

Acknowledgement

We are indebted to the Royal School of Artillery and to SD-Scicon, an operational research organization, for the data and documentation they have provided on calibration projects in the U.K.

Capt. R. L'Esperance, of the Directorate Land Armament and Electronics Engineering and Maintenance (DLAEEM) has been very helpful in providing information about the ACS project and artillery in general.

Executive Summary

The Directorate of Mathematics and Statistics (DMS) has been working with staff from the Directorate Land Requirements (DLR) and the Directorate Land Armament and Electronics Engineering and Maintenance (DLAEEM) to produce a statistical model that forecasts Howitzer Muzzle Velocities. Data from the C1, L5 and M109 guns has been collected over the past two years in order to produce a working model.

The computer model developed for this purpose is documented in this report. The ADGA Group of consulting engineers has been tasked to take the DMS model and incorporate it within a larger system called the Artillery Calibration System (ACS).

The statistical model proposed for use here is called a **Double Exponentially Weighted Moving Average (DEWMA)** Model. This model is a time series model that weights "fresh" data more heavily than older data. Eight other statistical models are discussed as well, including a model that simply uses the firing table muzzle velocities as the forecast.

To compare the various models described herein, a measure based upon the root mean square prediction error was developed. We then found the percentage improvement (across all charges) for a given model, relative to the firing table forecasts.

The following table shows the percentage relative improvement for the DEWMA model when compared to the firing table (FT) model — for the C1, L5 and M109 (non-illuminated projectiles) across all charges.

Gun/projectile category	Relative Improvement of DEWMA over the FTs (%)
C1 (non-illum)	38
L5 (non-illum)	54
M109 (non-illum)	37

For each gun/projectile category above, the DEWMA model also outperforms (a simulation of) the former procedure used to forecast muzzle velocities.

Contents

Introduction

1. It is surprising just how many known, or suspected factors influence the muzzle velocity of howitzers such as the M109 155mm, C1 105mm or the L5 105mm.

2. For example, when a gun barrel is new, the projectile fits inside very tightly and when fired, the propellant pressure is high, producing a high muzzle velocity. In a worn gun, the initial resistance to the motion of the projectile is less, therefore there is a decrease in the initial propellant (shot-start) pressure. The projectile can be rammed further forward in a worn barrel thus increasing the initial space available for the expansion of propellant gases. This results in a lowering of the pressure and a reduction in the muzzle velocity (reference [13]).

3. There is also what is known as the "hump" effect. During the first several hundred rounds fired from a new gun barrel, the muzzle velocity rises rapidly to a peak. It then falls to a level commensurate with the state of wear. The occurrence of this effect is uncertain and its magnitude is variable.

4. Even a battery of new barrels may exhibit a velocity spread of 5m/s between guns with the highest muzzle velocity (MV) and guns with the lowest muzzle velocity. As an example, for the M109 gun, at a middle charge (charge 5W), with target at 3000m, a difference of 5m/s leads to roughly a 60m difference in range. For a high charge (charge 7W), with target at 7000m, there would be a 90m difference in range. Since the "kill radius" for the M109 round is about 50m, it is operationally important to obtain the best possible estimate for the MV.

5. The temperature of the propellant, the weight of a projectile, the explosiveness of a particular lot of propellant, the temperature of the barrel (barrel expansion), the occasion (which is defined below) and other factors all influence the muzzle velocity to some degree.

6. This report describes various ways to forecast the muzzle velocities of howitzers such as the M109 155mm, the C1 105mm, and the L5 105mm. The purpose of the project is to improve upon the old Muzzle Velocity Monitoring Policy that was based upon firing four to six rounds for each charge from each gun; rejecting any rounds that were found to be outside a given tolerance band, and then taking the average of the remaining muzzle velocities. This policy was based upon a 1972 U.K. study. The problem with this approach is that there is "occasion-to-occasion" variation in the muzzle velocities, where an occasion is defined to be a (contiguous) series of similar projectiles, at a given charge, that are fired less than two hours apart. Taking the average of the muzzle velocities from this single occasion, and using it to predict the muzzle velocities for the occasions that follow is therefore not as accurate as it could be.

7. SD-Scicon, a U.K. consulting firm (see reference [17]) produced a series of documents that explained its approach to modelling muzzle velocities. The first phase document concluded that taking the average of the muzzle velocities across all barrels of a given charge would decrease the prediction error. This prediction approach was termed the Global Mean Model. We have developed a prediction algorithm that is slightly more sophisticated than the Global Mean Model. The U.K. operational analysts have also developed more sophisticated models themselves and outlined these in references [18] and [19]. To understand some of the misgivings we have about the Global Mean Model, we first need an understanding of how "age" is measured for a gun barrel. This leads us to the concept of the Effective Full Charge (EFC).

8. Firing one thousand rounds with a high charge is known to cause more wear and fatigue than firing one thousand rounds at a very low charge. This is analogous to running two cars the same distance, but running one car at 7000 RPM and the other car at 3000 RPM. The cylinder walls of the high RPM car will be more worn than those of the other car. In the firing tables for each of the guns we see a term called the Effective Full Charge (EFC). Whereas the highest charge is given an EFC rating of unity, the lesser charges are given EFC ratings of some fraction of one. Therefore, four rounds of a low charge with an EFC rating 0.25 will produce the same barrel wear as a single round fired at the highest charge.

9. If all barrels are of the same or nearly the same age in terms of wear, and there are scant data available on any given barrel, then taking the global mean muzzle velocity makes good sense, and provides a simple, understandable model. However, in the case of the Canadian Forces gun barrels, the "ages" as measured (albeit dubiously) by the Effective Full Charge (EFC) can vary substantially. Therefore, taking an average of the muzzle velocities across all barrels, and using that value as a forecast, will not "fit" the data well for relatively young or old barrels. Also, if a gun barrel needs to be replaced for whatever reason, then presumably, using a global mean model would underestimate the actual muzzle velocities for that particular barrel.

10. If there is barrel wear taking place (we take this as a given) then after a period of time some of the data will become "stale". The barrels will age (in terms of wear) at different rates. As a result, the dataset upon which the Global Mean Model is based will have to be culled of stale data, but this may be an analysis headache for the officer left in charge of the project. What data needs to be archived? If there is a mixture of young and middle-aged barrels, what then?

11. One could presumably attempt to group classes of barrels of about the same age, or of the same wear, but this would be too is problematic. Statistical analysis of barrel wear and EFCs have revealed many inconsistencies, rendering the collected data of dubious value. Our analysis indicates that some of the barrels are decreasing

in diameter — a highly unlikely event. Current EFC information is based on the Number One's recollection of the rounds fired during a practice camp — therefore we have very little trust in any of these age related measurements.

12. We have a very messy data situation and we must come up with an appropriate forecasting solution. One of the most serious problems that we have experienced is simply getting enough data from which to develop and test any forecasting procedure that we develop. There is quite a number of gun/charge combinations for which only a few rounds of data are available. Although a number of different procedures are discussed in this report, the one that we propose is called a **double exponentially weighted moving average** or DEWMA model. The model has characteristics reminiscent of the Global Mean Model, but it is a time (and barrel wear) sensitive model that gives more weight to fresh data than to older data. We discuss the technical details of the model in a later section, but first we need to discuss the various types of data that are available — messy or not.

Methodology

The data

13. Muzzle Velocity (MV) data are collected at the gun barrel using a radar called the Muzzle Velocity Indicator (MVI). The MVI collects data on each round fired and stores that data in a small buffer. At a convenient time, stored data are then downloaded to the Artillery Calibration System (ACS) computer for analysis.

14. As an example of the raw data, in figure 1 we provide a listing of all currently available data on the C1-105mm gun, at charge 1.

```
C1        21 HE 2.5 1  8.0 .0    186.3 18/10/1110:16 2303199213:33:21
C1        21 HE 2.5 1  8.0 .0    181.0 18/10/1110:12 2303199213:33:21
C1        21 HE 2.5 1  8.0 .0    186.2 18/10/1110:12 2303199213:33:21
C1       123 HE 2.5 1 16.0 .0    188.8 09/10/1111:29 2303199213:33:21
C1       123 HE 2.5 1 16.0 .0    190.4 09/10/1111:28 2303199213:33:21
C1       123 HE 2.5 1 16.0 .0    191.9 09/10/1110:01 2303199213:33:21
C1       123 HE 2.5 1 16.0 .0    187.5 09/10/11 9:43 2303199213:33:21
```

Figure 1: Raw data captured by MVI for C1 Gun, Charge 1

15. There are seven rows of data here; each row corresponds to one round fired. Reading across the first row, we see that this is a C1 gun; the barrel number is 21; the projectile type is HE; the weight in units of "squares" is 2.5; charge 1; propellant temperature is 8.0 degrees Celsius; lot factor is 0; Muzzle velocity is 186.3 m/s; date fired was 18/10/11; time fired was 10:16; the blank space after the time indicates that this round was not rejected by the MVI; the upload date to the ACS computer was 23-03-1992 at time 13:33:21. Most of these terms require some explanation, which we will give shortly. The reader should note first the obvious date-of-firing error, for this round and all other rounds for charge 1. This type of error is gunner input error and needs to be addressed by the Directorate of Land Requirements, DLR. At this point we are not too concerned with date errors, but, if left unchecked, this type of error will seriously degrade the performance of any statistical models that we develop.

Editing

16. This is an appropriate place to discuss other types of errors that we have encountered. We have strong reason to doubt the validity of the charges as recorded by the gunners. That is, it appears sometimes that charge 4G should have been 4W; 5G should have been 5W; and so on. It is almost impossible to tell in these situations whether the data are wrong or not, but it will be important for the gunners to realize that any input errors that they make will degrade the muzzle velocity forecasts. It is in their interest to make sure that propellant temperatures, charges, dates and so on are recorded accurately.

17. We have edited the data for the M109, C1 and L5 guns and removed any observations that were obviously erroneous. For instance, observations with muzzle velocities of over 800m/s have been automatically rejected. We have tolerated date-related errors, but again, this type of error will eventually cause problems, if left unchecked.

18. As a point of interest, it has been conjectured that these very large muzzle velocities are caused by an interaction effect between two guns/MVIs. It is possible that if two guns are positioned too closely, and fired nearly simultaneously, a MVI could pick up the signal of the round fired from the adjacent gun. This could lead to large muzzle velocities as we have observed.

Gun and Projectile types

19. There are three gun types considered here: the 155mm M109, the 105mm

C1 and the 105mm L5. The respective firing tables at references [2, 3, 4] provide standard muzzle velocity estimates for "illuminated" or IL and others such as High Explosive or HE projectiles.

20. In the case of the M109, there is also another class of projectiles coded AD, RA and DP. No firing table data have been obtained for these projectile types as Area Denial (AD), Remote Anti-armour mine system (RA) and Dual Purpose improved conventional munition (DP) projectiles are not currently used by the CF. Therefore, we have seven categories of muzzle velocity data. These categories are given in the table below.

Gun type	Projectile types	Gun/proj category code
M109	IL	MiL
	HE, WP, HC, CR, CG, CV, CY	M1
	AD, RA, DP	Mad
C1	IL	Cil
	HE, WP, HC, CR, CY, CV, CG	C1
L5	IL	LiL
	HE, WP, HC, CR, CY, CV, CG	L5

Table 1: Seven categories of gun/projectile types

We define the remaining projectile codes as follows:

- white phosphorus (WP);
- Base Ejection Coloured Smoke (white (HC), red (CR), green (CG), yellow (CY) and violet (CV)).

21. The illuminated projectile data will not be combined with data from one of the other categories. The reason for this is that illuminated rounds will have a different muzzle velocity from non-illuminated rounds (see firing tables). Data for the HE, WP, HC, CR, CG, CV and CY projectile types will be combined and treated homogeneously, however. The AD, RA and DP projectiles in the M109 gun/projectile category will also be treated homogeneously.

Weight of the projectile

22. Each projectile is weighed so that any differences from the stated standard weight can be taken into account. A heavier-than-standard projectile will have a slightly slower-than-standard muzzle velocity, everything else being constant. The

converse is also true. We will need to account for the effects of non-standard weight if our models are to be reliable.

23. The phrase "four square weight" is the term used to describe the standard weight of a projectile from the 155mm guns (see reference [20]). These weights are given in Table 2. The value $m_{sq_{std}}$ is the number of squares in the standard weight round. The value $m_{lb_{std}}$ is the equivalent weight in pounds for the standard weight. The notation m_s represents the standard projectile family weight in pounds; these terms are described also in the glossary of terms in annex G.

