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TITLE
 AN ANALYTIC BARRIER SURVEILLANCE MODEL FOR FIXED AND MOBILE PLATFORMS

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**DEPARTMENT OF NATIONAL DEFENCE
CANADA**



**OPERATIONAL RESEARCH AND ANALYSIS
DIRECTORATE OF MARITIME OPERATIONAL RESEARCH
ORA PROJECT REPORT PR695**

**AN ANALYTIC BARRIER SURVEILLANCE MODEL FOR
FIXED AND MOBILE PLATFORMS**

**by
B. A. Trenholm**

October 1994

OTTAWA, CANADA



Operational Research and Analysis

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OTTAWA, CANADA

OCTOBER 1994

ABSTRACT

This report documents the barrier surveillance model used in DMOR Project 21279-13 "Atlantic/ Pacific Undersea and Surface Surveillance Options". In this model surveillance is limited to the initial detection and cueing function. When very large areas are involved a less costly cueing solution is to provide one or more layered barriers rather than full area coverage. The classic Koopman random search model is adapted to the problem of barrier surveillance and applied to fixed systems as well as mobile search systems. Barrier surveillance is modelled as area search subject to a time constraint which depends on the minimum time for a target to transit the barrier. A special variant of the model is developed to meet the unique requirements of maritime patrol aircraft. The depth of a barrier depends on the capability of the surveillance systems. For some of the systems considered, the barriers may be sufficiently wide that significant area coverage is achieved.

RÉSUMÉ

Le présente rapport explicite le modèle de surveillance par couches utilisé dans le projet 21279-13 du DRO(M) qui porte sur les possibilités de surveillance sous-marine et en surface dans l'Atlantique et le Pacifique. Dans ce modèle, la surveillance se limite à la fonction de détection initiale et de repérage. Lorsque de très grandes étendues sont à surveiller, une solution de repérage moins coûteuse consiste à prévoir une ou plusieurs couches superposées plutôt qu'à offrir une couverture complète de la zone. Le modèle classique de recherche aléatoire de Koopman convient au problème de la surveillance par couches, et il s'applique aux systèmes de recherches tant fixes que mobiles. La surveillance par couches est modélisée comme une recherche par zone soumise à une contrainte temporelle qui est fonction du temps minimal mis par une cible pour traverser la couche. Une variante particulière de ce modèle est mise au point pour satisfaire aux exigences propres aux aéronefs de patrouille maritime. La profondeur d'une couche est fonction des capacités des systèmes de surveillance. Dans certains des systèmes étudiés, les couches peuvent être larges au point d'offrir une couverture significative de la zone.

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**An Analytic Barrier Surveillance Model for
Fixed and Mobile Platforms**

EXECUTIVE SUMMARY

This report documents the barrier surveillance model used in DMOR Project 21279-13 "Atlantic/ Pacific Undersea and Surface Surveillance Options". In this model surveillance is limited to the initial detection and cueing function. When very large areas are involved a less costly cueing solution is to provide one or more layered barriers rather than full area coverage. The classic Koopman random search model is adapted to the problem of barrier surveillance and applied to fixed systems as well as mobile search systems. Barrier surveillance is modelled as area search subject to a time constraint which depends on the minimum time for a target to transit halfway through the barrier. A special variant of the model is developed to meet the unique requirements of Maritime Patrol Aircraft (MPA). The depth of a barrier depends on the capability of the surveillance systems. For some of the systems considered, the barriers may be sufficiently wide that significant area coverage is achieved.

The hierarchy of maritime coastal defence surveillance functions comprises four basic levels:

- a. **Intelligence and Warning.** The top level of surveillance is the function of advanced Intelligence & Warning or preconditioning (also known as Indication & Warning). Typical subfunctions include intelligence methods, such as Human Intelligence (HUMINT), Electronic Intelligence (ELINT) and Electronic Support Measures (ESM).
- b. **Initial Alert and Cueing.** The second level of surveillance is strategic initial alert and cueing, usually accomplished by remote systems. This function may provide rough estimates of target position and classification, which contributes to picture compilation and the direction of reactive surveillance assets.
- c. **Reconnaissance.** The third level of surveillance is reconnaissance, to gather missing information in order to complete picture compilation when and if required.
- d. **Reaction.** The fourth level of surveillance is supplied by follow-on reactive vehicles. The type of vehicle would be matched to the requirement, depending on available reaction time, whether continued surveillance should be overt or covert, whether close tracking and monitoring is required, and if physical interdiction is required.

The model presented in this report focuses on "level b", the initial alert and cueing function provided by a single barrier of sensors, which may be either fixed or mobile. The force levels calculated are based on 24-hour, year-round surveillance. It is further assumed that the assets conducting the cueing function are dedicated solely to this function. Contacts requiring continued tracking would become the responsibility of other units.

The coastal surveillance problem can be subdivided into surveillance zones. Each zone is defined by distance from land, and may have a different required detection probability which may vary depending on the target type. Three zones have been suggested by the Director Maritime Force Development (DMFD) [Ref. 1] based on the Maritime Defence Strategy [Ref. 2]. One or more surveillance cueing barriers could be positioned within these zones, or on boundaries between these zones, as required:

- a. An Inner Zone, extending from 0 to 30 nm from shore. This zone would typically require very high probabilities of detection, particularly for military targets;
- b. A Middle Zone, extending from 30 to 250 nm. This zone would be of interest for defence of the Exclusive Economic Zone (EEZ) and defence of shipping routes; and
- c. An Outer Zone, extending typically from 250 to 1000nm. This zone would be of interest primarily for detection of military targets.

This report presents a model of a single surveillance cueing barrier, established at an arbitrary distance from the shore. The model is based on an analytic formula for area search developed by Koopman. The following modifications have been made to the model:

- a. Initial Illuminated Area. The searcher swept area includes an initial area at the beginning of each search cycle, corresponding to the area illuminated (or insonified) when the searcher arrives at the patrol area, and turns on its search sensor. This initial illumination area permits the model to be used for fixed sensors. It also provides a better representation of long-range ship sensors such as SURTASS (Ship Surveillance Towed Array Sonar System) which are not usually operated during high speed transit.
- b. Variable Barrier Width. In order to model a barrier as an area search problem, the total length of the search area is specified, but the width of the search area is allowed to adapt to the capability of the search platform. Fixed platforms and ships are assumed to form a barrier of the

specified length, with the barrier width equal to the searcher sweep width. Maritime Patrol Aircraft (MPA) are assumed to form a deep barrier subdivided into sections of maximum width and length for a single aircraft, with each MPA searching a square area. The MPA deep barrier tactic is designed to exploit MPA characteristics of limited time on station combined with high speed.

c. Time Constraint. A time constraint is placed on the search, based on the minimum time for a target to transit halfway through the barrier. The original search model requires that targets remain in a fixed area; this constraint controls the rate at which new targets enter the area and old targets depart. The time constraint also allows additional time for the searcher to investigate real as well as false contacts.

Simple analytic formulas have been developed which can be implemented on a desktop computer using a spreadsheet program. Two forms of the equations are given. The first is suitable for fixed surveillance systems and surveillance ships. The second is suitable for MPA. Several examples are given for barriers consisting of auxiliary ships with Surveillance Towed Array Sonar Systems (SURTASS), fixed underwater arrays, land-based High Frequency Surface Wave Radar (HFSWR), land-based Microwave Radar, and MPA equipped with Maritime Surface Surveillance Radar (MSSR).

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LIST OF ABBREVIATIONS AND ACRONYMS

CP140	Aurora Patrol Aircraft
CP140A	Arcturus Patrol Aircraft
DARMR	Director Air Requirements Maritime and Rotary
DGMD	Director General Maritime Development
DMES	Director Maritime Engineering Support
DMFD	Director Maritime Force Development
DMOR	Directorate of Maritime Operational Research
DNR	Director Naval Requirements
EEZ	Exclusive Economic Zone
ELINT	Electronic Intelligence
ESM	Electronic Support Measures
HF	High Frequency
HFSWR	High Frequency Surface Wave Radar
HUMINT	Human Intelligence
MPA	Maritime Patrol Aircraft
MSSR	Maritime Surface Surveillance Radar (airborne)
SONAR	Sound Navigation and Ranging
SURTASS	Surveillance Towed Array Sonar System
SWATH	Small Waterplane Area Twin Hull
TACTASS	Tactical Towed Array Sonar System
TAGOS	Towed Array Ocean Surveillance System
VTSR	Vessel Traffic Services Radar (Coast Guard)

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LIST OF VARIABLES

A	Search area (sq. nm) containing target
A_0	Initial swept area (sq.nm.) for single search platform at time $t=0$
AMPA	Search area (sq. nm) assigned to one MPA
A_N	Area swept by N search units on station, in time interval T_C
A_T	Total Area (sq.nm) swept by single search platform in time interval T_C
APF	Availability Planning Factor
B	Side of square MPA patrol area (nm)
D	Distance to patrol station (nm) from base
D_T	Distance (nm) travelled by target during search time T_C
E	Effectiveness of Surveillance Barrier
E	Elliptical integral of the second kind
ETF	Effective Time on Station Factor
FAR	Fuel availability ratio
FCR_0	Fuel consumption rate (lb/hr) during transit to station
FCR_S	Fuel consumption rate (lb/hr) while on station
FPF	Force Level Planning Factor
G	Fuel payload (lb)
G_{OS}	Fuel available (lb) for on-station patrol
H	Search effort factor as a function of k
k	Ratio of W_0 to W_Y
k_X	Ratio of W_X to R_X
k_Y	Ratio of W_Y to R_Y
L	Total searcher path length during time T_C (nm)
L_0	Required barrier length (nm)
L_i	Barrier length of i-th fixed barrier (nm)
L_S	Total length of fixed barriers (nm)
N	Number of Search Units On Station (including idle units)
N_{OS}	Number of Search Units On Station (fully employed)
N_{FL}	Total Force Level
Φ	Search Density (Ratio of swept area to search area)
P_D	Probability of Detection
RTF	Required Time on Station Factor