Gun type	Projectile types	$m_{lb_{std}}$	$m_{sq_{std}}$	m_s
M109	IL	95.0 lbs.	4	91.5 lbs.
	HE, WP, HC, CR, CG, CV, CY	95.0 lbs.	4	95.0 lbs.
	AD, RA, DP	103.5 lbs.	4	103.5 lbs.
C1, L5	IL	33.0 lbs.	2	32.7 lbs.
	HE, WP, HC, CR, CG, CV, CY	33.0 lbs.	2	33.0 lbs.

Table 2: Standard Weights for the Projectiles

24. One square weight difference for the M109-155mm guns is 1.1 lbs. Thus for the M109 guns, the parameter k is 1.1 pounds per square weight. This parameter will be used shortly in equation 5. Therefore, as an example, the nominal weight for a 5 square M109-HE projectile would be $95.0 + 1.1 = 96.1$ pounds.

25. For the 105mm C1 and L5 guns, the system is slightly different. The codephrase "two-square weight" is used to describe the standard weight of these projectiles. The two-square weight for all 105 mm projectiles is 33.0 pounds. One square weight difference is 0.6 pounds. Thus for the C1 and L5 guns, the parameter k is 0.6 pounds per square weight. Therefore, as an example, the nominal weight for a 2.5 square 105 mm projectile would be $33.0 + \frac{1}{2}0.6 = 33.3$ pounds.

Charges

26. For the C1 and L5 guns, there are seven different charges. These are simply denoted charge 1, charge 2,..., charge 7. Charge 1 is the lower charge, producing the least force of expulsion.

27. For the M109 guns there are eleven charges. In order of increasing charge we have charges: 1G, 2G, 3G, 3W, 4G, 4W, 5G, 5W, 6W, 7W and 8.

Temperature

28. The temperature of the propellant will also affect the explosiveness of the charge. Standard temperature is 70° Fahrenheit or 21.1° Celsius. Therefore, if the propellant temperature was 25 degrees Celsius, we would expect that the actual muzzle velocity would be higher than what it might have been if the propellant was at standard temperature, all other things being equal.

Lot factor

29. Propellant is made up in large batches or lots. Even with careful technique, some lots of propellant may be more or less explosive than "standard". To account for this phenomenon, a lot factor is assigned to each bag of propellant. A lot factor of zero (0) implies that the propellant has the same explosive properties as the standard. A lot factor of 1.0 would imply that the expected MV will be one meter per second more than for a bag of propellant from a standard lot, everything else being equal. Negative lot factors are possible, but rare.

30. As an example, assume that under standard conditions of temperature and weight and so on, that the forecasted Muzzle velocity is 200m/s. If a bag of propellant is labelled lot factor 1, then the revised estimate for the MV will be: $200 + 1 = 201$ m/s. Notice that the contribution here is additive, not multiplicative — making the term Lot *Factor* somewhat misleading.

Barrel Wear Data

31. Measurements of the diameter of some of the barrels have also been obtained. Unfortunately, this type of data is of little use. The measurement process appears to be fairly crude and there are cases where the barrel diameters are getting smaller with age (instead of larger as one would expect). Until the measurements become more accurate, no barrel wear data should be used to construct a forecasting tool for muzzle velocities. Instead, we will develop the model using the velocity data only.

Muzzle Velocity Correction Algorithms

32. We have discussed in some detail what available input data there are and why we need this data. What we must do now is explain the process of compensating

for the effects of non-standard weights, temperatures and lot factors on the muzzle velocities.

33. The current forecast for a given gun/projectile type and charge will be that muzzle velocity expected under standard conditions of temperature, weight and propellant. These conditions will rarely be met on a round-to-round basis, and therefore the forecast will have to be revised or corrected to take into account these non-standard conditions.

34. Similarly, the observed muzzle velocities reflect the (usually) non-standard conditions for a given round. To build a forecasting tool for standard conditions we must first standardize (or "reduce") the data. The word "reduce" does not necessarily mean that the value is diminished in magnitude. The terminology presumably stems from computer science jargon where the phrase "data reduction" implies a transformation of raw data into a more useful form. It is the reduced muzzle velocities (RMV) that we will model, and *not* the observed MVs.

35. DLAEEM has provided an algorithm for correcting the muzzle velocities prior to firing the round (to adjust for weight, temperature and lot factor) at reference [21]. This algorithm is coded and used within the MiliPAC (artillery) fire control system. Later we will describe how we have "inverted" the algorithm for our purposes here.

MiliPAC MV Correction Equations

36. Here, the algorithm is used by gunners *before* a given round is fired from the gun. We assume that a predicted muzzle velocity exists for standard conditions — the forecast for standard conditions for a given charge ($V_\circ$). Gunners however, may have a projectile with non-standard weight, non-zero lot factor, and a bag of propellant at some non-standard temperature. Therefore the forecast must be revised to correct for these conditions. The procedure now in use is given in the equations below. The notation has been changed from that in reference[12].

$$\begin{aligned} V^{\sim\lambda} &= V_\circ + \lambda \\ &\ \text{[standard MV corrected for lot factor]} \end{aligned} \tag{1}$$

$$F = \frac{9}{5}\mathrm{C} + 32 \quad \text{[Celsius to Fahrenheit]} \tag{2}$$

$$\tau = a(F-70) + b(F-70)^2 + c(F-70)^3 \quad \text{[temp correction]} \tag{3}$$

$$V^{\sim\lambda\tau} = V^{\sim\lambda} + \tau \quad \text{[standard mv corrected for lot factor \& temp]} \tag{4}$$

$$m = (m_{sq} - m_{sq_{std}})k + m_{lb_{std}} \quad \text{[actual weight in lbs]} \tag{5}$$

$$\omega = V^{\sim\lambda\tau}(\frac{m}{m_s} - 1)N \quad \text{[weight correction]} \tag{6}$$

$$\begin{aligned} V^{\sim\lambda\tau\omega} &= V^{\sim\lambda\tau} + \omega \\ &= V^{\sim\lambda\tau}(1 + (\tfrac{m}{m_s} - 1)N) \\ &\quad \text{[standard mv corrected for lot factor, temp and weight]} \end{aligned} \tag{7}$$

37. The revised estimate for the actual muzzle velocity is then taken to be $V^{\sim\lambda\tau\omega}$. The $\sim$ symbol here denotes "corrected for", thus $V^{\sim\lambda\tau}$ could be spoken as: standard muzzle velocity forecast, corrected for the lot factor and temperature conditions of this round. After the round is fired the actual muzzle velocity is recorded by the MVI. Any difference between the observed value and $V^{\sim\lambda\tau\omega}$ will be due to random error, and a host of other things such as barrel temperature perhaps, that we are unable to measure or model yet.

38. For convenience, we have included a glossary of all these terms in annex G.

MVI MV Reduction Algorithm

39. The muzzle velocity indicator (MVI) determines by radar the actual muzzle velocity of the round fired. This quantity is called Λ and is stored in a memory buffer in the MVI along with other pertinent information. The MVI also can standardize these Λ using what is referred to as the reduced muzzle velocity algorithm.

40. The standardization algorithm (or muzzle velocity reduction algorithm) is used to estimate what the muzzle velocity of the round would have been, had the round been fired under standard conditions. The word reduced here means "corrected" or "modified", but not necessarily "decreased in value". The algorithm takes the actual observed muzzle velocity (Λ) and then, depending upon the non-standard temperature, weight or lot factor conditions of that round, produces a reduced muzzle velocity (RMV). The RMV can then be used to develop models for muzzle velocity prediction. One possible approach to developing this algorithm is simply to invert the MiliPAC algorithm. Therefore, it follows that if perchance the observed muzzle velocity was identical to the expected muzzle velocity for this charge corrected for non-standard conditions, then, the RMV will equal the *a priori* forecast for standard conditions. Thus, we arrive at the following three equations:

$$\Lambda^{\sim\omega} = \frac{\Lambda}{1 + (\frac{m}{m_s} - 1)N} \tag{8}$$

$$\Lambda^{\sim\omega\tau} = \Lambda^{\sim\omega} - \tau. \tag{9}$$

$$\mathrm{RMV} = \Lambda^{\sim\omega\tau\lambda} = \Lambda^{\sim\omega\tau} - \lambda \tag{10}$$

where the lot factor, weight and temperature corrections are as defined in the glossary.

41. As an example of the standardization procedure, let the observed muzzle velocity, Λ be 186.3 m/s for the C1 gun; charge 1; HE; 2.5 square projectile weight; 8.0 degrees Celsius propellant temperature; with lot factor zero (0). Here the actual weight in pounds is:

$$m = ((2.5 - 2) \times 0.6) + 33 = 33.3$$

and the N factor taken from Table 9 is (-0.48). Therefore the unitless weight correction divisor is:

$$1 + ((\frac{33.3}{33} - 1)(-0.48)) = 0.9956.$$

42. From Table 7 we find the three temperature coefficients: $a = 0.0559$, $b = 1.81 \times 10^{-4}$, $c = 0$. Using these, we calculate the temperature correction, for $F = (1.8 \times 8.0) + 32 = 46.4$ degrees Fahrenheit, as:

$$\tau = 0.0559(F - 70) + 1.81 \times 10^{-4}(F - 70)^2 + (0)(F - 70)^3 = -1.2184$$

43. Finally, we can substitute these values into the set of three standardization equations to get:

$$\Lambda^{\sim\omega} = \frac{186.3}{0.9956} = 187.1233$$

$$\Lambda^{\sim\omega\tau} = 187.1233 - (-1.2184) = 188.3417$$

$$\mathrm{RMV} = \Lambda^{\sim\omega\tau\lambda} = 188.3417 - 0 \approx 188.3.$$

In figure 2 we have replicated the charge 1 data from figure 1 adjoining the reduced muzzle velocities at the end of each row.

44. We can see how well the algorithm removes the temperature and weight effects from the muzzle velocities by calculating a measure of association between weight and temperature, with the MV and RMV. Taking one of the larger groups as an example, such as gun/projectile combination C1, charge 5, we calculate Kendall's τ_b. This measure of association (or correlation) between two variables is robust in the sense that it is insensitive to outliers, and, it is not dependent upon distributional

```
C1        21 HE 2.5 1  8.0 .0   186.3 18/10/1110:16 2303199213:33:21  188.3
C1        21 HE 2.5 1  8.0 .0   181.0 18/10/1110:12 2303199213:33:21  183.0
C1        21 HE 2.5 1  8.0 .0   186.2 18/10/1110:12 2303199213:33:21  188.2
C1       123 HE 2.5 1 16.0 .0   188.8 09/10/1111:29 2303199213:33:21  190.1
C1       123 HE 2.5 1 16.0 .0   190.4 09/10/1111:28 2303199213:33:21  191.7
C1       123 HE 2.5 1 16.0 .0   191.9 09/10/1110:01 2303199213:33:21  193.2
C1       123 HE 2.5 1 16.0 .0   187.5 09/10/11 9:43 2303199213:33:21  188.8
```

Figure 2: Raw data captured by MVI for C1 Gun, Charge 1, with RMV included at the end of each row

assumptions. It is the relative magnitude rather than the absolute magnitude of the variables under study that is important here. A value of Kendall's τ_b near zero is indicative that the two variables are independent; it will tend to -1 if the association is negative (the two variables are in perfect discordance); and it will tend to +1 if the association is positive (the two variables are in perfect concordance).

45. Kendall's τ_b for weight in squares and MV is -0.22 while the measure is -0.09 for the weight in squares and RMV case. The association is less with RMV, and therefore to some degree the algorithm has removed the effect of weight. (Observe that the association is negative — that is, as weight goes up, velocity goes down.)

46. Kendall's τ_b for temperature and MV is 0.28 while the measure of association between temperature and RMV is 0.10. Again the algorithm has been successful to some degree. Observe that the association here is positive — as temperature goes up, velocity goes up.

47. We found that RMV (as compared with MV) was *less* associated with weight in 12/12 cases. Not all gun/charge combinations were used because of small sample sizes. The MV was also found to be negatively associated with weight —as one would expect— in 12/12 cases.

48. The RMV (as compared with MV) was *less* associated with temperature in 10/18 cases. The MV was positively associated with temperature —as one would expect— in 13/18 cases.