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r	Ratio of V_E to V_{MAX}
R	Underwater array average detection range (nm)
R_1	Underwater array detection range (nm) downslope (seaward)
R_2	Underwater array detection range (nm) upslope (landward)
R_x	Detection range (nm) along search axis, or parallel to barrier
R_y	Detection range (nm) perpendicular to search axis, or normal to barrier
σ	Parameter (radians) used in calculation of V_E
T_C	Cutoff Time (hr) for search pattern
T_M	Total Mission Time (hr)
T_{OS}	Time On Station (hr)
T_T	Total Travel Time (hr) to and from search area
u	Ratio of V_{MIN} to V_{MAX}
V_0	Searcher Transit Speed (kt) to reach search station
V_E	Effective speed of encounters, or resultant search speed (kt) for an open ocean scenario, combining both searcher and target motion
V_{MIN}	Minimum of searcher and target speeds (kt)
V_{MAX}	Maximum of searcher and target speeds (kt)
V_S	Searcher Patrol Speed (kt) while on station
V_T	Target Speed (kt)
W_0	Required Barrier width (nm)
W_x	Sensor sweep width (nm) along search axis, or parallel to barrier
W_y	Sensor sweep width (nm) perpendicular to search axis, or normal to barrier
YFR	Maximum yearly flying rate (hr) per aircraft

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**An Analytic Barrier Surveillance Model for
Fixed and Mobile Platforms**

Introduction

1. **Purpose of Report.** This report documents the barrier surveillance model used in DMOR Project 21279-13 "Atlantic/ Pacific Undersea and Surface Surveillance Options". This model is limited to the initial detection and cueing function. When surveillance of very large coastal areas is required, a less costly cueing solution is to provide one or more layered barriers rather than full area coverage. The classic Koopman random search model is adapted to the problem of barrier surveillance and applied to fixed systems as well as mobile search systems. Barrier surveillance is modelled as area search subject to a time constraint which depends on the minimum time for a target to transit halfway through the barrier. A special variant of the model is developed to meet the unique requirements of Maritime Patrol Aircraft (MPA). The depth of a barrier depends on the capability of the surveillance systems. For some of the systems considered, the barriers may be sufficiently wide that significant area coverage is also achieved. The model calculates the force levels required to maintain continuous cueing surveillance of a specified length of coastline.

2. **Hierarchy of Maritime Surveillance Functions.** There are many roles and functions which come under the general umbrella of surveillance. In a maritime surface surveillance scenario, surveillance functionality can be viewed as a hierarchy of functions, as shown in Figure 1. There are four basic levels of maritime surveillance:

a. **Intelligence and Warning.** The top level of surveillance is the function of advanced Intelligence & Warning or preconditioning (also known as Indication & Warning). This function can raise or lower alertness levels to meet the changing strategic situation. Typical subfunctions include intelligence methods, including Human Intelligence (HUMINT), Electronic Intelligence (ELINT) and Electronic Support Measures (ESM). Advanced warning permits other surveillance resources to be reallocated, and readiness levels to be adjusted.

b. Initial Alert and Cueing. The second level of surveillance is strategic initial alert and cueing, usually accomplished by remote systems. This function may provide rough estimates of target position and classification, which contributes to picture compilation and the direction of reactive surveillance assets. It is assumed in the model that this function is required to operate on a year-round 24-hour basis.

c. Reconnaissance. The third level of surveillance is reconnaissance, to gather missing information in order to complete picture compilation when and if required. Additional information and situation assessment will usually be required to determine the type of follow-on reaction required. Typical maritime reconnaissance for surface vessels would involve aircraft reacquisition and overflight to establish or confirm classification and intent, and to refine estimates of location and heading. Indirect reconnaissance support, to reduce the required level of effort, could also include the concept of "electronic license plates" for registered ships, to determine the identities and positions of known friendly vessels.

d. Reaction. The fourth level of surveillance is supplied by follow-on reactive vehicles. The type of vehicle would be matched to the requirement, depending on available reaction time, whether continued surveillance should be overt or covert, whether close tracking and monitoring is required, and if physical interdiction is required. Although physical interdiction is a sovereignty enforcement function rather than a surveillance function, the choice of reactive asset would normally take this possible additional requirement into consideration. Due to the high cost of sending out reactive vehicles, it is essential that the all of the foregoing functions be highly capable in filtering out the majority of false alarms and false contacts.

3. (U) Focus on Initial Alert and Cueing by a Sensor Barrier. The model presented in this report focuses on the initial alert and cueing function provided by a single barrier of sensors, which may be either fixed or mobile. The force levels calculated are based on 24-hour, year-round surveillance. It is assumed that forces assigned to the cueing function are dedicated assets. Any distractions from their assigned duties should be minimal, in order to maintain the integrity of the barrier. The prime duty of the barrier forces is to provide an initial detection. Additional information such as contact range, bearing, classification, identification, status, speed and heading would assist in reducing false alarms, but should not interfere with the detection role.

HIERARCHY OF SURVEILLANCE FUNCTIONS

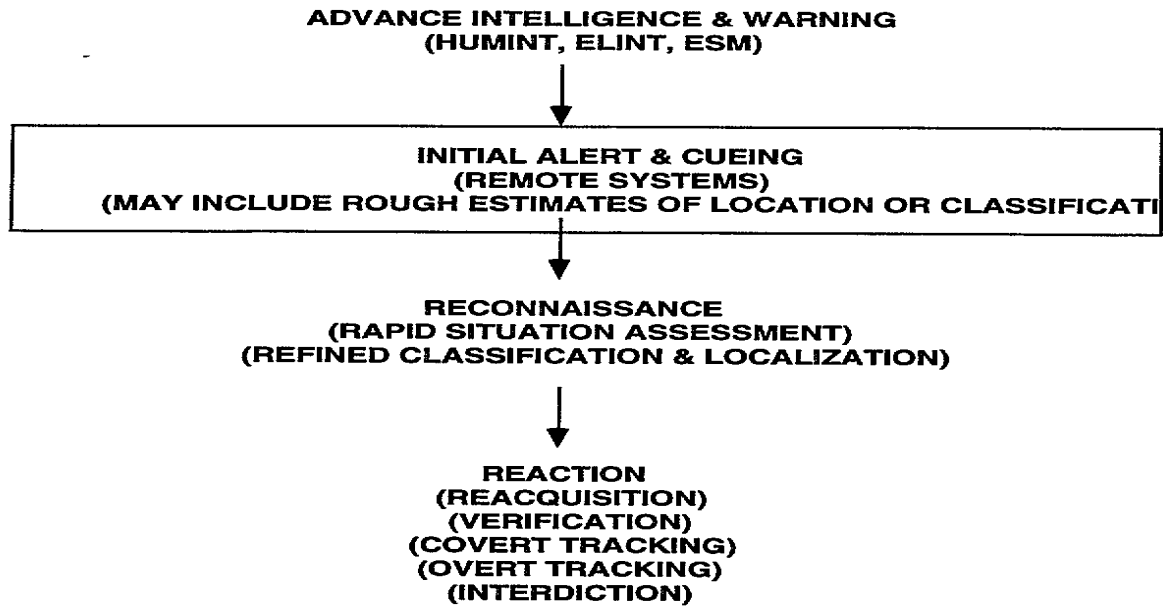


Fig. 1. Hierarchy of Maritime Surveillance Functions

4. Coastal Defence Zones. The Coastal defence area may be subdivided into zones, as illustrated in Figure 2. The Director of Maritime Force Development (DMFD) has suggested a requirement for three layered defence zones [Ref.1] , consistent with the Maritime Defence Strategy [Ref. 2]:

- a. An Inner Zone, extending from 0 to 30 nm from shore. This zone would typically require very high probabilities of detection, particularly for military targets;
- b. A Middle Zone, extending from 30 to 250 nm. This zone would be of interest for defence of the Exclusive Economic Zone (EEZ) and defence of shipping routes; and
- c. An Outer Zone, extending typically from 250 to 1000nm. This zone would be of interest primarily for detection of military targets.

Targets are assumed to be distributed over all three zones. Targets may be stationary, transiting, or conducting complicated maneuvers. For example, all three modes of operation are typical of fishing vessels. One or more barriers could be positioned within these zones, or on boundaries between zones. The barriers would provide the initial alert and cueing function. Figure 3 illustrates a single barrier located at a distance just beyond the middle zone. Each barrier would provide detections on contacts operating within the barrier patrol area (illustrated by the cross-hatched area in Figure 3), as well as contacts which are transiting through the barrier.

5. Organization of This Report. In the remainder of this report, the application of the Koopman search model is explained and adaptations of the basic model are derived, beginning with the ship barrier model. The effects of searcher speed, target speed, and searcher detection range are explained and illustrated with examples which relate fleet size requirements to detection capability. It is then shown that the case of fixed sensor barriers can be treated as a degenerate case of the ship barrier model, with ship speed set to zero. Examples are given for fixed underwater arrays, land-based High Frequency Surface Wave Radar (HFSWR) and land-based Microwave Radar. It is further shown that the equations for fixed sensor barriers can be reduced to a very simple form which is convenient for estimating the effectiveness of a barrier consisting of a variety of fixed platforms. A final variation of the barrier search model is introduced for MPA search barriers. Examples are given which relate aircraft fleet size to the required barrier length, aircraft speed, time on station, sensor detection capability, and target speed.

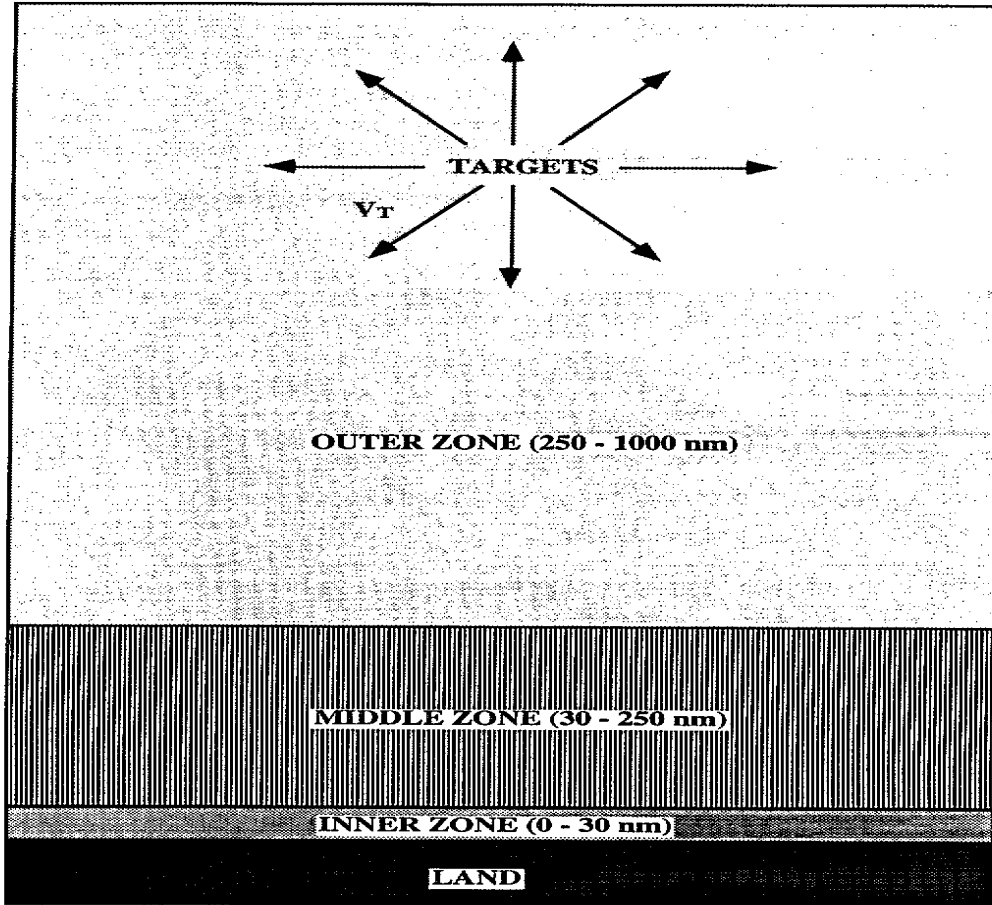


Fig. 2. Schematic representation of coastal defence zones

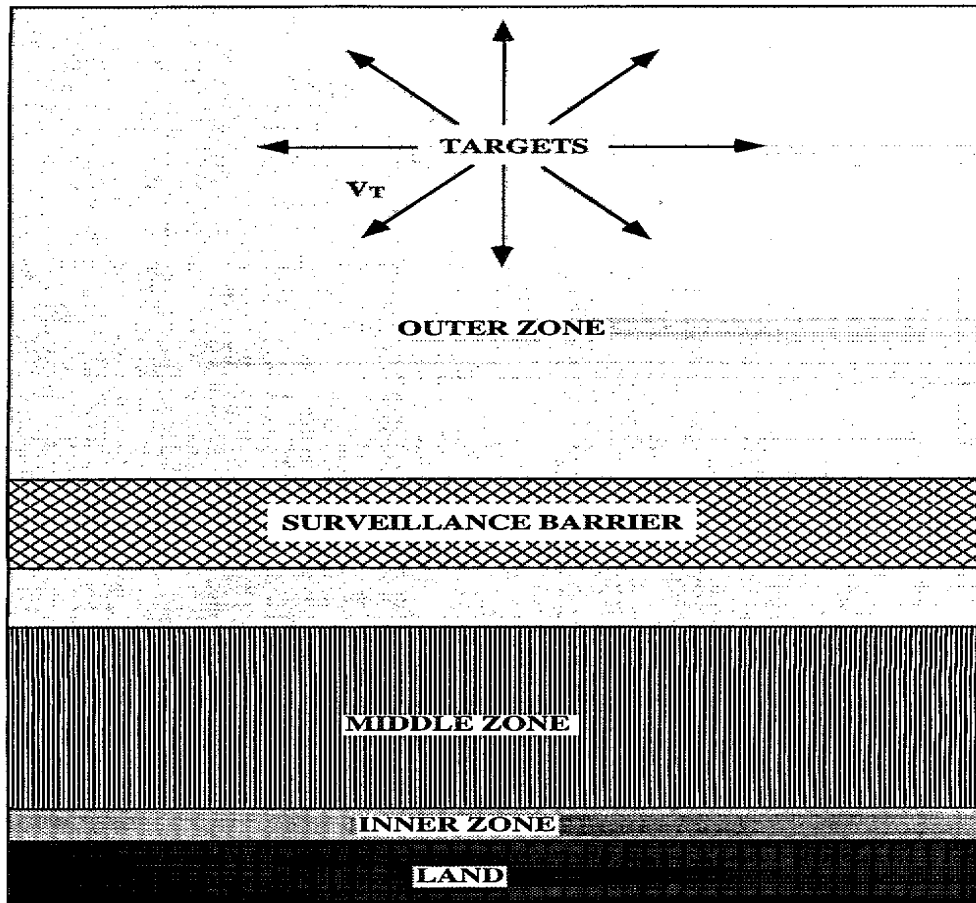


Fig. 3. Schematic representation of a surveillance barrier in the outer zone. One or more barriers could be established within zones or on boundaries between zones, as required.

General Methodology

6. Random Search Theory. The classic Koopman random search model [Ref. 3] has traditionally been used to model area search. The Koopman model provides an analytic formula which is convenient for broad brush analysis, where the fine details of stochastic detection and environmental variation are not of primary importance. In this formula, the cumulative probability of detecting a target, P_D , is related to the search density Φ , by the following expression:

$$P_D = 1 - e^{-\Phi} \quad [1]$$

where search density is defined as the ratio of the searched area A_N to the area A containing the target:

$$\Phi = A_N / A \quad [2]$$

The searched area, A_N , is the total area in square nautical miles, swept out by a fleet of one or more searchers. The search density Φ may be greater than unity, for an intensive search by many searchers, or repeated or closely spaced sweeping of the area by a single searcher. If the desired detection probability is specified, then the required search effort can be calculated by inverting equation 1 to obtain:

$$\Phi = -\ln(1 - P_D) \quad [3]$$

Figure 4 illustrates the relationship between the search density Φ and the probability of detection P_D .

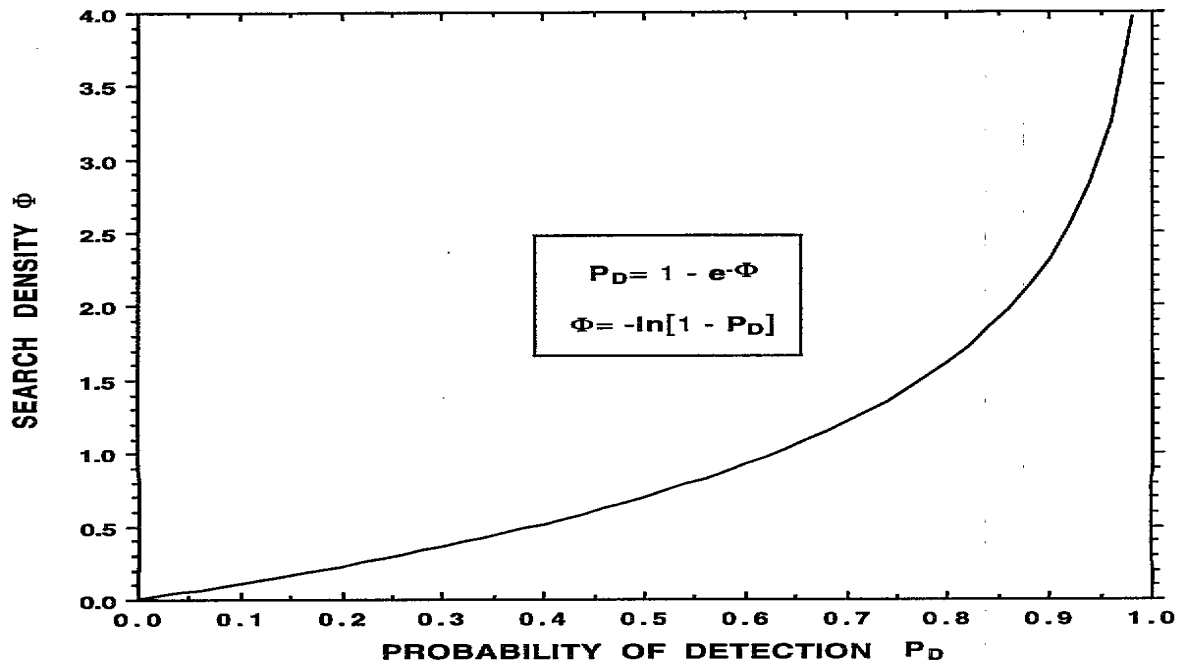


Fig. 4. Relationship between search density and probability of detection

7. Assumptions of Random Search Model. The Koopman theory of random search is an analytical and probabilistic model based on the following assumptions [Ref. 4], although in practice it has been found to agree with real data from a wide variety of situations [Ref. 5]:

- a. Targets are uniformly distributed within the search area;
- b. Target headings are uniformly distributed from 0 to 360°;
- c. Targets do not react to the searcher's presence;
- d. Targets move in a random fashion with average speed V_T , may change speed, and may backtrack, but do not leave the area;
- e. The searcher speed V_S is constant and the searcher moves either randomly or in an organized way, searching continuously. The *relative* motion of target and searcher is random [Ref. 6], hence the designation *random search*; and
- f. The searcher sweep width may be approximated by an effective sweep width which is given by a constant, but which usually represents integration over time and space [Ref. 7]. Detection in this probabilistic model is not perfect, however, since 100% "coverage" of an area ($\Phi = 1$, or a single sweep) gives only a 64% probability of detection ($P_D = 1 - e^{-1} = .64$). Thus the input sweep width should be chosen with these parameters in mind. Imperfect detection can be used to represent many actual problems of short- and long-term variations due to changes in operator attention, target aspect, the environment, and interference from clutter [Ref. 8].