49. We conclude from this analysis that the RMV algorithm is more effective at removing the effects of weight than it is at removing the effects of temperature. In six months to a year, this particular algorithm should be re-evaluated to ensure it is operating as one expects. It may then be possible to develop an improved algorithm,

provided that gunners have been conscientious about recording lot factors, weights and temperatures accurately.

A Test for Randomness

50. Let us first consider how the data arise. Usually a series of rounds will be fired within a short span of time. A series of rounds fired that are all the same projectile category, same charge, and fired within a time span of less than two hours apart has been earlier defined to be an occasion. The shots fired within that occasion are referred to as rounds within occasion, or just rounds. Our choice of model depends fairly heavily on whether there is any non-random pattern to the RMV of rounds within an occasion.

51. To determine if there was any evidence of non-random behaviour in the time-ordered rounds within each occasion, we conducted a non-parametric test called the Wald-Wolfowitz runs test (see reference [5]). We can define a run as follows. Suppose the time-ordered RMVs are lined up in a row. Above each RMV value we put an A or a B depending upon whether the RMV is above or below the median for that occasion. A Run is therefore a succession of identical letters which is preceded and followed by different letters, or no letters at all. The total number of runs is an indicator of non-randomness in this series. Too few runs may indicate clustering, and too many runs may indicate a systematic alternating pattern. To test the hypothesis, we have:

H_o : RMV fluctuate randomly about the median value for each occasion.

versus

H_a : RMV fluctuate non-randomly about the median value for each occasion.

where H_o is the null hypothesis and H_a is the alternative.

52. We show the results of fourteen tests in table 3 below. A small P-value is evidence in support of the alternative hypothesis, Ha. Here we take a P-value of 0.05 (the *de facto* standard cut-off) or less as evidence in support of the alternative hypothesis. Eleven of the fourteen tests show no evidence of non-random behaviour. For this reason we have provisionally assumed that the RMVs vary randomly within an occasion. Caution is advised however because not all gun/charge combinations are represented. The possibility remains that whatever models we develop here, could be further improved by taking into account the (possible) non-random behaviour of the RMV.

53. In figure 3 we show a plot for one of the occasions mentioned in table 3, i.e. gun/projectile type L5, charge 6, barrel 4891. There are 24 rounds fired in this occasion. This plot is rather interesting because this is one of the three cases noted above for which there is evidence of non-random fluctuations about the median value.

54. As another example, in figure 4 we show a plot of one occasion from gun/projectile type M1 (M109 non-illuminated), charge 2G, barrel 3314. Here there are 23 rounds fired, and there is more variation about the median value than in the previous example.

Gun Type	Charge	Barrel	n	P-value	Decision
M109	2G	3314	23	0.019	Reject Ho
M109	2G	4387	15	0.003	Reject Ho
M109	4W	4395	15	0.802	Fail to Reject Ho
M109	5G	4099	15	0.802	Fail to Reject Ho
M109	7W	3117	27	0.329	Fail to Reject Ho
L5	5	4159	21	0.508	Fail to Reject Ho
L5	6	4152	18	0.331	Fail to Reject Ho
L5	6	4173	16	0.301	Fail to Reject Ho
L5	6	4891	24	0.037	Reject Ho
C1	4	21	15	0.502	Fail to Reject Ho
C1	5	15	22	0.190	Fail to Reject Ho
C1	5	59	18	0.145	Fail to Reject Ho
C1	5	121	24	0.404	Fail to Reject Ho
C1	6	121	18	0.627	Fail to Reject Ho

Table 3: Results of the Runs Tests

Outliers or wild observations

55. One of the first jobs was to "debug" the raw data. There were date-related errors as we have said, but some of the muzzle velocities also seemed out of line with the others. For reasons we have already discussed, some of the muzzle velocities were over 800 m/s. In this case it is easy to determine that an observed muzzle velocity is outlying the credible range of other MV from that gun/charge. However, there was also a need to determine outlying observations that were still outside the credible range, but perhaps not obviously so.

56. For RMVs from a new set of data, if the difference between the latest forecast and a RMV is too large, then that particular RMV is a potential outlier.

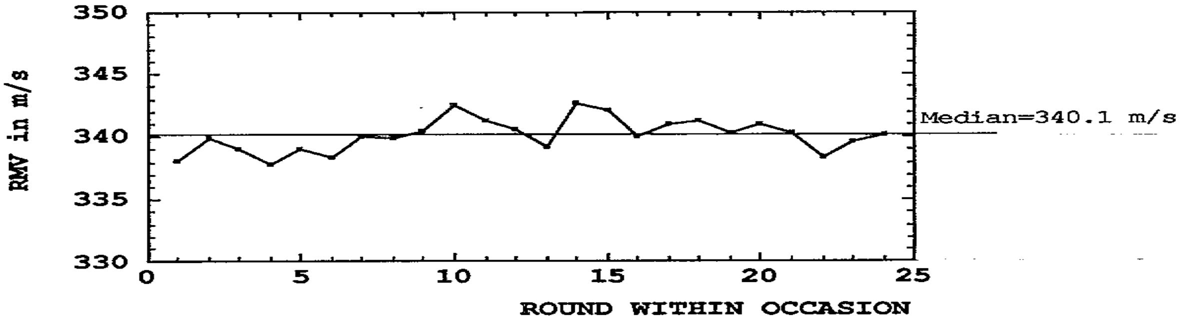


Figure 3: A single occasion from Gun/projectile L5, Charge 6, Barrel 4891, with 24 rounds fired

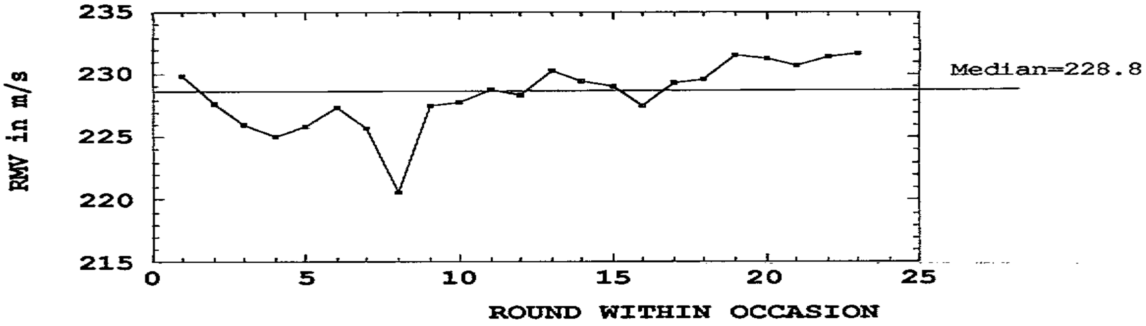


Figure 4: A single occasion from Gun/projectile M1, Charge 2G, Barrel 3314, with 23 rounds fired

The criterion of how large is too large is based upon the standard deviation of the RMV. If the absolute value of the difference between the RMV and the latest forecast is larger than some multiple of the standard deviation of the RMV, then that RMV is declared to be an outlier. In mathematical language, reject the observation if:

$$|\mathrm{RMV} - \mathrm{FCST}| > \delta \sqrt{\mathrm{Variance(RMV)}}$$

where δ is a number which is at least 1.0. This approach is discussed in more detail later in the paper. As a crude outlier detector, this approach does seem to work quite well.

Some Possible Models

57. Given the provisional assumption of random behaviour of the RMV, there are a number of simple models that we might consider. As a baseline model we consider using the firing table values — which we call the MV0FT for each gun/projectile category and charge. We express this model simply as:

$$FCST = MV0FT. \qquad \text{[Model 1]}$$

We have given each model a number that will be used later to identify related values like the forecasts from that model. That is, forecasts from Model 1 will be identified as FCST(1), and so on. There are nine models in all, and the model number itself has no relevance other than to identify the different models.

58. Another possible model is the average RMV across all barrels of a given charge. We call this Model 2.

59. We might also take the median (or middle value) of the RMV across all barrels of a given charge. We call this Model 3.

60. We might also take the mean or the median for each barrel at a given charge. This provides Model 4 and Model 5, respectively.

Single Occasion Mean

61. In the past, gunners have fired a four to six round series for each charge; rejected rounds outside four probable errors (of the muzzle velocity), and calculated an average of the acceptable rounds. This prediction approach was based upon a 1972 UK study.

62. We can simulate this method by taking the average of a single occasion where at least four rounds have been fired, and then use that mean to predict the RMV of the following occasions. This is Model 6, and therefore any forecasts from it will be termed FCST(6).

Stein estimate

63. Another interesting model is based upon empirical Bayes (statistical) methodology. We will draw heavily from the references on Bayes techniques ([6, 10, 11]) and from a DMS paper by Mr. E. Emond (reference [1]). The details can be studied from these books and papers.

64. Let the current barrel mean RMV (for a given gun/projectile category and charge) be denoted as $\bar{x}_b$ and assume that $\bar{x}_b$ is distributed normally with mean μ and known variance σ_b^2. Let the prior distribution for μ also be normal with known mean μ_o and variance τ^2. It can be shown that the posterior mean, for the distribution of μ given the data is:

$$\frac{\sigma_b^2}{\sigma_b^2 + \tau^2}\mu_o + \frac{\tau^2}{\sigma_b^2 + \tau^2}\bar{x}_b \qquad \text{[Model 7]}$$

65. That is, the posterior mean is a weighted average of the prior mean and the current barrel mean. The weights, which are functions of the parameters, are estimated from the data (empirically).

66. In the outlier detection routine we use the variance of the RMV which we have estimated from the currently available data. Our estimate for the variance of $\bar{x}_b$ is also found using analysis of variance techniques and is a function of the estimated variance within occasions and the variance between occasions (see reference [16] and Annex B).

67. Using the methodology in reference [1], we estimate μ_o by the average of the barrel means (for a given charge) and we estimate τ^2 empirically using the sample variance of the barrel means.

68. The posterior mean is now a weighted average of:

- the average of all barrel means at this charge; and
- the individual barrel mean at this charge.

69. This estimate which we will loosely refer to as the Stein estimate, has the property that it draws strength from information about other barrels, when scant data are available for a particular barrel. As more data becomes available, n increases, and the forecast is effectively just the barrel mean for that charge.

Double Exponentially Weighted Moving Average (DEWMA) Model

70. We have also studied another slightly more sophisticated model called an exponentially weighted moving average (EWMA) model. This is a barrel-specific model, therefore, each barrel has its own forecasted RMV for a given projectile category/charge. Let the occasions be indexed by t, $t = 1, 2, 3, \ldots T$ and let the average of the RMV from the tth occasion be denoted by $\bar{x}_t$. Then, given data up to the tth occasion, the forecasted RMV will be denoted by z_t. Similarly, the forecast for data up to the $t-1$th occasion is z_{t-1}. The EWMA equation is

$$z_t = r\bar{x}_t + (1-r)z_{t-1}$$

with the weight r between zero and one. For example, if the weight was set to $r = 1$ then the forecasted RMV would simply be the mean RMV of the last occasion. (In this particular case, we would have one form of a single occasion mean model.) To show how the formula works we take $r = 0.1$ and assume that only two occasions are available. We get,

$$z_1 = 0.1\bar{x}_1 + (1-.1)z_0$$

$$z_2 = 0.1\bar{x}_2 + (1-.1)z_1$$

The last equation can be expanded by substituting in the value of z_1:

$$\begin{aligned} z_2 &= 0.1\bar{x}_2 + 0.9(0.1\bar{x}_1 + 0.9z_0) \\ &= 0.1\bar{x}_2 + 0.09\bar{x}_1 + 0.81z_0 \end{aligned}$$

71. A double exponentially weighted moving average (DEWMA) model is simply a re-application of the EWMA. A good reference here is Kendall, Stuart and Ord (see §**50.40** of reference [8]). We take

$$\tilde{z}_t = rz_t + (1-r)\tilde{z}_{t-1} \qquad (11)$$

72. The DEWMA forecast, given data up to the tth occasion is $\tilde{z}_t$. The DEWMA model is a time series model which is sensitive to linear trend.

73. When there are only a few occasions available (as is true for the ACS project at present) the so-called "starting value" z_0 becomes quite important, since it carries so much weight. Since there is little data for some barrels and charges we have used the Stein estimate discussed above. We use the same startup value for the DEWMA series. Therefore, even though there may be little data available for one particular barrel at first, the model draws strength from the information about other barrels. As more data for an individual barrel become available, more weight tends to be given to the individual barrel data.