8. The Effects of Target Speed. An increase in motion by the target can lead to two different and opposite effects:

- a. An increase in target speed may increase the rate of occurrence of detection opportunities. For the special case of open ocean transit, or a case in which searchers and targets *move in straight lines at steady speed*, Koopman [Ref. 9] suggests a resultant speed of encounter or effective search speed V_E , which depends on both V_S and V_T . It is shown in Annex B that the resultant search speed in this case is approximately equal to the maximum of the searcher speed and the target speed. The resultant search speed for this case is at most 27% greater than the maximum of the searcher and target speeds. The maximum value of V_E occurs when searcher speed and target speed are equal. The case of open ocean transit is of limited interest for the barrier surveillance scenarios of interest in Project 21279-13, but may be of interest in other applications. Details are given in Annex B. To implement this option, the value of V_E would be calculated and substituted for V_S in the model.

b. In the barrier model, and for the maritime cases of interest, it will be seen that a much larger and opposite effect of target speed, is that a time constraint may be placed on the time the target can be expected to remain in the area of the barrier. Increases in target speed will therefore limit the search cycle time and hence reduce the probability of detection.

9. Adaptation for Barrier Search. The simplicity of the Koopman random search formula is attractive for analysis. The assumptions of the random search model can accommodate barrier search, provided a constraint is placed on the available search time. Targets may be assumed to be within the patrol area, but this assumption is reasonable only for a limited time. In this barrier search model, targets are not always constrained to operate inside the barrier patrol area. Some of the targets may remain in the patrol area, while others may transit on a random heading. One of the goals of the barrier is to detect any targets which transit through the barrier. The worst case target would transit in a straight line at right angles to the barrier. This worst case scenario is given sufficiently high priority that it is used to determine the limit on available search time. By introducing a worst-case time constraint, all transiting targets are eventually contained within an area equal to the patrol area, as required by the condition in paragraph 7d, and the random search model may therefore be applied to the barrier problem. The initial patrol of the area may miss targets which are already in transit and approaching the boundaries of the area, but subsequent repetitive patrol leads to a steady-state condition which provides the required detection coverage.

10. Adaptation for MPA Barrier Search. The barrier model can be applied directly to ship barrier patrols and barriers consisting of fixed systems. The formulas may also be applied to Maritime Patrol Aircraft (MPA). However, MPA have different operating characteristics, such as shorter mission duration, lower percentage of time on station, and a preference for a schedule which may be matched to day or night flying as required. In the case of MPA, best practical use can be made of MPA resources by searching a barrier which is as wide and as deep as possible, so that the area remains sanitized for a longer period of time, and flights can be made less frequently. A variant of the barrier model will be presented for MPA, in which the width and depth of the barrier are equal, giving a square search area for a single MPA.

Barrier Search Model for Ships

11. Barrier Dimensions. Figure 5 illustrates the geometry of a surveillance barrier. The barrier has length L_0 nm, and width W_0 nm, with a total area A in square nautical miles, given by:

$$A = L_0 \cdot W_0 \quad [4]$$

Targets are assumed to be uniformly distributed, both within the area of the surveillance barrier, and in the areas outside of the barrier.

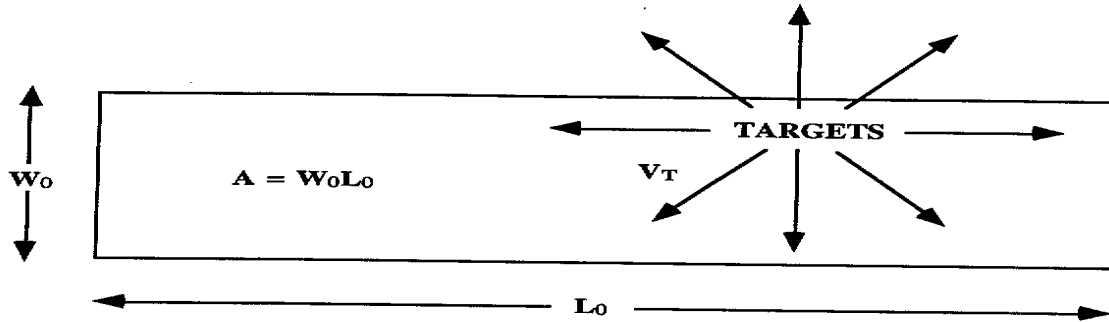


Fig. 5. Geometry of surveillance barrier

12. Swept Area. The area swept out by a single searcher is assumed to consist of an initial area plus a swept area. The initial area corresponds to the area illuminated (or insonified) when the searcher arrives at the patrol area, and turns on its search sensor. The introduction of an initial area is a modification of the standard Koopman random search model and allows representation of fixed sensors as well as special sensors such as SURTASS (Ship Surveillance Towed Array Sonar System) which are not operated until the searcher reaches its patrol station and slows down to the required operational speed. The dimensions of the illuminated area are assumed to be given approximately by a rectangle, with dimension W_x nm along the direction of the searcher track, and dimension W_y nm normal to the searcher track. These dimensions are assumed to be related to the nominal sensor detection ranges R_x and R_y along the searcher track and normal to the searcher track, respectively:

$$W_x = k_x \cdot R_x \quad [5]$$

$$W_y = k_y \cdot R_y \quad [6]$$

The constants k_x and k_y determine the ratio of sweep width to detection range in directions x and y respectively. For a surface ship with a sonar sensor, the sweep width is typically twice the detection range:

$$k_x = 2 \quad [7]$$

$$k_y = 2 \quad [8]$$

The path length travelled by the searcher is L nm, travelling at a speed of V_S kt during a specified cutoff time T_C hours:

$$L = V_S \cdot T_C \quad [9]$$

The initial illuminated area A_0 , in sq. nm., is given by the product:

$$A_0 = W_x \cdot W_y \quad [10]$$

The area A_T swept by a single searcher in the time interval T_C is the sum of the initial illuminated area plus the swept area:

$$A_T = A_0 + W_y \cdot L \quad [11]$$

Figure 6 illustrates the geometry of a surveillance barrier for a single patrol ship. The ship is assumed to patrol parallel to the length of the barrier, reversing direction at time intervals T_C .

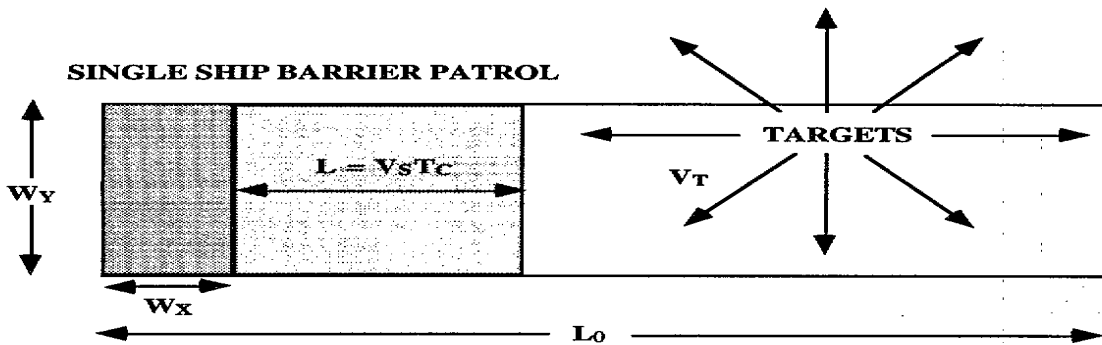


Fig. 6. Geometry of surveillance barrier for a single patrol ship

13. Optimal Barrier Width. For the ship barrier patrol it is assumed that the overall barrier width W_0 is chosen to be equal to the ship sweep width W_Y normal to the barrier:

$$\begin{aligned} W_0 &= k \cdot W_Y \\ &= W_Y \end{aligned} \quad [12]$$

This means that barrier width is allowed to depend entirely on the capability of the search sensor. No additional credit is given for a very wide barrier, although a wide barrier would give more opportunity for early warning, and additional holding and tracking time. In the context of simply providing an initial alert, a narrow barrier is deemed to be as good as a wide barrier. Full area coverage, while intuitively desirable, can be costly, both as a result of the cost of the larger search area, as well as the cost of reactive vehicles, particularly when there are many false alarms. Narrow barriers can have advantages as well, if they produce fewer false contacts, and are used only in critical locations. It is shown in Annex A that $k=1$ is optimal for this ship barrier tactic, for normal ship scenarios, i.e. those scenarios in which

$$T_{OS} > 0.5 W_Y / V_T \quad [13]$$

where T_{OS} is the searcher time on station. Annex A also shows that the model results are insensitive to the choice of k , if the initial search area A_0 is zero. If the initial search area A_0 is large, then force levels increase linearly with k .

14. Effective Time On Station Factor. The total mission time T_M hours, for the ship, is a required input. For a typical 14 day mission, $T_M=14 \times 24=336$ hr. The effective time on station factor, ETF, is also a required input. ETF will decrease with increasing distance D to the patrol area. D is the average of the distances to and from the patrol area at the beginning and end of the on-station search. For example, $ETF=0.85$ at a distance of 800nm, for an auxiliary TAGOS SWATH ship [Ref. 10]. The resulting ship time on station, T_{OS} , in hours is given by:

$$T_{OS} = ETF \cdot T_M \quad [14]$$

15. Constraints on Search Time. The cutoff time T_C allowed for the search pattern is constrained by the ship time on station, and also by the minimum time it would take a target, travelling at speed V_T kt, to penetrate halfway through the barrier :

$$T_C = \min (0.5 W_0 / V_T , T_{OS}) \quad [15]$$

The choice of the halfway point allows additional time for holding and tracking the target, if a detection occurs, and also allows for the back and forth motion of the search ship. In the worst case of a target transiting the barrier at right

angles, if an initial detection occurs during time interval T_C , then an equal opportunity exists for a second sighting during the subsequent time interval of length T_C , with the ship travelling in the opposite direction and sweeping the same area. These two data points, if properly correlated, would provide the bare minimum of data to establish a target track, and thereby assist in the reduction of false alarms.

16. Required Time On Station Factor. In calculating force levels, it is necessary to consider the required time on station factor, RTF. In most ship scenarios, this factor is unity. In other words, the ship is required to be on station almost continuously in order to provide an effective barrier. This factor is based on the distance D_T travelled by the target during the time interval T_C :

$$D_T = V_T \cdot T_C \quad [16]$$

and the ratio of this distance to one-half the barrier width:

$$RTF = D_T / (0.5W_0) \leq 1 \quad [17]$$

This ratio will be unity unless the ship time on station T_{OS} is unusually short, or the sweep width W_0 is unusually large. When RTF is significantly less than unity, the MPA barrier tactic may be more appropriate.

17. Calculation of Required On Station Force Level. To calculate the number of searcher units required on station, it is necessary to first compute the required search density Φ as given previously in equation [3]:

$$\Phi = -\ln(1-P_D) \quad [3]$$

From equation [2], we have the ratio of swept area to the area containing the target:

$$A_N / A = \Phi \quad [2]$$

The total swept area A_N is assumed to increase linearly with the total number N of units on station:

$$A_N = N \cdot A_T \quad [18]$$

Equations [2] and [18] may then be solved to obtain the total number of units on station, N :

$$N = \Phi \cdot A / A_T \quad [19]$$

Bringing in the required time on station factor, RTF, the actual number of fully employed units required on station, N_{OS} , is given by:

$$N_{OS} = N \cdot RTF \quad [20]$$

18. Availability Planning Factor. The availability planning factor, APF, is a required input to the barrier model for ships. APF is the number of ships required in the fleet to guarantee that one ship is available to sail, taking into account time for ship maintenance and crew rest. For example, if ships are available typically 50% of the time [Ref. 10], then:

$$\begin{aligned} APF &= 1 / (\text{fraction of time available}) \\ &= 1 / 0.5 \\ &= 2 \end{aligned} \quad [21]$$

19. Force Planning Factor. The total force planning factor, FPF, is the number of ships required in the fleet in order to provide one ship continuously on station at a specified distance D. FPF takes into account the availability planning factor, APF, and the effective time on station factor, ETF:

$$FPF = APF / ETF \quad [22]$$

For example, if $APF=2$, and $ETF=0.85$, then $FPF=2/0.85=2.35$.

20. Calculation of Required Total Force Level. The final calculation of the required total force level, N_{FL} , is the product of the number of fully employed ships required on station, N_{OS} , and the force planning factor, FPF:

$$N_{FL} = N_{OS} \cdot FPF \quad [23]$$

21. Examples for Ship Surveillance Platform. Table I shows four examples of ship barrier calculations in a spreadsheet format. The required barrier length L_0 in these examples is 1000 nm. The ship patrol speed is 4 kt, which is typical of a Surveillance Towed Array Sonar System (SURTASS) auxiliary ship. (A Tactical Towed Array Sonar System (TACTASS) auxiliary ship might operate at higher speeds.) Detection probability, detection ranges, and target speed are varied arbitrarily to indicate the influence of these parameters on required searcher force levels:

a. In Example 1, the ship detection range is 400nm and the target speed is 10 kt. To achieve 64% detection probability, a search density of 1.0 is required. In the calculated solution, each surveillance ship patrols a length of 160 nm, reversing direction every 40 hours. A total of 1.1 ships would be required on station to provide initial detection. To maintain year-round 24-hour surveillance of this barrier, a fleet of 2.5 ships would be required.

b. In Example 2, the detection range is decreased to 200 nm, which reduces the area search rates. Each surveillance ship patrols a length of 80 nm, reversing direction every 20 hours. A total of 2.1 ships would be required on station to provide initial detection. For year-round surveillance, a fleet of 5 ships would be required.

c. In Example 3, the detection range remains at 200 nm, but the target speed is increased to 15 kt. This reduces the available search time. Each surveillance ship patrols a length of 53 nm, reversing direction every 13.3 hours. A total of 2.3 ships would be required on station. For year-round surveillance, a fleet of 5.3 ships would be required.

d. Example 4 is identical to Example 3, except that the required detection probability is increased to 80%, which requires a search density of 1.6. Force levels therefore increase by a factor of 1.6, leading to a requirement for 3.6 ships on station, and a total fleet size of 8.4 ships.

TABLE I
Examples of Ship Barrier Calculations

	Example 1	Example 2	Example 3	Example 4
Input				
P _D :	0.64	0.64	0.64	0.80
L ₀ (nm):	1000	1000	1000	1000
R _X (nm):	400	200	200	200
R _Y (nm):	400	200	200	200
k _X :	2.00	2.00	2.00	2.00
k _Y :	2.00	2.00	2.00	2.00
V _S (kt):	4	4	4	4
V _T (kt):	10	10	15	15
T _M (hr):	336	336	336	336
ETF:	0.85	0.85	0.85	0.85
APF:	2.00	2.00	2.00	2.00
Intermediate Calculations				
Φ	1.0	1.0	1.0	1.6
W _X (nm):	800	400	400	400
W _Y (nm):	800	400	400	400
W ₀ (nm):	800	400	400	400
A (sq.nm):	800,000	400,000	400,000	400,000
A ₀ (sq.nm):	640,000	160,000	160,000	160,000
T _{OS} (hr):	286	286	286	286
T _C (hr):	40.0	20.0	13.3	13.3
L (nm):	160	80	53	53
A _T (sq.nm):	768,000	192,000	181,333	181,333
RTF:	1.00	1.00	1.00	1.00
N _{OS} :	1.1	2.1	2.3	3.6
FPF:	2.4	2.4	2.4	2.4
Total Force Level				
N _{FL} :	2.5	5.0	5.3	8.4

Barrier Search Model for Fixed Surveillance Platforms

22. Fixed Platform Barrier Model as a Degenerate Case of the Ship Barrier Search Model. The case of a barrier of fixed sensors may be regarded as a degenerate case of the ship barrier model. The same equations apply, but with searcher speed set to zero, and effective time on station and availability planning factors set to unity:

$$V_S = 0 \quad [24]$$

$$ETF = 1 \quad [25]$$

$$APF = 1 \quad [26]$$

The total mission time T_M should be input as a large number, e.g. 30 daysx24hr = 720 hr., to ensure that the required time on station factor is also unity:

$$RTF = 1 \quad [27]$$

The sweep width constants k_X and k_Y will depend on the shape of the sensor detection areas and their orientation with respect to the barrier. It will be shown further on that the force level results depend only on the value of W_X , or the coverage parallel to the barrier, and are insensitive to the value of W_Y , or the extent of the coverage normal to the barrier.

23. Examples for Fixed Underwater Arrays. For an underwater array, a schematic diagram of a typical detection area is shown in Figure 7. The detection range downslope (seaward) is given by R_1 . The detection range upslope (landward) is given by R_2 . The detection area is represented as a circle with range R equal to the average of the downslope and upslope ranges, and with the centre of the circle shifted seaward by $0.5(R_1 - R_2)$ as shown:

$$R = (R_1 + R_2) / 2 \quad [28]$$

The sweep width constants in this case assume that the circle is equivalent to a rectangle with the same maximum dimensions:

$$k_X = 2 \quad [7]$$

$$k_Y = 2 \quad [8]$$

Figure 8 shows a schematic representation of fixed underwater arrays arranged as barriers. Table II shows four examples of barrier calculations, in a spreadsheet format, for fixed underwater arrays. The required barrier length is fixed at 1000nm. In Example 1, the detection range is 400nm. This is reduced to 200nm in Example 2. Note in comparing Examples 2 and 3, that the results for fixed sensors are insensitive to changes in target speed, if all other parameters remain unchanged. In Example 4, the detection probability is increased to 80% resulting in a corresponding increase in required force levels.