74. With the many very short time series that we have, the stein-estimate-as-startup provides us with the ability to produce reasonable DEWMA forecasts. There is a loose similarity here between the Global Mean Model and the DEWMA model; both models attempt to gather strength from information about the other barrels to produce reasonable forecasts for any single barrel. We can monitor the effect of the startup on the forecasts, and modify this approach, if required. For example, it is easy to modify the algorithm to use the first occasion mean as the startup instead of the stein estimate.

75. To summarize this section on the DEWMA model, if there are T occasions available for analysis to date, then we take the forecast RMV for a given gun/projectile category, charge, and barrel combination as:

$$FCST = \tilde{z}_T \qquad \text{[Model 8]}$$

with weight $r = 0.10$. This weight is chosen judgmentally for now (experience suggests weights between 0.1 and 0.3 are most common). Analysts can optimize the weight, using the Model Performance Checks to be discussed shortly, when more data are available, and barrel wear related trend becomes apparent.

76. As more data become available, the models can all be updated accordingly, but some will do better than others. If there is no barrel wear, and therefore, the RMV mean value is constant, then all the above models will be satisfactory. If there is barrel wear over time, then the dynamic model, or time-sensitive (DEWMA) model will adapt to the new "situation" and it is expected that the forecasts will be better than with the other models. How do we measure whether one model provides more accurate forecasts than another? That is the topic of the next section.

A Hybrid Model

77. Just before going on to the Model Performance section, we should mention that there are many possible hybrid models that we could develop; indeed the

DEWMA model, with the startup value that we have chosen, is a hybrid of the preceding models. Another model that we have looked at is one in which the first round of an occasion is forecasted using the DEWMA forecast; then all subsequent rounds in that occasion are forecasted using the actual RMV of the first round. We will refer to this model as Model 9.

78. This model is impractical at present, for it presumes the existence of a MVI for each barrel; we have more barrels than MVIs.

Model Performance

79. There are many ways to compare the forecasting procedures discussed above. The description of one such method is facilitated using the following terminology. If we let RMV_{bij} be the reduced muzzle velocity for the bth barrel, the ith occasion and the jth round within the occasion, for a given charge/projectile category, then we may calculate the squared error

$$(\mathrm{RMV}_{bij} - \mathrm{FCST}_b)^2$$

for each RMV. For a given gun barrel, at a given charge we might plot the RMV as in figure 5.

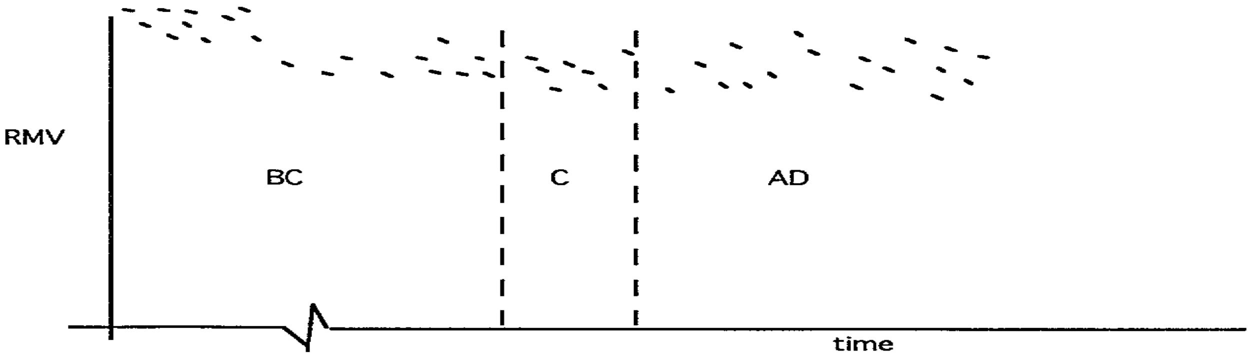


Figure 5: A plot of RMV versus time, showing the data partitions

80. In this plot we have partitioned the data for a fictitious barrel into three sets. The set BC is composed of all rounds of this charge/projectile category fired

in the past but were not measured by the muzzle velocity indicator; the set $\mathcal{C}$ is composed of all rounds fired from the oldest occasion where data is available; and the set $\mathcal{AD}$ is composed of all other rounds (for the most recent occasions) where data is available. We do not know what the RMV are in the first set, but what if we did?

81. Ideally, if knowledge of the RMV in set $\mathcal{BC}$ was available, we could use data from the sets $\mathcal{BC}$ and $\mathcal{C}$ to develop our forecasts for each of the models, and see how well each model predicts the RMV in the few occasions of set $\mathcal{AD}$. For example, the DEWMA forecast in model 8 would be based upon all data in sets $\mathcal{BC}$ and $\mathcal{C}$ while the forecast from model 6 (single occasion forecast) would be based only upon the data in set $\mathcal{C}$. We could calculate the sum of squares prediction errors for each model as:

$$\mathrm{SSPE}(8) = \sum\sum\sum(\mathrm{RMV}_{bij} - \mathrm{FCST}(8)_b)^2$$

and

$$\mathrm{SSPE}(6) = \sum\sum\sum(\mathrm{RMV}_{bij} - \mathrm{FCST}(6)_b)^2$$

where the summations are over all barrels, at a given charge, for RMV within set $\mathcal{AD}$. We would calculate similar sums of squares for the other models. Note that the number in brackets () represents the model number for a particular forecast or sums of squares.

82. The rub is, we have no data for set $\mathcal{BC}$.

83. If we try to get around this problem by calculating the forecasts using data only from set $\mathcal{C}$, then the performance of models: 2, 3, 4, 5, 7, 8, and 9 will suffer badly because they all depend upon having a sizable amount of historical data. We are stumped, unless we assume that the characteristics of the data in the few occasions prior to set $\mathcal{C}$ were not too different from the few occasions that we do have in set $\mathcal{AD}$. This does not seem too unreasonable. We assume that we will get a reasonable idea of the relative performance of each of these models.

84. We then use set $\mathcal{AD}$ as a substitute for set $\mathcal{BC}$ and do the sums of squares prediction errors as before.

85. As we mentioned earlier, model 6 — the single occasion model— is a simulation of the forecasting approach that has been used by the army in the past. Four to six rounds are fired and the RMVs are then averaged. That mean was then adopted for a period of time to predict muzzle velocity.

86. We have calculated the forecasts for Model 6 but then taken a subset of those barrels for which there were at least four rounds in the first available occasion (set $\mathcal{C}$), and at least one occasion in the set $\mathcal{AD}$. For these remaining barrels in the subset, the SSPE was then calculated on the rounds within set $\mathcal{AD}$. For gun/projectile

category M1 (these categories were explained in table 1), there were 127 rounds in the $\mathcal{AD}$ partition; for C1 there were 147 rounds; and for L5 there were 213 rounds.

87. This subset of barrels, used to calculate the SSPE for Model 6, is also used to calculate the SSPE for all the other models. This has been done to ensure that the SSPE are comparable across models.

88. In figures 6, 7 and 8, we list the results of our calculations for each of the nine models. We have tabulated the mean square prediction errors (MSPE) which is just the SSPE divided by the number of rounds in set $\mathcal{AD}$ for this charge, across all barrels in the subset. For example, the MSPE, on 60 rounds, for gun C1, charge 5 for Model 8 (DEWMA) is MSPE(8)=7.04. The Mean squared prediction error when the firing table MVs are used as forecasts (model 1) is MSPE(1)=10.70.

89. Taking the square root of the MSPE yields the *root* mean square prediction error, or RMSPE. We define the percentage relative improvement using the mth model, relative to the RMSPE of model 1 (firing table forecasts) as:

$$\mathrm{Re(m)} = \frac{\mathrm{RMSPE(1)} - \mathrm{RMSPE(m)}}{\mathrm{RMSPE(1)}} \times 100\% \qquad (12)$$

90. We emphasize that all comparisons are made relative to the firing table forecasts, so that Re(1) will equal zero for all charges. We have tabulated the relative improvement for the C1 gun/projectile category in figure 6, for the L5 gun/projectile category in figure 7 and for the M1 gun/projectile category in figure 8. In each of the figures, Re(i) is given as REi.

91. There are very few rounds available to calculate the MSPE for some charges and we should therefore draw our conclusions about the relative merits of the models from those charges with large sample sizes. Models 2, 3, 4, 5 and 7 are all based upon means or medians. The startup value for the DEWMA model is also an average and therefore we should not be too surprised to see similar results for all of these models, across charges. Also, some barrels and charges have been ignored in the present analysis, if the sample size was too small (eg. C1: charge 1).

```
                    C1 : Mean Square Prediction Error by charge

OBS CHARGE NN MSERR1 MSERR2 MSERR3 MSERR4 MSERR5 MSERR6 MSERR7 MSERR8 MSERR9

 1     2     8  16.03    3.32    3.48    5.07   4.41    8.68    4.01    4.04  17.85
 2     3    10  19.32   10.22   10.54    9.20   8.77   10.11    9.06    9.06  35.73
 3     4    44  18.48   11.79   11.83    4.61   4.93    5.96    4.63    4.63  14.69
 4     5    60  10.70    8.64    7.35    6.90   5.89   24.34    6.98    7.04   6.57
 5     6    13  14.10    6.13    7.51    3.50   2.38   11.44    3.13    3.17   1.90
 6     7    12  14.95   11.76   12.51    6.25   9.94   12.10    3.22    3.25   4.01

                    C1 : Mean Square Prediction Error overall

             C        C        C        C        C        C        C        C        C
             U        U        U        U        U        U        U        U        U
   C         M        M        M        M        M        M        M        M        M
   U         E        E        E        E        E        E        E        E        E
O  M         R        R        R        R        R        R        R        R        R
B  N         R        R        R        R        R        R        R        R        R
S  N         1        2        3        4        5        6        7        8        9

1  147   14.55     9.43     9.13     5.92     5.74    14.88     5.61     5.64    10.98

                    C1 :relative Error,  re(i) for all i

OBS  CHARGE  NN    RE1    RE2    RE3    RE4    RE5    RE6    RE7    RE8    RE9

 1      2     8    0.0   54.5   53.4   43.8   47.5   26.4   50.0   49.8   -5.5
 2      3    10    0.0   27.3   26.1   31.0   32.6   27.7   31.5   31.5  -36.0
 3      4    44    0.0   20.1   20.0   50.0   48.4   43.2   50.0   49.9   10.8
 4      5    60    0.0   10.1   17.1   19.7   25.8  -50.9   19.2   18.9   21.6
 5      6    13    0.0   34.1   27.0   50.1   58.9    9.9   52.9   52.5   63.3
 6      7    12    0.0   11.3    8.5   35.3   18.5   10.0   53.6   53.4   48.2

              Gun/Projectile category: C1, cumulative relative error

 OBS    CRE1    CRE2    CRE3    CRE4    CRE5    CRE6    CRE7    CRE8    CRE9

  1      0.0    19.5    20.8    36.2    37.2    -1.1    37.9    37.7    13.1
```

Figure 6: MSPE, Overall MSPE, and Relative improvement for the C1 Gun: DEWMA weight=0.10

```
                       L5 : Mean Square Prediction Error by charge

 OBS CHARGE NN MSERR1 MSERR2 MSERR3 MSERR4 MSERR5 MSERR6 MSERR7 MSERR8 MSERR9

  1     1    4   0.77   4.90   3.54   0.78   1.30   1.69   2.99   2.91   1.18
  2     2   18   7.47   1.81   1.96   1.91   1.77   5.53   0.89   0.91   0.75
  3     3   20   4.60   4.68   4.59   6.20   9.97  12.47   4.67   4.69   3.67
  4     4   44  29.64   4.71   5.05   4.35   4.38   7.32   4.83   4.94   3.96
  5     5   76   6.43   8.31   8.50   4.95   5.47  17.39   7.20   7.23  12.33
  6     6   29  41.76   9.67   9.82   5.53   6.78  15.04   5.62   5.65   5.19
  7     7   22  94.75   5.43   5.38   3.81   3.46   7.36   3.84   3.87   6.99

                       L5 : Mean Square Prediction Error overall

               C      C      C      C      C      C      C      C      C
               U      U      U      U      U      U      U      U      U
     C         M      M      M      M      M      M      M      M      M
     U         E      E      E      E      E      E      E      E      E
 O   M         R      R      R      R      R      R      R      R      R
 B   N         R      R      R      R      R      R      R      R      R
 S   N         1      2      3      4      5      6      7      8      9

 1  213    24.97   6.50   6.63   4.57   5.25  12.20   5.30   5.34   7.07

                       L5 :Relative Error,  re(i) for all i

 OBS  CHARGE  NN   RE1    RE2    RE3    RE4    RE5    RE6    RE7    RE8    RE9

  1     1      4   0.0   -153   -115   -0.9  -30.0  -48.3  -97.5  -94.7  -23.9
  2     2     18   0.0   50.7   48.8   49.4   51.3   13.9   65.6   65.2   68.3
  3     3     20   0.0   -0.9    0.1  -16.1  -47.2  -64.6   -0.7   -1.0   10.8
  4     4     44   0.0   60.1   58.7   61.7   61.5   50.3   59.6   59.2   63.4
  5     5     76   0.0  -13.7  -15.0   12.2    7.7  -64.4   -5.8   -6.0  -38.4
  6     6     29   0.0   51.9   51.5   63.6   59.7   40.0   63.3   63.2   64.8
  7     7     22   0.0   76.1   76.2   80.0   80.9   72.1   79.9   79.8   72.8

                 Gun/Projectile category: L5, cumulative relative error

 OBS    CRE1    CRE2    CRE3    CRE4    CRE5    CRE6    CRE7    CRE8    CRE9

  1      0.0    49.0    48.5    57.2    54.1    30.1    53.9    53.7    46.8
```

Figure 7: MSPE, Overall MSPE, and Relative improvement for the L5 Gun: DEWMA weight=0.10

```
                      M1 : Mean Square Prediction Error by charge

OBS  CHARGE  NN  MSERR1  MSERR2  MSERR3  MSERR4  MSERR5  MSERR6  MSERR7  MSERR8  MSERR9

 1     2G    65   41.17   19.13   19.40   16.78   16.68   24.44   19.67   19.56   23.37
 2     3G    25   42.88   12.27   10.51   10.84   13.53   12.78   10.42   10.37   20.10
 3     4G     6   20.85   28.07   21.94    7.85    4.16   14.83    1.41    1.44    1.33
 4     4W    12   94.18   27.47   34.90   29.94   37.72   47.47   17.19   17.25   25.25
 5     5G    14    7.87    8.41   13.43    5.71    5.70    6.08    6.32    6.30   12.39
 6     7W     5   56.88   59.20   66.15   62.56   87.26   86.52   61.49   61.56   37.33

                      M1 : Mean Square Prediction Error overall

               C        C        C        C        C        C        C        C        C
               U        U        U        U        U        U        U        U        U
      C        M        M        M        M        M        M        M        M        M
      U        E        E        E        E        E        E        E        E        E
O     M        R        R        R        R        R        R        R        R        R
B     N        R        R        R        R        R        R        R        R        R
S     N        1        2        3        4        5        6        7        8        9

1   127    42.51    19.39    20.42    17.01    19.03    24.29    16.93    16.87    21.20

                      M1 :Relative Error,  re(i) for all i

OBS  CHARGE   NN    RE1     RE2     RE3     RE4     RE5     RE6     RE7     RE8     RE9

 1     2G     65    0.0    31.8    31.4    36.2    36.3    23.0    30.9    31.1    24.7
 2     3G     25    0.0    46.5    50.5    49.7    43.8    45.4    50.7    50.8    31.5
 3     4G      6    0.0   -16.0    -2.6    38.6    55.3    15.7    74.0    73.7    74.8
 4     4W     12    0.0    46.0    39.1    43.6    36.7    29.0    57.3    57.2    48.2
 5     5G     14    0.0    -3.3   -30.6    14.8    14.9    12.1    10.4    10.6   -25.5
 6     7W      5    0.0    -2.0    -7.8    -4.9   -23.9   -23.3    -4.0    -4.0    19.0

               Gun/Projectile category: M1, cumulative relative error

OBS     CRE1     CRE2     CRE3     CRE4     CRE5     CRE6     CRE7     CRE8     CRE9

 1       0.0     32.5     30.7     36.7     33.1     24.4     36.9     37.0     29.4
```

Figure 8: MSPE, Overall MSPE, and Relative improvement for the M1 Gun: DEWMA weight=0.10

92. We provide two examples to show how the data in the three figures may be interpreted. On 60 rounds of data, the DEWMA model forecasts for gun/projectile category C1, charge 5, yield a 18.9% relative improvement, while the single-occasion-mean model (6) has a -50.9% "improvement". Large negative relative improvement of course implies inferior performance for the *m*th model, relative to the firing tables.

93. On 65 rounds of data for the M1 gun/projectile category at charge 2G, the DEWMA model yields a 31.1% improvement over the firing table forecasts.

An overall measure of performance, across charges

94. We can also get a measure of the overall performance of each model by taking a weighted average of the MSPE across charges. Using the number of rounds per charge in set $\mathcal{AD}$ (N) as the weight, we arrive at a "cumulative" or overall measure for the *m*th model:

$$\mathrm{CUMERR}(m) = \frac{\sum N \ \mathrm{MSPE}(m)}{\sum N}$$

where the summation is over all charges.

95. Again not surprisingly, since the DEWMA model startup value is based upon the barrel (and charge) mean RMV, we see that the performance of the DEWMA model is quite similar to models 2, 3, 4, 5, and 7. Each of these latter models is essentially a means or medians based approach. The DEWMA model consistently has smaller overall MSPE than model 1 (firing tables), Models 2 and 3 (means and medians, by charge only), model 6 (single occasion forecast), or model 9 (hybrid).

96. Here N is the number of RMV considered for a given charge. (On the computer outputs it is shown as NN.) The sum of N across all charges is equal to CUMNN. MSERR8 is the Mean Squared Error for the DEWMA model and CUMERR8 is the mean squared error across all charges for the DEWMA model. MSERR1 is the mean squared error when the firing table muzzle velocities are used as the forecast. The other code-words are summarized for convenience below:

- MSERR1, CUMERR1, and RE1 refer to the model which uses the firing table (standard) muzzle velocities as the RMV forecast.
- MSERR2, CUMERR2, and RE2 refer to the model which uses the average RMV across all barrels of a given charge as the forecast.
- MSERR3, CUMERR3, and RE3 refer to the model which uses the median RMV across all barrels of a given charge as the forecast.

- MSERR4, CUMERR4, and RE4 refer to the model which uses the average RMV for a given barrel at a given charge as the forecast.
- MSERR5, CUMERR5, and RE5 refer to the model which uses the median RMV for a given barrel at a given charge as the forecast.
- MSERR6, CUMERR6, and RE6 refer to the single occasion model for a given barrel at a given charge.
- MSERR7, CUMERR7 and RE7 refer to the Stein estimate model for a given barrel at a given charge.
- MSERR8, CUMERR8, and RE8 refer to the DEWMA model with weight equal 0.10.
- MSERR9, CUMERR9, and RE9 refer to a hybrid model which uses the DEWMA forecast for the first round of an occasion, and then uses the first-round RMV as the forecast for all subsequent rounds.

97. We can re-apply equation 12 to the cumulative mean squared errors to get an overall relative percentage improvement, across all charges. This variable is given the code **CREi**, i=1,9, for each of the nine models. As before, each model is compared to the firing-tables-as-forecast model. Table 4 gives the results of these calculations for the DEWMA and Single-Occasion models.

	percentage relative improvement	
Gun/Proj	Model 8 (DEWMA)	Model 6 (Single Occasion)
C1	37.7%	-1.1%
L5	53.7%	30.1%
M109 non-illum.(M1)	37.0%	24.4%

Table 4: overall relative percentage improvement

98. Therefore we see from this table that the simulation of the single-occasion forecasting procedure can yield some improvement over the firing tables. However, the DEWMA model has higher overall percentage relative improvement. The other means and medians based models generally yield similar results to the DEWMA model.

Model Choice and Justification

99. The DEWMA model (8) provides a forecast of the RMV for a particular

gun/projectile category combination, on a barrel by barrel basis, for a given charge.

100. The DEWMA model has been selected on the basis of the relative performance results above, and for the reasons reiterated below.

101. The data are ordered in time. The DEWMA model is a time series model that weights recent data more heavily than old data. Thus as the effect of barrel wear becomes more significant, the model will again tend to provide better estimates for the RMV than a model that weights old data the same as new data.

102. As the barrels age, some may wear faster than others causing a practical difference between old and new barrels. Therefore any model that is not barrel specific will eventually become inaccurate.

103. At present, there is very little data for some charges/barrels. The DEWMA model uses a startup value that "draws strength" from other barrels for a given charge, allowing the forecasts, initially, to be at least as good as a model that averages data across barrels.

104. Although the DEWMA forecasting performance will have to be monitored, the problem of "when to discard stale data" should not be as big a problem as it might be with static means and medians models.

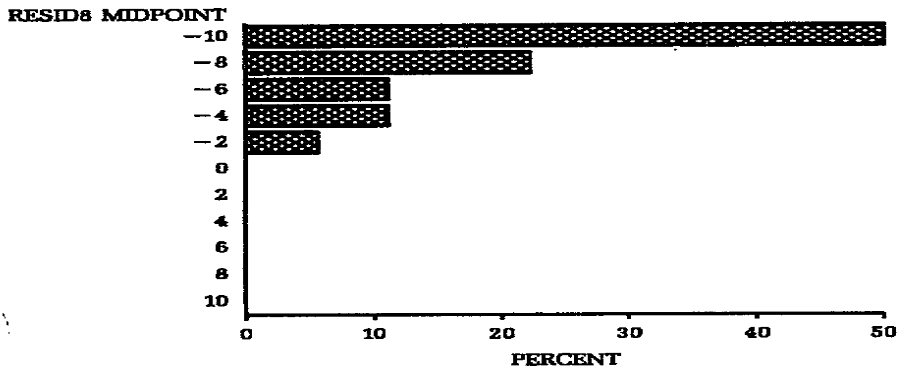
105. The DEWMA model smooths out the effects of the occasions. Therefore, even if the last few occasions have been "irregular", the forecast for a given situation should still be reasonable. The model has been set to adjust slowly to changes in the muzzle velocity. Thus, short term changes (like occasion effects) will not have too much effect on the forecast. The model is however still sensitive to longer term trends due to barrel wear, as it should be.

Frequency charts of residuals

106. One important analysis in checking a model is to chart the residuals, that is: the values (RMV-FCST). If the model is appropriate, the residuals should be distributed closely about zero. We have produced the frequency charts of the residuals (across all barrels, not just the subset of barrels used above) for each gun/projectile category/charge combination. However, there are a very large number of these charts and most are excluded here. These residuals are reasonably symmetric about zero as they should be, except for charge 8, for the M1 gun/projectile category. This singular case is discussed below.

107. The chart of charge 8 residuals (see figure 9) across all barrels for the M1

gun/projectile combination has a cumulative frequency of 18. These residuals show that the model does not fit the data all that well here. The reason is that all eighteen observations are from the same gun barrel. When there is only one barrel, the startup value for the DEWMA model is the firing table MV instead of the stein estimate. Once another barrel fires charge 8 and those data are processed along with the present data, this forecast should improve.



RESID8 MIDPOINT	FREQ.	CUM. FREQ.	PCT.	CUM. PCT.
−10	9	9	50.00	50.00
−8	4	13	22.22	72.22
−6	2	15	11.11	83.33
−4	2	17	11.11	94.44
−2	1	18	5.56	100.0
0	0	18	0.00	100.0
2	0	18	0.00	100.0
4	0	18	0.00	100.0
6	0	18	0.00	100.0
8	0	18	0.00	100.0
10	0	18	0.00	100.0

Figure 9: Histogram of residuals, Gun/projectile category M1, charge 8

Summary and Conclusions

108. A simple algorithm has been created to improve the accuracy of muzzle velocity forecasts for the C1, L5 and M109 cannons. A double exponentially weighted moving average or DEWMA model is used to predict the muzzle velocities under standard conditions. Forecasting is done on a barrel-by-barrel basis, for each charge of a gun/projectile category.