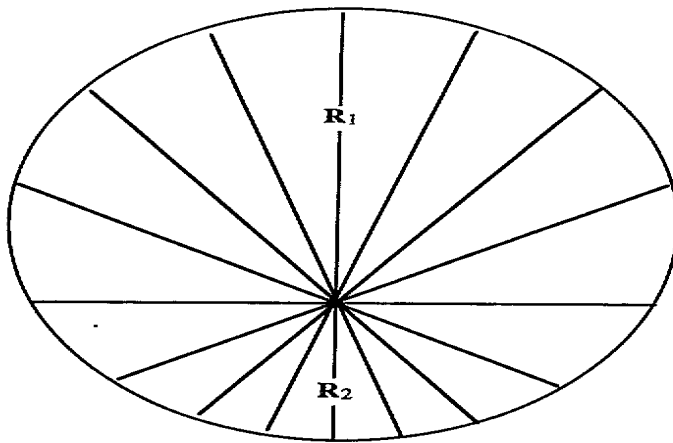


Fig. 7. Simple model of fixed underwater array detection area

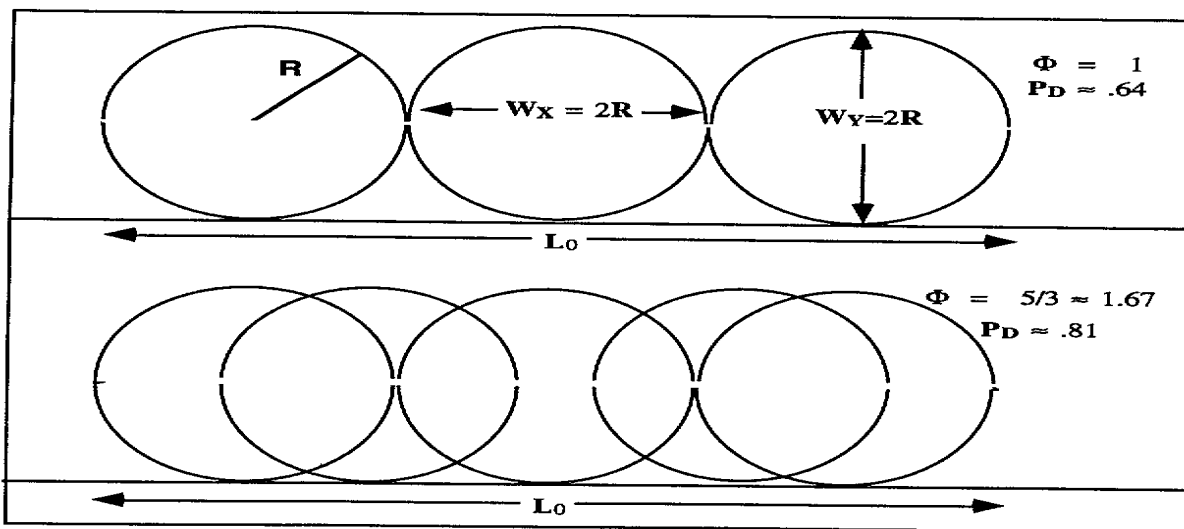


Fig. 8. Schematic representation of fixed underwater array barriers

TABLE II
Examples of Underwater Array Barrier Calculations

	Example 1	Example 2	Example 3	Example 4
Input				
P _D :	0.64	0.64	0.64	0.80
L ₀ (nm):	1000	1000	1000	1000
R _X (nm):	400	200	200	200
R _Y (nm):	400	200	200	200
k _X :	2.00	2.00	2.00	2.00
k _Y :	2.00	2.00	2.00	2.00
V _S (kt):	0	0	0	0
V _T (kt):	10	10	15	15
T _M (hr):	720	720	720	720
ETF:	1.0	1.0	1.0	1.0
APF:	1.0	1.0	1.0	1.0
Intermediate Calculations				
Φ	1.0	1.0	1.0	1.6
W _X (nm):	800	400	400	400
W _Y (nm):	800	400	400	400
W ₀ (nm):	800	400	400	400
A (sq.nm):	800,000	400,000	400,000	400,000
A ₀ (sq.nm):	640,000	160,000	160,000	160,000
T _{OS} (hr):	720	720	720	720
T _C (hr):	40.0	20.0	13.3	13.3
L (nm):	0	0	0	0
A _T (sq.nm):	640,000	160,000	160,000	160,000
RTF:	1.00	1.00	1.00	1.00
N _{OS} :	1.3	2.6	2.6	4.0
FPF:	1	1	1	1
Total Force Level				
N _{FL} :	1.3	2.6	2.6	4.0

24. Examples for Land-Based High Frequency Surface Wave Radar. In these examples, the High Frequency Surface Wave Radar (HFSWR) of interest is a land-based system, with the axis of maximum sensitivity pointed seaward. The field of view is limited to $\pm 60^\circ$ from the main axis, as shown in Figure 9. The sweep width constants are adjusted to reflect the shape of the coverage area for this case:

$$k_x = 2 \sin(60^\circ) \approx 1.73 \quad [7a]$$

$$k_y = 1 \quad [8a]$$

Table III contains four examples of barrier calculations, in a spreadsheet format, for land-based HFSWR detection of surface vessels. In Example 1, the detection range is 250nm, decreasing to 125nm in Example 2. In Example 3, the target speed is increased from 10 to 15 kt, but the model results are insensitive to this parameter, if all other parameters are unchanged. In Example 4, the detection probability is increased to 80%, requiring a search density of 1.6 and a corresponding increase in force levels.

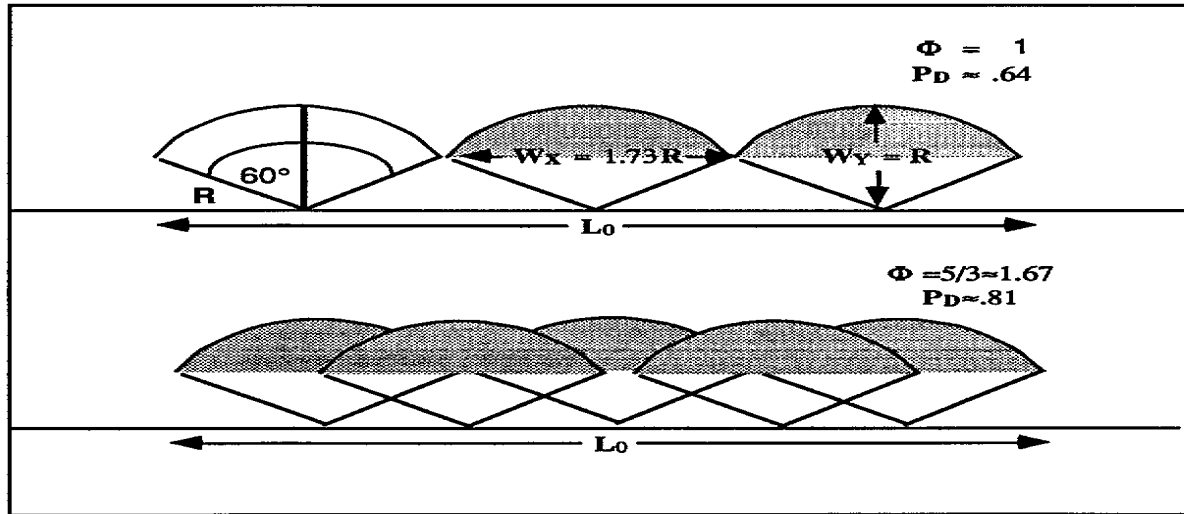


Fig. 9. Schematic representation of land-based HFSWR barriers

TABLE III
Examples of Land-Based HF Radar Calculations

	Example 1	Example 2	Example 3	Example 4
Input				
P _D :	0.64	0.64	0.64	0.80
L ₀ (nm):	1000	1000	1000	1000
R _X (nm):	250	125	125	125
R _Y (nm):	250	125	125	125
k _X :	1.73	1.73	1.73	1.73
k _Y :	1.00	1.00	1.00	1.00
V _S (kt):	0	0	0	0
V _T (kt):	10	10	15	15
T _M (hr):	720	720	720	720
ETF:	1.0	1.0	1.0	1.0
APF:	1.0	1.0	1.0	1.0
Intermediate Calculations				
Φ	1.0	1.0	1.0	1.6
W _X (nm):	433	216	216	216
W _Y (nm):	250	125	125	125
W ₀ (nm):	250	125	125	125
A (sq.nm):	250,000	125,000	125,000	125,000
A ₀ (sq.nm):	108,125	27,031	27,031	27,031
T _{OS} (hr):	720	720	720	720
T _C (hr):	12.5	6.3	4.2	4.2
L (nm):	0	0	0	0
A _T (sq.nm):	108,125	27,031	27,031	27,031
RTF:	1.0	1.0	1.0	1.0
Nos:	2.4	4.7	4.7	7.4
FPF:	1	1	1	1
Total Force Level				
N _{FL} :	2.4	4.7	4.7	7.4

25. **Examples for Land-Based Microwave Radar.** Land-based microwave radars can provide line-of-sight coverage and are of most interest for inner zone barriers. The Vessel Traffic Services Radars (VTSR) operated by the Coast Guard are microwave radars designed for detection of surface vessels. The field of view permits full coverage $\pm 90^\circ$ of the seaward directions as shown in Figure 10, assuming a straight coastline with no intervening hills. The sweep width constants are adjusted to reflect the shape of the coverage area:

$$k_x = 2 \quad [7b]$$

$$k_y = 1. \quad [8b]$$

Table IV contains four examples of barrier calculations, in a spreadsheet format, for land-based microwave radars. In Example 1, the detection range is 30nm, decreasing to 15nm in Example 2. In Example 3, target speed is reduced from 15 kt to 10 kt, but results are insensitive to changes in target speed if all other parameters remain unchanged. In Example 4, the detection probability is increased to 80%, requiring a search density of 1.6, and a corresponding increase in required force levels.

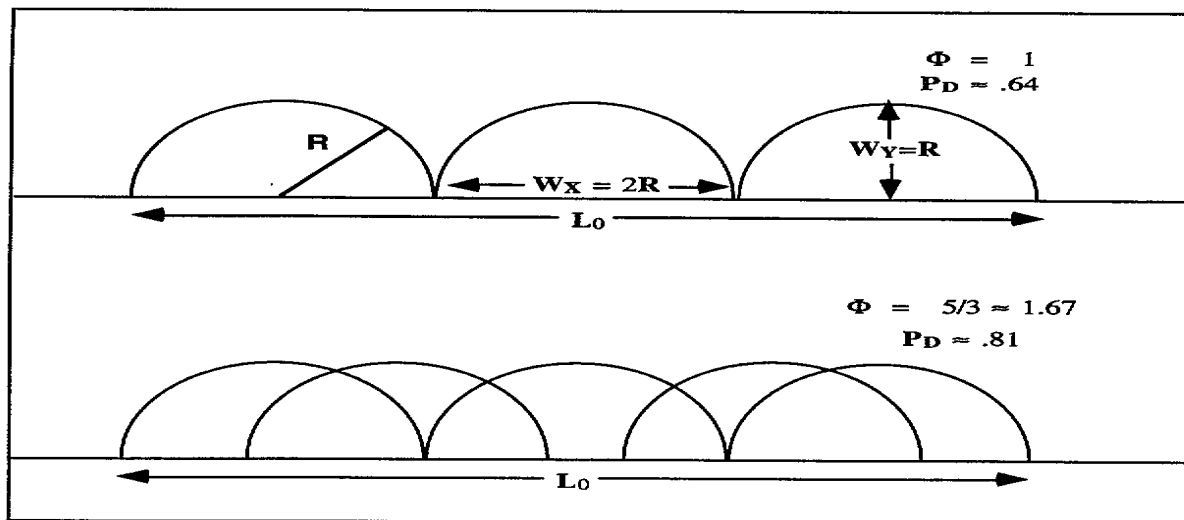


Fig. 10. Schematic representation of land-based microwave radar barriers

TABLE IV
Examples of Land-Based Microwave Radar Calculations

	Example 1	Example 2	Example 3	Example 4
Input				
P _D :	0.64	0.64	0.64	0.80
L ₀ (nm):	1000	1000	1000	1000
R _X (nm):	30	15	15	15
R _Y (nm):	30	15	15	15
k _X :	2.00	2.00	2.00	2.00
k _Y :	1.00	1.00	1.00	1.00
V _S (kt):	0	0	0	0
V _T (kt):	10	10	15	15
T _M (hr):	720	720	720	720
ETF:	1.0	1.0	1.0	1.0
APF:	1.0	1.0	1.0	1.0
Intermediate Calculations				
Φ	1.0	1.0	1.0	1.6
W _X (nm):	60	30	30	30
W _Y (nm):	30	15	15	15
W ₀ (nm):	30	15	15	15
A (sq.nm):	30,000	15,000	15,000	15,000
A ₀ (sq.nm):	1,800	450	450	450
T _{OS} (hr):	720	720	720	720
T _C (hr):	1.5	0.8	0.5	0.5
L (nm):	0	0	0	0
A _T (sq.nm):	1,800	450	450	450
RTF:	1.0	1.0	1.0	1.0
N _{OS} :	17.0	34.1	34.1	53.6
FPF:	1	1	1	1
Total Force Level				
N _{FL} :	17.0	34.1	34.1	53.6