109. A set of four programs has been written to implement this algorithm. The ACSDATA.SAS program inputs raw Muzzle Velocity Indicator (MVI) data from the

regiments and calculates the reduced muzzle velocities for each round fired. The OUTLIER.SAS program determines if any RMVs lie outside a given tolerance band, and if so, extracts these rounds from the "working" dataset. The UPDATE.SAS program takes the new data in the working dataset and appends it to a dataset of historical data — for each gun/projectile category. Finally, the DEWMA.SAS program inputs the historical data and calculates revised or "fresh" forecasts for the entire suite of gun barrels. Each of these programs is outlined in the annexes which follow. There is also documentation on each program within the source code.

110. These four programs, written in the SAS language, have been incorporated in a larger set of programs by ADGA Consulting Engineers. The ACS program has been developed on an IBM PC, but it has the potential to be run on any machine that runs SAS. The main output from the suite of programs will then be the standardized muzzle velocity forecast for each gun/projectile/charge/barrel combination. These forecasts can be updated as new data (from a gun camp say) arrive for analysis.

111. This completes the documentation for the proposed new muzzle velocity algorithm. A number of utility programs can be (or are already) devised to do follow-on analysis of the forecasting.

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References

[1] Emond, E. *An Alternative Approach to the Problem of Estimation, Ranking and Multiple Comparison of Mean Values*, DMS Staff Note, December 1988.

[2] Firing Tables for Cannon, 155mm Howitzer (M109) etc., FT-155-AM-2, Headquarters, Department of the (U.S.) Army, 31 March 1983.

[3] *Firing Tables for Gun Equipment*, 105mm, L5, (UK) Ministry of Defence, 1972.

[4] Firing Tables for Cannon, 105mm Howitzer (C1), FT-105-H-7, Headquarters, Department of the (U.S.) Army, May 1971.

[5] Freund, R. J., *Mathematical Statistics*, 4th edition

[6] Hogg, R.V. and Craig, A.T., *Introduction to Mathematical Statistics*, McMillan Publishing Co., 1978.

[7] Hunter, J. S. (1986), *The Exponentially Weighted Moving Average*, Journal of Quality Technology, 18, pp. 203-210.

[8] Kendall, Sir Maurice, Stuart, A. and Ord, J.K., *The Advanced Theory of Statistics: Volume 3, Design and Analysis, and Time Series*, MacMillan Publishing Co., Inc., New York, 1983.

[9] Larson, H.J., *Introduction to Probability Theory and Statistical Inference*, John Wiley and Sons, 2nd edition, 1974.

[10] Lee, P.M. *Baysian Statistics: An introduction*, Oxford University Press, 1989.

[11] Maritz, J.S. and Lwin, T. *Empirical Bayes Methodology*, 2nd edition, Chapman and Hall, 1989.

[12] MiliPAC SRS, WP1 9313:1 Volume 2 of 4, issue 1.0, 30 March 1994.

[13] National Defence Manual, B-GL-306-006/FP-001 Interim 1, Field Artillery, Volume 6, Ballistics and Ammunition.

[14] Montgomery, D.C. (1976), *Design and Analysis of Experiments*, John Wiley & Sons, New York.

[15] Montgomery, D.C. (1985), *Statistical Quality Control*, John Wiley & Sons, New York.

[16] Neter, John, Wasserman, William, and Kutner Michael H. (1990). *Applied Linear Statistical Models*. Homewood, IL, Richard D. Irwin Inc.

[17] SD-Scicon Company, *Study of Muzzle Velocity Variation: Phase 1 - Report on the Establishment of the Future RA Gun Calibration Policy*, Doc Reference C20337.00/1, Issue 1.0, June 1989. U.K. Restricted.

[18] SD-Scicon Company, *Study of Muzzle Velocity Variation: Phase 2 - Report on the Methods for Producing a Continuously Updated Muzzle Velocity*, Doc Reference C20337.00/2A, Issue 1.0, November 1989. U.K. Restricted.

[19] SD-Scicon Company, *Study of Muzzle Velocity Variation: Phase 3 - Report on the Feasibility of Predicting the Next Round's Muzzle Velocity Using an Adaptive Empirical Model*, Doc Reference C20337.00/3A, Issue 1.0, November 1989. U.K. Restricted.

[20] 7150-1 DLAEEM 3-5 Sec MSG 14 August 1989

[21] 20-01-1993 Memo from ADGA, Corrected Reduced Muzzle Velocity Algorithm, cc: to DMS.

Annex A Program ACSDATA.SAS

1. This annex provides a simple overview of the ACSDATA.SAS program.

2. This program first reads in raw data which may include all types of guns, and all types of charges.

3. The program calculates the occasion (OCCASION) and round within occasion (ROUND) variables. Reference [13] states that "the time span of an occasion has yet to be determined but has been taken to be a day until better information becomes available". In consultation with DLAEEM 8-4-3, we have defined an OCCASION to include any contiguous series of rounds (fired no more than 2 hours apart), of the same projectile category, and of the same charge. This time span can be changed easily by changing the variable TIMEDIF in the SAS programs.

4. The program then calculates the reduced muzzle velocity (RMV) as a function of lot factor, charge temperature and projectile weight (in squares).

5. Data from each gun/projectile category is collated and output to a separate file. Thus there are seven output files stored in the library GUNS: M1, MIL, Mad, C1, CIL, L5 AND LIL. The GUNS.Mad file stores records from gun type M109 and projectiles 'AD', 'RA' and 'DP'. As illustrated in figure 10, the GUNS.C1 dataset is created from the raw data in this program.

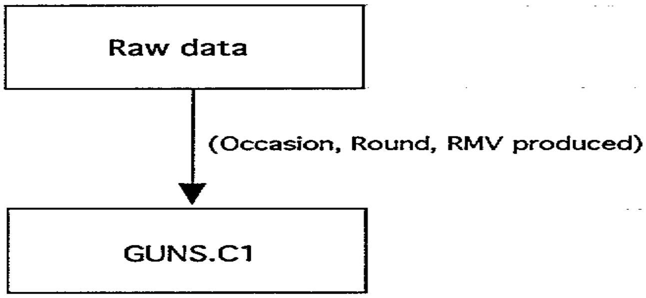

Figure 10: The GUNS.C1 file is created from the raw data

Annex B Program OUTLIER.SAS

1. This annex provides substantial detail about the methodology used to detect outlying or aberrant RMV data in the working dataset.

2. To produce valid and reasonable forecasts in the later DEWMA step, the outliers must be removed from the "working" dataset. An RMV OUTLIER by definition is any reduced muzzle velocity value that is unlikely to have arisen naturally from the process. As an example, an outlier might be an observed RMV of 800m/s for charge 1, gun L5. Since the standard muzzle velocity for this situation is about 183 m/s there is little doubt that the observation should be discarded. Unfortunately, not all situations are so obvious. As we discussed in the main body of the report, it appears that gunners sometimes mistakenly record, say, charge 4W as 4G. This leads to observed muzzle velocities that look "questionable" and we need a statistical criterion with which to judge whether to keep or reject the observation.

3. Observations or records that are rejected will be output and stored in a file called HISTORIC.RJxx. There is one RJxx file for each gun/projectile category (xx=C1, L5, M1, CIL, LIL, MIL, MAD).

4. When we first began to analyse data for this project we developed a general linear model to determine what effect the occasion had upon the reduced muzzle velocities. From the U.K. reports, we strongly suspected that there would be variation in RMV between the occasions, as well as variation in RMV within an occasion. With very little data, there is no noticeable trend due to wear, and therefore we were not uncomfortable using a "stationary mean" model to analyse the variation in RMV. This model provided the basis for our outlier detection algorithm. The model chosen was:

$$\mathrm{RMV}_{ij} = \mu + \theta_i + \varepsilon_{ij} \tag{13}$$

where μ is the overall mean, θ_i is the (random) effect from the i'th occasion, and ε_{ij} is the random error term. The index i runs from 1 to k and the index j runs from 1 to n_i. The occasion effect is assumed to be normally independentally distributed (NID) with mean zero and variance σ_θ^2. The random error, (essentially the round within occasion effect) ε_{ij} is assumed to be NID$(0, \sigma^2)$. We can estimate all the variance components here.

5. The variance of the RMV, call it σ_x^2 say, can now be written as: $\sigma_\theta^2 + \sigma^2$.

6. We digress briefly to show how these variance parameters may be estimated. The mean squared error (within occasions),

$$MSE = \frac{\sum\sum(x_{ij} - \bar{x}_{i.})^2}{\sum(n_i - 1)}$$

is an estimate for σ^2. The mean square between occasions,

$$MSO = \frac{\sum\sum(\bar{x}_{i.} - \bar{x}_{..})^2}{k - 1}$$

is an estimate for $\sigma^2 + n'\sigma_\theta^2$. Here, n' is a function of the n_i (see Montgomery, 1976). We can now estimate σ_θ^2 as:

$$\frac{MSO - MSE}{n'}$$

7. Using Normal Distribution theory, we know that

$$\Pr(\mu - \delta\sigma_x < X < \mu + \delta\sigma_x) = 1 - \alpha.$$

where X is the random variable (in this case the RMV), and δ is the $1 - \frac{\alpha}{2}$ quantile from the standard normal distribution. We can now write an appropriate criterion for determining whether an observed RMV is an outlier or not. Any observation that does not meet the criterion:

$$|rmv - \mu| \leq \delta \sqrt{\text{Variance(RMV)}}$$

is rejected. Taking $\delta = 1.96$ say, will lead us to reject "good" data 5% of the time. Taking a larger value like $\delta = 5.4$ leads us to reject "good" data only about once in 100,000 observations. In the latter case, there is also a higher chance of accepting data which should be rejected.

8. This would be fine if the mean of the data was static; that is the mean was not expected to change with time. With the continual wear of the barrel, this is unlikely. As more occasions are observed, the overall mean will change, thereby inflating the between occasions variance. The end result of all this is that the tolerance band for acceptable RMV would continually get larger as more data are collected.

9. We can make some simple modifications of outlier criterion above to produce a crude outlier detector which is not susceptible to this inflation problem, and takes into account the current mean RMV for a particular barrel. This outlier detector also takes into account the two types of variation that we have discussed: the between occasion and the within occasion variation.

10. We have one part, the within occasion variation, that is relatively insensitive to these changes in mean. The other part, the between occasion variation will become affected by this non-stationarity (gradual change in mean).

11. We can estimate σ_θ^2 and σ^2 from the *currently* available data. And, instead of μ we can use the forecast RMV (FCST) in the outlier criterion. We arrive at the modified criterion: Reject any RMV such that

$$|rmv - \mathrm{FCST}| > \delta \sqrt{\hat{\sigma}_\theta^2 + \hat{\sigma}^2}.$$

12. In essence, we found a tolerance bound from the currently available data and have assumed that this bound will *remain* reasonable for determining if RMVs lie "too far away" from the current forecast.

13. In the OUTLIER.SAS program we refer to $\hat{\sigma}^2$ as MVERR. We have calculated the ratio of $\hat{\sigma}_\theta^2$ to $\hat{\sigma}^2$ and saved this value as PSI. Therefore, ((PSI+1)MVERR) is our estimate in the computer program for the variance of the RMV. The constant, δ, at the time of writing is 5.4 and was chosen to ensure that the data that appeared reasonable, by inspection, were not rejected. In the computer source code, δ is coded as DELTA.

14. Now when the above procedure is used there may be some outliers discovered. As an example, valid observations for the C1 non-illuminated rounds appear in a directory/file called: GUNS.Ac1. Rejected observations for the C1 non-illuminated rounds appear in a directory/file called: GUNS.RJc1 (see figure 11). The artillery calibration system officer can review the rejected rounds to ensure that no "credible" observations have been rejected. The values given to δ, PSI and MVERR can be re-evaluated from time to time to determine if the outlier detection algorithm is working appropriately.