Simplified Formulas for Fixed Barriers

26. Simplified Equations. The equations for the ship barrier model can be simplified considerably for the case of stationary search sensors. Equations [24] to [27] may be substituted in the appropriate equations to obtain a simplified version of equation [23]:

$$\begin{aligned} N_{FL} = N_{OS} = N &= \Phi \cdot A / A_T \\ &= \Phi \cdot L_0 / W_X \end{aligned} \quad [23a]$$

Conversely, if the number of sensors N_{OS} is given, then the search density is obtained by inverting equation [23a] to obtain:

$$\Phi = N_{OS} \cdot W_X / L_0 \quad [2a]$$

27. Measure of Effectiveness for a mix of barrier systems. During the course of Project 21279-13, it was necessary to develop a method for quantifying the performance of existing and proposed systems of fixed sensors without full disclosure of sensor locations, in particular underwater array locations. The systems consisted of various mixes of sensor types. Equation [2a] indicates that search density for such systems could be given as the ratio of total linear barrier coverage to the desired barrier length. The associated "detection probability" would be only an approximate measure of effectiveness, since the distribution of search effort over the barrier might not be uniform, depending on the exact locations of the fixed sensors along the coast, and their distances away from the coast. It would nevertheless provide a first-order estimate of existing capabilities, and assist in explaining the impact of adding or subtracting sensors. To arrive at the formal measure of effectiveness, it is necessary to introduce a few additional definitions. L_i is defined as the coverage of the i -th sensor parallel to the coastal barrier, i.e. its sweep width W_{Xi} along the coast, and L_S is defined as the sum of all sweep widths:

$$L_i = W_{Xi} \quad [29]$$

$$L_S = \sum L_i \quad [30]$$

The effective barrier depth W_0 is taken to be the minimum sweep width normal to the barrier:

$$W_0 = \min (W_{Yi}) \quad [12a]$$

Then the total area A of the barrier is defined as before, where L_0 is the length of coastline to be defended:

$$A = L_0 \cdot W_0 \quad [4]$$

The area covered by the i -th sensor is given by the product of sensor coverage along the barrier, and the effective barrier depth:

$$\begin{aligned} A_{Ti} &= W_{Xi} \cdot W_0 \\ &= L_i \cdot W_0 \end{aligned} \quad [31]$$

The total area coverage of all sensors is assumed to be the sum of their individual contributions, although this coverage might be nonuniform:

$$\begin{aligned} A_T &\approx \sum A_{Ti} \\ &\approx W_0 \cdot \sum L_i \\ &\approx W_0 \cdot L_S \end{aligned} \quad [11a]$$

This results in the following simplified expression for approximate search density, as the ratio of total linear coverage to the desired barrier length:

$$\begin{aligned} \Phi &\approx A_T / A \\ &\approx L_S / L_0 \end{aligned} \quad [2b]$$

However, the fixed sensors may not be uniformly distributed along the barrier, and the barrier may no longer be a straight line, particularly if parts of the barrier are at different distances from the coastline. In such cases, the random search formula for detection probability may be optimistic. However, the analogous formula for P_D can be used as an approximate first-order measure of effectiveness, E :

$$\begin{aligned} E &= 1 - e^{-\Phi} \\ &= 1 - e^{-(L_S / L_0)} \end{aligned} \quad [32]$$

28. Example of Barrier Effectiveness. Table V contains an example of an effectiveness calculation for a mixed barrier. The barrier consists of five land-based microwave radars, two land-based HFSWR, and two fixed underwater arrays. The length of coastline to be defended is 2000nm. In this example, the ratio L_S / L_0 is calculated to be 0.84, and the resulting effectiveness is 0.57.

TABLE V
 Examples of Effectiveness Calculation for a Given Mixed Barrier

	Land-based Microwave Radars	Land-based HF Surface- Wave Radars	Fixed Underwater Arrays	Mixed Barrier
Input				
L ₀ (nm):				2000
V _T (kt):				15
N units:	5	2	2	
R _X (nm):	25	125	250	
R _Y (nm):	25	125	250	
k _X :	2.00	1.73	2.00	
k _Y :	1.00	1.00	2.00	
Intermediate Calculations				
W _X (nm):	50	216	500	
W _Y (nm):	25	125	500	
W ₀ (nm):				25
A ₀ (sq.nm):				50,000
T _C (hr):				0.8
L _S (nm):	250	433	1000	1683
L _S / L ₀ :	0.13	0.22	0.50	0.84
Approximate Effectiveness of Barrier				
E:	0.12	0.19	0.39	0.57

Barrier Search Model for MPA

29. Special Characteristics of MPA. The ship barrier model can be used for MPA calculations, but in practice the distinctive characteristics of MPA are better adapted to a variation of the ship barrier tactic. An MPA has shorter mission duration, lower percentage of time on station, and possible preference for a schedule which may be matched to day or night flying as required. The ratio of search speed to target speed is also much larger. In the case of MPA, best practical use can be made of MPA resources by searching a barrier which is as deep and wide as possible, so that the area remains sanitized for a longer period of time, and flights can be made less frequently. A variant of the barrier model is presented for MPA, in which the width and depth of the barrier are equal, giving a square search area for a single MPA. This deep barrier tactic is normally suitable when searcher speed is much greater than target speed, or when the initial search area A_0 is large.

30. MPA Search Pattern. Figure 11 shows the geometry of a deep surveillance barrier of length L_0 and depth B nm. A single MPA is assigned to search part of this barrier. The MPA search area is a square with length and depth equal to B . The value of B will be chosen to optimize MPA performance. A square area gives the maximum area for a fixed search time, and meets the MPA preference for flight segments which are straight lines [Ref. 11]. Several MPA would be required to maintain the complete barrier. A is the area of the complete barrier, while A_{MPA} is the area assigned to one MPA:

$$A = L_0 \cdot B \quad [33]$$

$$A_{MPA} = B^2 \quad [34]$$

31. Swept Area for MPA. The MPA sweep widths along the search path (W_X) and normal to the search path (W_Y) are defined as before, where k_X and k_Y are constants chosen to represent the shape of the sensor detection area:

$$W_X = k_X \cdot R_X \quad [5]$$

$$W_Y = k_Y \cdot R_Y \quad [6]$$

For the case of a sensor detection area which is nearly circular, such as a Maritime Surface Surveillance Radar (MSSR), the constants may be chosen very approximately to represent a rectangle with the same maximum dimensions. [Or, alternatively, $k_X = \pi/2$ and $k_Y = 2$ would represent a circle exactly]:

$$k_X = 2 \quad [7]$$

$$k_Y = 2 \quad [8]$$

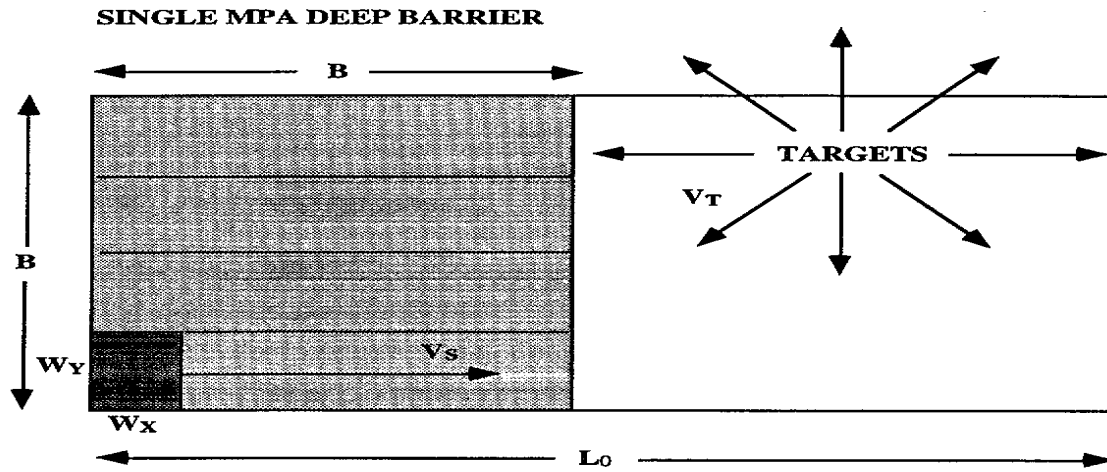


Fig. 11. Geometry of a deep surveillance barrier for a single MPA

The initial area coverage upon arrival at the search area is given as the product of W_x and W_y :

$$A_0 = W_x \cdot W_y \quad [10]$$

while the searcher speed V_s , path length L and swept area A_T are defined as before, and depend on the search cutoff time T_C :

$$L = V_s \cdot T_C \quad [9]$$

$$A_T = A_0 + W_y \cdot L \quad [11]$$

The search cutoff time T_C remains to be specified, and will be derived further on to obtain maximum MPA performance.