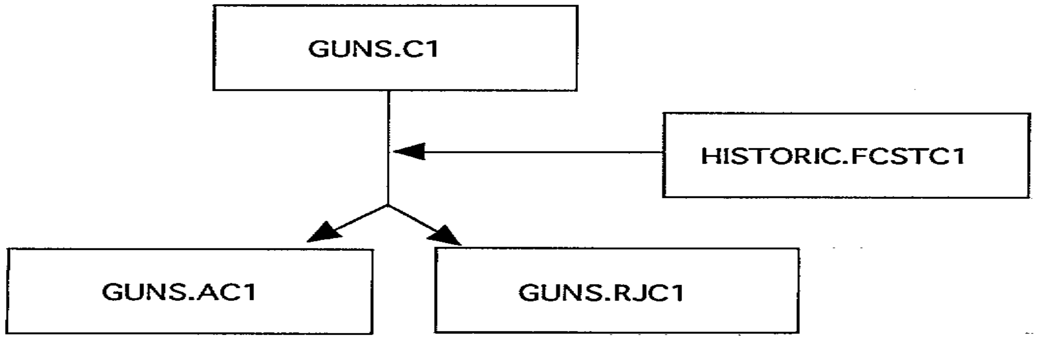

Figure 11: The GUNS.C1 file is separated into two files

Annex C Program UPDATE.SAS

1. After all aberrant observations are rejected, the historical data will be updated.

2. The new and valid data from the GUNS directory must be combined with historical data. A PROC DATASETS is used to do the append. For example we would append GUNS.AC1 data to HISTORIC.C1 (see figure 12). It is this HISTORIC datafile from which forecasts are generated in the DEWMA program.

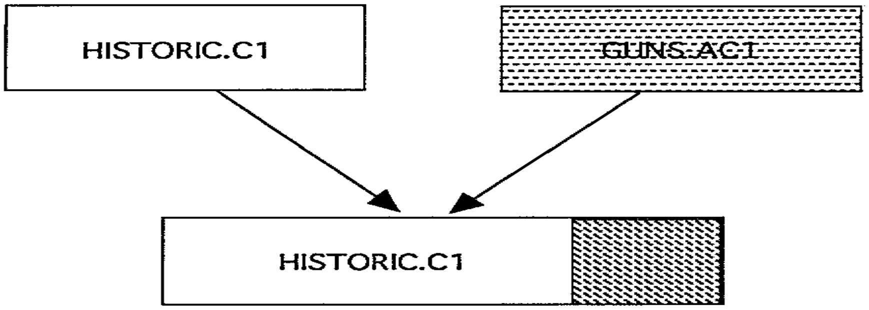

Figure 12: The GUNS.AC1 file is appended to the historical data

Annex D
To Project Report 678
Dated September 1994

Annex D Program DEWMA.SAS

1. This annex provides an outline of the DEWMA.SAS program.

2. The double exponentially weighted moving average models all require a starting value. This starting value — called STARTUP in the source code — in calculated in a subroutine (macro) called JSMACRO. The DEWMA forecasts for each gun/projectile category/barrel and charge are calculated in the subroutine LOOPMA. Finally, the forecasts are output to a file called HISTORIC.FCSTxx, where xx is C1, L5, M1 and so on (see figure 13). The current estimate of within occasion variance of the RMV (MSER) is provided here as well. These estimates are themselves quite variable because of the small sample size available to us. The number of rounds available for a particular gun/projectile category/barrel and charge is denoted as BARLRMVN.

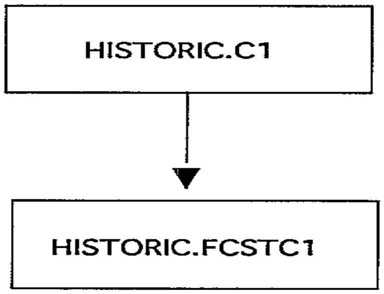

Figure 13: The HISTORIC.C1 file is used to produce forecasts for RMV

Annex E MiliPAC/RMV Algorithm parameters

1. The coefficients *a* *b* and *c* for the temperature correction equation are given in Table 5 below. These coefficients are for the M109 - 155mm cannon and for the HE, WP, IL, HC, CR, CG, CV and CY projectiles. This and other data in this annex have been taken from reference D.

Charge	a	b	c
1G	0.0262	-4.4E-5	5.0E-7
2G	0.0315	-4.6E-5	5.7E-7
3G	0.0358	-4.8E-5	6.8E-7
4G	0.0425	-5.6E-5	7.8E-7
5G	0.0498	-6.4E-5	8.9E-7
3W	0.0484	-3.0E-5	8.3E-7
4W	0.0559	-3.2E-5	9.0E-7
5W	0.0633	-4.2E-5	1.11E-6
6W	0.0780	-5.0E-5	1.36E-6
7W	0.0902	-6.8E-5	1.59E-6
8	0.2186	2.28E-4	1.33E-6

Table 5: Temperature Coefficients for M109-155mm, Projectiles:HE, WP, IL, HC, CR, CG, CV and CY.

2. The coefficients *a* and *b* for the temperature correction equation are given in Table 6 below. The coefficient $c = 0$ throughout. These coefficients are for the M109 - 155mm cannon, for projectiles: AD, RA and DP.

3. The coefficients *a* and *b* for the temperature correction equation are given in Table 7 below. The coefficient $c = 0$ throughout. These coefficients are for both the C1 and the L5 105mm cannons, for projectiles: HE, IL, WP, HC, CR, CY, CV and CG.

4. Table 8 provides the *N* factor of the weight correction equation. These factors are for the M109-155mm Cannon, for projectiles: DP, AD and RA.

Charge	a	b
1G	0.0	0.0
2G	0.0	0.0
3G	0.0326	2.15E-5
4G	0.0359	-3.58E-5
5G	0.0419	-5.4E-6
3W	0.0577	4.72E-5
4W	0.0475	3.88E-5
5W	0.0760	8.41E-5
6W	0.0992	-1.04E-5
7W	0.1085	-3.13E-5
8	0.1899	1.019E-4

Table 6: Temperature Coefficients for M109-155mm, Projectiles: AD, RA, DP

Charge	a	b
1	0.0559	1.81E-4
2	0.0617	1.78E-4
3	0.0701	1.61E-4
4	0.0737	1.17E-4
5	0.1014	2.17E-4
6	0.1385	2.26E-4
7	0.1489	1.97E-4

Table 7: Temperature Coefficients for the C1 and L5 guns, all Projectiles

Charge	N
1G	0.00
2G	0.00
3G	-0.48
4G	-0.48
5G	-0.48
3W	-0.41
4W	-0.41
5W	-0.41
6W	-0.41
7W	-0.41
8	-0.35

Table 8: N Factors for the M109-155mm Cannon, for Projectiles: DP, AD and RA

Charge	N
1	-0.48
2	-0.48
3	-0.45
4	-0.37
5	-0.34
6	-0.31
7	-0.30

Table 9: N Factors for C1 and L5 guns, all Projectiles.

Charge	N
1G	-0.50
2G	-0.50
3G	-0.49
4G	-0.49
5G	-0.48
3W	-0.44
4W	-0.43
5W	-0.42
6W	-0.41
7W	-0.41
8	-0.30

Table 10: N Factors for the M109-155mm Cannon, for Projectiles: HE, WP, HC, CR, CG, CY, CV, and IL

5. Table 9 provides the N factor of the weight correction equation. These factors are for the 105mm C1 and L5 Cannons, for all projectiles.

6. Table 10 provides the N factor of the weight correction equation. These factors are for the M109-155mm Cannon, for projectiles: HE, WP, HC, CR, CG, CY, CV, and IL.

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Annex F The DEWMA Forecasts

```
                    Mil-FORECASTS (M109 illuminated)
                          projectile: IL

 OBS     CHARGE     BARREL      FCST     BARLRMVN       MSER

  1        2G        3314      239.0        1            0.0
  2        3G        4387      280.0        3            0.6
  3        4W        3314      355.1       10            4.0
  4        4W        4096      355.9        2            0.6
  5        4W        4395      355.9        3            9.9
```

```
                          M109-FORECASTS
              projectiles: HE, WP, HC, CR, CG, CV, CY

OBS     CHARGE     BARREL       FCST     BARLRMVN        MSER

  1       2G         3314      231.8        47             5.2
  2       2G         3361      231.1        23            12.2
  3       2G         4096      232.7        36             8.7
  4       2G         4099      233.4         2             0.7
  5       2G         4387      230.4        15             6.1
  6       2G         4388      232.3        20            10.8
  7       2G         4395      230.8        46             8.9
  8       2G         4486      233.4        15             3.3
  9       2G         4487      232.5        13             3.6
 10       3G         3261      270.6         2             0.0
 11       3G         3314      272.6         8             2.0
 12       3G         3361      271.7        12            10.8
 13       3G         4096      273.2         5             2.2
 14       3G         4099      273.1         7            25.7
 15       3G         4244      272.1         1             0.0
 16       3G         4387      271.9         3             0.1
 17       3G         4388      271.4        10            12.2
 18       3G         4391      270.7        10            14.2
 19       3G         4395      270.7        30             4.7
 20       3G         4486      271.2         7             0.2
 21       3G         4487      271.8         2             5.2
 22       3G        23217      272.9         1             0.0
 23       3W        26086      297.0         5             4.1
 24       4G          260      321.1         3            56.7
 25       4G         3361      317.9        14             1.5
 26       4G         4099      322.1         9           111.3
 27       4G         4211      315.6         1             0.0
 28       4G         4388      316.5         8             5.9
 29       4G         4391      316.7         1             0.0
 30       4G         4395      312.7         9             1.1
 31       4G         4486      316.1         3             0.4
 32       4G         4487      317.2        10             1.5
 33       4G        24093      323.1         8            93.5
 34       4W         3314      343.6        26            33.3
 35       4W         3361      344.6         5            14.0
```

M109-FORECASTS
projectiles: HE, WP, HC, CR, CG, CV, CY

OBS	CHARGE	BARREL	FCST	BARLRMVN	MSER
36	4W	4096	343.4	18	5.9
37	4W	4395	344.9	18	3.0
38	4W	26086	344.5	3	2.7
39	5G	260	376.3	6	1.0
40	5G	3261	374.8	6	3.8
41	5G	3314	374.5	6	0.3
42	5G	3361	374.3	9	2.3
43	5G	4096	376.6	8	0.7
44	5G	4099	381.0	18	62.1
45	5G	4209	373.6	12	0.5
46	5G	4244	375.4	5	5.2
47	5G	4388	376.6	17	5.8
48	5G	4391	375.7	2	0.2
49	5G	23217	376.7	6	11.5
50	5W	1547	394.2	3	0.2
51	5W	3314	391.7	10	1.7
52	5W	3361	395.1	1	0.0
53	5W	4096	392.9	9	2.3
54	5W	4099	393.7	1	0.0
55	5W	4204	394.9	4	0.4
56	5W	4244	394.3	4	18.5
57	5W	4395	394.0	10	3.3
58	5W	4486	391.9	5	1.2
59	5W	4487	392.5	1	0.0
60	5W	23217	395.1	1	0.0
61	6W	1547	475.2	1	0.0
62	6W	3314	475.3	1	0.0
63	6W	3361	475.1	12	1.2
64	6W	4099	475.2	13	1.6
65	6W	4211	474.8	1	0.0
66	6W	4244	475.0	2	0.0
67	6W	4388	476.1	10	4.5
68	6W	4391	476.0	3	2.2
69	6W	4395	476.2	2	0.2
70	6W	4487	472.9	5	5.7

```
                         M109-FORECASTS
             projectiles: HE, WP, HC, CR, CG, CV, CY

OBS     CHARGE     BARREL      FCST     BARLRMVN        MSER

 71       6W        26086     477.0          7           1.8
 72       7W         3117     567.2         27           1.2
 73       7W         3361     566.9         10           0.4
 74       7W         4099     565.0          4           1.1
 75       7W         4388     569.8          5          54.1
 76       7W         4391     570.8          3           0.0
 77       7W        24487     567.2          2           0.0
 78       8          3117     683.9         18          11.5
```

```
                      c1-FORECASTS
                 projectiles: HE, WP, HC, CR, CG, CV, CY

OBS     CHARGE     BARREL      FCST     BARLRMVN      MSER

  1       1           21      187.3         3          9.3
  2       1          123      190.2         4          3.7
  3       2           21      207.8         2          0.0
  4       2           22      208.1         5          2.0
  5       2           50      206.9         4          4.5
  6       2          282      211.0         6          3.4
  7       2          545      207.3        19          5.5
  8       3            3      226.3         6          5.8
  9       3            9      225.9         4          0.7
 10       3           18      228.9         1          0.0
 11       3           21      229.4         6          1.7
 12       3           22      227.6         5          2.4
 13       3           25      229.4         6         13.7
 14       3           31      229.8         6          2.5
 15       3           50      227.7         5          8.7
 16       3           52      229.9         6          7.8
 17       3           68      228.9         8         11.3
 18       3          107      228.6         5          1.3
 19       3          121      230.5        13          9.2
 20       3          169      227.2         5         11.1
 21       3          174      229.1         7          6.4
 22       3          191      231.4         6          2.8
 23       3          194      230.6         1          0.0
 24       3          256      227.7         7         27.0
 25       3          282      229.4         5          1.0
 26       3          306      229.8        12          7.7
 27       3          316      229.2         5          4.1
 28       3          318      230.5        10          2.3
 29       3          333      230.8         3          0.3
 30       3          349      232.4         7          2.9
 31       3          390      231.7         5          2.3
 32       3          423      230.7         4          0.8
 33       3          442      229.3         5          3.0
 34       3          466      229.5         9         11.2
 35       3          501      228.7        14          2.0
```

```
                              c1-FORECASTS
                  projectiles: HE, WP, HC, CR, CG, CV, CY

OBS      CHARGE      BARREL         FCST      BARLRMVN         MSER

 36        3           503         229.5          6            22.6
 37        3           514         232.6          5             1.8
 38        3           540         224.7          5             2.1
 39        4             3         255.9          4             0.4
 40        4             4         263.9          3             0.7
 41        4             9         256.8          5             0.2
 42        4            18         257.8         11             5.0
 43        4            21         256.2         20             3.2
 44        4            22         258.5          6             3.8
 45        4            31         258.7          7             1.4
 46        4            33         259.0         11             1.7
 47        4            50         259.0          5             2.0
 48        4            54         258.8          3             0.2
 49        4            78         259.7          6             2.1
 50        4            82         260.0          6             2.3
 51        4           121         261.1         13             2.0
 52        4           147         262.2          6             5.8
 53        4           174         263.6         10             1.2
 54        4           191         263.4          8             2.0
 55        4           223         256.3          5             2.2
 56        4           237         259.6          4             1.2
 57        4           253         263.3         10             2.2
 58        4           256         254.7          5             7.0
 59        4           282         261.6         19             2.9
 60        4           292         258.7          3             0.2
 61        4           303         265.9          2             0.0
 62        4           316         261.5          4            11.1
 63        4           333         264.3          2             0.0
 64        4           349         264.1          2             0.9
 65        4           390         262.7          4             0.1
 66        4           423         259.5          6             3.4
 67        4           465         259.9          1             0.0
 68        4           466         259.2         17            12.9
 69        4           503         257.6          4             0.8
 70        4           514         264.5          5             2.0
```

c1-FORECASTS
projectiles: HE, WP, HC, CR, CG, CV, CY

OBS	CHARGE	BARREL	FCST	BARLRMVN	MSER
71	4	540	253.0	4	1.2
72	5	3	299.5	4	0.9
73	5	9	302.5	4	1.3
74	5	15	301.1	22	3.9
75	5	18	297.6	4	1.1
76	5	21	301.0	9	2.6
77	5	22	301.9	6	2.4
78	5	25	304.6	6	1.3
79	5	31	304.8	5	2.4
80	5	34	301.3	7	2.0
81	5	50	302.7	15	4.1
82	5	52	304.3	5	1.4
83	5	59	299.2	18	8.3
84	5	68	304.7	6	2.4
85	5	107	304.4	6	2.9
86	5	109	303.9	9	3.9
87	5	121	303.4	39	1.3
88	5	123	301.8	5	6.6
89	5	169	303.5	6	0.3
90	5	170	302.9	15	2.5
91	5	177	303.6	11	4.1
92	5	191	303.2	8	1.0
93	5	195	298.4	2	1.1
94	5	206	302.7	8	2.0
95	5	256	298.3	4	2.4
96	5	282	302.2	10	4.3
97	5	296	301.8	9	1.6
98	5	303	305.7	20	9.8
99	5	306	305.6	6	1.5
100	5	318	304.2	5	4.7
101	5	322	301.4	7	6.2
102	5	330	299.7	14	5.8
103	5	442	305.5	7	0.9
104	5	472	301.5	10	2.8
105	5	476	304.9	7	1.9

c1-FORECASTS
projectiles: HE, WP, HC, CR, CG, CV, CY

OBS	CHARGE	BARREL	FCST	BARLRMVN	MSER
106	5	503	300.1	5	7.2
107	5	540	295.5	6	5.0
108	5	4217	302.5	4	2.2
109	6	3	366.5	5	1.1
110	6	9	367.0	13	1.4
111	6	18	366.9	6	1.9
112	6	22	367.5	8	9.4
113	6	31	369.6	14	2.3
114	6	50	370.2	6	1.4
115	6	54	369.5	5	5.5
116	6	78	369.9	7	3.3
117	6	82	370.0	1	0.0
118	6	106	371.5	10	4.0
119	6	107	370.1	7	3.0
120	6	118	369.4	6	1.9
121	6	121	370.0	21	2.7
122	6	128	364.9	2	15.0
123	6	147	372.0	6	0.5
124	6	170	365.3	2	0.7
125	6	177	369.2	5	3.9
126	6	191	368.6	5	1.3
127	6	206	367.1	5	4.2
128	6	223	365.2	5	5.8
129	6	237	367.5	5	6.6
130	6	256	366.4	5	15.8
131	6	282	369.7	5	2.9
132	6	304	371.4	6	3.5
133	6	306	371.6	8	2.4
134	6	318	370.6	5	0.6
135	6	333	372.0	8	4.5
136	6	349	367.2	11	16.7
137	6	423	372.5	8	4.7
138	6	442	371.6	6	1.2
139	6	465	370.0	14	2.2
140	6	469	367.5	1	0.0

c1-FORECASTS
projectiles: HE, WP, HC, CR, CG, CV, CY

OBS	CHARGE	BARREL	FCST	BARLRMVN	MSER
141	6	476	368.4	5	1.3
142	6	503	366.9	4	1.3
143	6	540	362.2	5	11.5
144	7	3	466.3	4	1.5
145	7	4	468.5	13	4.5
146	7	9	466.7	5	0.3
147	7	18	460.2	6	2.8
148	7	21	464.9	4	1.0
149	7	22	466.4	6	0.6
150	7	33	468.0	6	4.0
151	7	50	464.3	12	1.0
152	7	52	467.8	6	2.8
153	7	54	466.7	5	9.5
154	7	78	469.6	6	1.6
155	7	82	464.9	8	0.3
156	7	106	469.6	1	0.0
157	7	123	465.1	5	0.4
158	7	128	464.2	4	0.0
159	7	147	469.4	5	2.3
160	7	191	468.5	4	2.5
161	7	195	463.2	7	2.8
162	7	256	464.8	4	0.6
163	7	282	468.0	6	0.7
164	7	292	468.7	6	5.7
165	7	306	468.7	5	1.3
166	7	318	468.9	7	1.1
167	7	333	467.9	6	1.9
168	7	342	469.6	3	6.3
169	7	442	469.2	7	4.8
170	7	503	465.3	4	4.6
171	7	540	459.5	4	0.1

```
                    L5-FORECASTS
             projectiles: HE, WP, HC, CR, CG, CV, CY

OBS      CHARGE      BARREL        FCST     BARLRMVN        MSER

  1        1          4137        183.0         1            0.0
  2        1          4143        184.2        12            3.4
  3        1          4173        182.6         3            0.1
  4        1          4176        182.3         6            3.7
  5        2          4137        201.9         8            2.2
  6        2          4143        203.3        11            0.4
  7        2          4152        204.5         6            1.3
  8        2          4159        202.1         8            3.0
  9        2          4165        201.8         7            0.2
 10        2          4173        202.5        10            0.8
 11        2          4176        202.1        13            0.6
 12        2          4196        202.3         3            0.1
 13        2          4217        200.4        13            1.3
 14        3          4137        219.4         5            0.3
 15        3          4143        219.5        11            1.3
 16        3          4152        219.2         6            0.3
 17        3          4159        219.4         6            0.4
 18        3          4165        218.9        16            0.5
 19        3          4173        219.1        15            2.4
 20        3          4176        219.5         8            0.6
 21        3          4196        219.6         6            1.2
 22        3          4217        219.4        17            0.9
 23        4          4137        246.1        10            0.8
 24        4          4143        245.8         6            0.6
 25        4          4152        245.2         6            2.3
 26        4          4159        246.0         6            0.2
 27        4          4165        244.0        34            1.6
 28        4          4173        244.8        19            2.3
 29        4          4176        245.2        17            1.0
 30        4          4196        246.0         6            0.4
 31        4          4217        245.5         6            6.6
 32        5          4137        282.3        16            1.3
 33        5          4143        282.2        13            2.8
 34        5          4152        283.3         6            0.3
 35        5          4159        280.8        43            2.3
```

```
                              L5-FORECASTS
                projectiles: HE, WP, HC, CR, CG, CV, CY

 OBS     CHARGE     BARREL      FCST     BARLRMVN      MSER

  36        5        4165      281.8        13          1.2
  37        5        4173      280.9        11          2.0
  38        5        4176      281.1        17          1.4
  39        5        4196      282.9         6          1.8
  40        5        4512      282.8         6          0.2
  41        6        4137      340.0         9          2.3
  42        6        4143      339.7        10          1.1
  43        6        4152      342.8        18          3.7
  44        6        4159      340.1         9          1.0
  45        6        4165      342.1         6          1.7
  46        6        4173      342.5        24          2.2
  47        6        4176      339.8        16          2.3
  48        6        4196      342.3         6          1.0
  49        6        4217      340.2         6          8.6
  50        6        4891      340.6        24          1.7
  51        7        4137      432.9        15          3.1
  52        7        4143      433.6        17          1.1
  53        7        4152      436.2         4          0.9
  54        7        4159      435.1        12          1.7
  55        7        4165      431.5        12          2.9
  56        7        4173      435.4        12          2.5
  57        7        4176      434.1        12          4.8
  58        7        4196      435.4         5          1.8
  59        7        4512      435.9         6          0.1
```

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Annex G Glossary

Notation used in MiliPAC and RMV formulae

- F is the temperature in degrees Fahrenheit;
- C is the temperature in degrees Celsius;
- a, b and c are coefficients of the linear, quadratic and cubic components, respectively, in a cubic equation (to correct for non-standard temperature). The coefficients are charge dependent and are found in either Table B-5 of reference B, or in Tables 5 to 7;
- N is a unitless weight correction constant, dependent upon charge. See Table B-6 in reference B, or Tables 8 to 10 in this report;
- k the square to pound conversion factor. This is the weight in pounds per unit square increment. See Table B-25 in reference B, or the section on weight of the projectile, below;
- m_{sq} is the entered projectile weight in squares;
- $m_{sq_{std}}$ standard projectile weight in squares, as per Table B-25 in reference B, or Table 2 in this report;
- $m_{lb_{std}}$ standard projectile weight in pounds, as per Table B-26 in reference B, or Table 2 in this report;
- m_s standard projectile family weight in pounds, as per Table B-26 in reference B, or Table 2 in this report;
- m actual weight of the round, in pounds;
- V_o standard muzzle velocity, forecast for standard conditions, for a given charge;
- λ lot factor correction, in meters/second;
- $V^{\sim\lambda}$ standard muzzle velocity, corrected for lot factor only, for a given charge;

- τ the temperature correction, in units m/s;
- ω the weight correction, in meters/second;
- $V^{\sim\lambda\tau}$ standard muzzle velocity, corrected for lot factor and temperature, for a given charge;
- $V^{\sim\lambda\tau\omega}$ standard muzzle velocity, corrected for lot factor, temperature and weight. This is the expected muzzle velocity for this charge, taking into account the non-standard conditions of this particular round;
- Λ is the observed muzzle velocity, as recorded by the muzzle velocity indicator;
- $\Lambda^{\sim\omega}$ is the observed muzzle velocity, standardized or "reduced" to account for the weight;
- $\Lambda^{\sim\omega\tau}$ is the observed muzzle velocity, standardized or "reduced" to account for the non-standard weight and temperature;
- $\Lambda^{\sim\omega\tau\lambda}$ is the observed muzzle velocity, standardized or "reduced" to account for the non-standard weight, temperature and lot factor. This quantity is commonly called the reduced muzzle velocity, or simply the RMV.

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This project report describes a number of ways in which muzzle velocity can be forecasted for howitzers such as the M109 155mm, C1 105mm or the L5 105mm. A simple time series model is selected and proposed for use with the Canadian Forces Artillery Calibration System (ACS) project. The proposed model is called the double exponentially weighted moving average (DEWMA) model.

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Muzzle Velocity

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