32. Dimensions of MPA Search Area. From equation [3], the required search density Φ depends on the specified detection probability:

$$\Phi = -\ln(1-P_D) \quad [3]$$

For a single MPA, the search density in the assigned area is defined as follows for the deep barrier tactic:

$$\begin{aligned} \Phi &= A_T / A_{MPA} \\ &= A_T / B^2 \end{aligned} \quad [35]$$

Equation [35] is then inverted to solve for the appropriate dimension of the MPA search area:

$$B = (A_T / \Phi)^{1/2} \quad [36]$$

It should be noted that A_T depends upon the optimal search cutoff time, T_C , which remains to be specified. A number of calculations must be performed first, including calculation of MPA time on station.

33. Effective Time on Station Factor for MPA. The MPA is assumed to transit to the patrol area at speed V_0 kt, and then to patrol at speed V_S kt. The distance D in nm is the average of the distances to and from the search area. (In some cases the aircraft may depart from one base and land at another, or enter and leave the search area at different points.) The total two-way transit time T_T is given by:

$$T_T = 2D / V_0 \quad [37]$$

The total fuel payload is G lb. The fuel availability ratio FAR allows for reserve fuel and may also be used to limit mission length based on personnel considerations. If the fuel consumption rate during transit is FCR_0 lb/hr then the amount of fuel available for use on station G_{OS} in lb is given as follows:

$$G_{OS} = FAR \cdot G - T_T \cdot FCR_0 \quad [38]$$

The MPA time on station, T_{OS} , in hr is then calculated based on the fuel consumption rate FCR_S in lb/hr at the patrol speed V_S :

$$T_{OS} = \max(0, G_{OS} / FCR_S) \quad [39]$$

The total mission duration T_M in hr is the sum of the two-way transit time plus the time on station:

$$T_M = T_T + T_{OS} \quad [40]$$

The effective time on station factor, ETF, is the ratio of the time on station to the total mission duration:

$$ETF = T_{OS} / T_M \quad [41]$$

Figure 12 shows the typical MPA effective time on station factor as a function of distance to the patrol area, using parameters for a CP140A Arcturus aircraft [Ref. 11]. The parameters chosen correspond to high altitude flying (26,000 to 28,000 ft) to give best area coverage for initial detection of surface vessels. At a distance of 800nm, the effective time on station factor is approximately 61% .

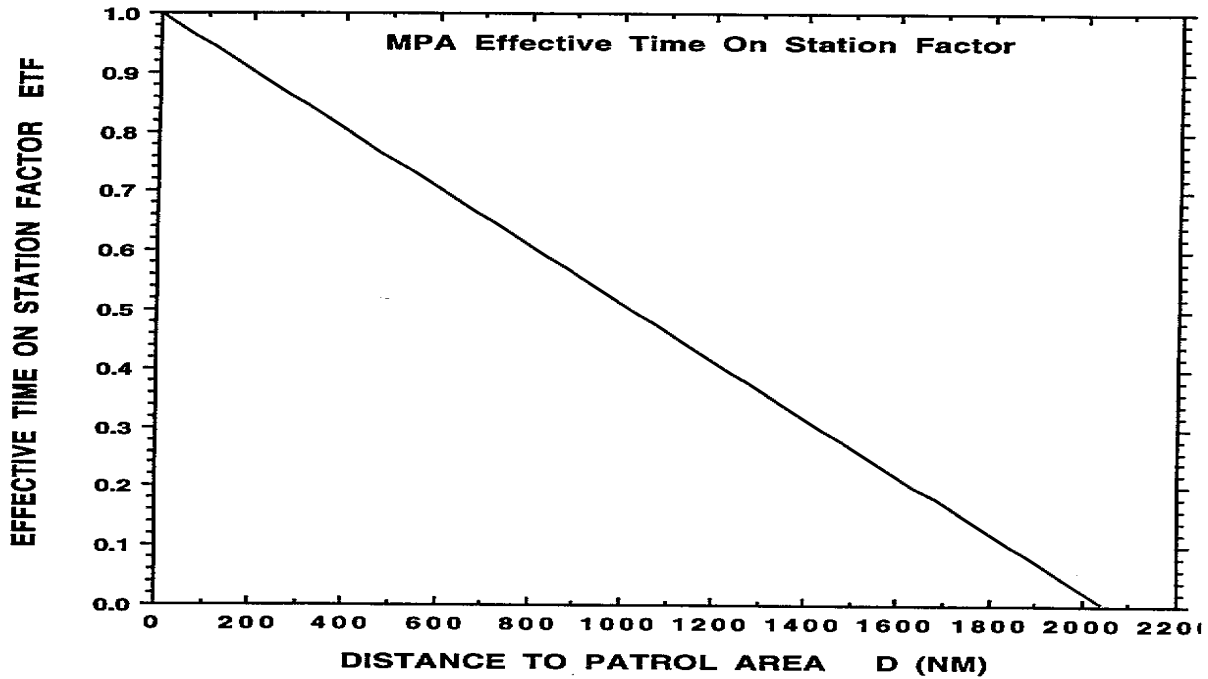


Fig. 12. Typical MPA effective time on station factor, ETF, as a function of distance to the patrol area. [G=60000 lb , FAR=.80, FCR₀ = FCR_S = 4100 lb/hr, V₀ = V_S = 350kt]

34. Constraints on MPA Search Time. The search cutoff time T_C is constrained by the available time on station. As for the ship barrier model, we will also require that the distance D_T travelled by a target during the search:

$$D_T = V_T \cdot T_C \quad [42]$$

is less than one-half the barrier depth B . Selection of the halfway point allows for a minimum of two data points, obtained over two aircraft missions, on a worst-case target which transits the barrier at right angles. Two points, if properly correlated, would be the minimum to establish a target track, and would assist in the reduction of false alarms. Selection of a worst-case transiting target also ensures the eventual steady-state coverage of all transiting targets within the patrol area. (See discussion of the ship barrier time constraint in paragraph 15.) The MPA may also lose some time in the investigation of false targets, if it descends to lower altitude or otherwise deviates from the planned search pattern. To determine the limit on MPA search time, it is therefore necessary to solve for the time T_{Cmax} for which $D_T=0.5B$:

$$\begin{aligned} V_T \cdot T_{Cmax} &= 0.5 B \\ &= 0.5 (A_T / \Phi)^{1/2} . \end{aligned} \quad [43]$$

Squaring equation [43], and substituting for A_T evaluated at $T_C = T_{Cmax}$ gives:

$$\begin{aligned} (V_T \cdot T_{Cmax})^2 &= 0.25 (A_T / \Phi) \\ &= 0.25 (A_0 + W_Y \cdot L) / \Phi \\ &= 0.25 (A_0 + W_Y \cdot V_S \cdot T_{Cmax}) / \Phi . \end{aligned} \quad [43']$$

The terms in equation [43'] are rearranged to obtain:

$$4 \Phi (V_T \cdot T_{Cmax})^2 - (A_0 + W_Y \cdot V_S \cdot T_{Cmax}) = 0 . \quad [43'']$$

Solving for the positive root of this quadratic equation gives the solution:

$$T_{Cmax} = [W_Y \cdot V_S + (W_Y^2 \cdot V_S^2 + 16 \Phi \cdot A_0 \cdot V_T^2)^{0.5}] / [8 \Phi \cdot V_T^2] . \quad [44]$$

The search cutoff time T_C is then the minimum of T_{OS} and T_{Cmax} :

$$T_C = \min (T_{OS} , T_{Cmax}) . \quad [45]$$

35. MPA Required Time On Station Factor. The MPA required time on station factor, RTF, is the ratio of the distance D_T travelled by the target during the search time T_C , to one-half the barrier depth:

$$RTF = D_T / (0.5B) \leq 1 . \quad [46]$$

If the target is proceeding very slowly, and the MPA can search a large area while on station, then the MPA would be required on station a small fraction of the time.

36. Calculation of Required On Station Force Level for MPA. The number of MPA search stations, N , depends on the ratio of the total search area, to the area assigned to one MPA:

$$\begin{aligned} N &= (A / A_{MPA}) \\ &= (L_0 \cdot B) / (B^2) \\ &= (L_0 / B) \end{aligned} \quad [47]$$

The actual number of fully employed MPA on station, N_{OS} , depends on the required time on station factor RTF:

$$N_{OS} = N \cdot RTF \quad [48]$$

37. Availability Planning Factor for MPA. The availability planning factor APF is the number of aircraft required in the fleet in order to guarantee that one aircraft is ready to fly. This may be calculated as follows, given the maximum yearly flying rate in hr, YFR, for a single aircraft:

$$APF = (365) \cdot (24) / YFR \quad [49]$$

For example, if YFR=1200 hours, then APF=7.3 .

38. Force Planning Factor. The final force planning factor, FPF is the number of aircraft required in the fleet in order to maintain one aircraft station at a specified distance D . As in the ship barrier model, FPF depends on the availability planning factor and the effective time on station factor as follows:

$$FPF = APF / ETF \quad [22]$$

For example, if APF=7.3 and ETF=0.61 at a range of 800nm, then FPF=12 .

39. Calculation of Required Total Force Level. The number of aircraft required in the fleet, to maintain the specified barrier at the specified probability level, depends on the required number on station and the force planning factor:

$$N_{FL} = N_{OS} \cdot FPF \quad [23]$$

40. Examples for MPA. Table VI contains four examples of barrier calculations for MPA. The required barrier is 1000nm in length, at an average distance of 800nm from the aircraft base. The Effective Time on Station Factor is 0.61 for all four examples. Mission time is 11.7 hours, with 7.1 hours spent on station. Detection ranges and target speeds are varied arbitrarily to show the effect of these parameters on force level calculations. Different detection ranges and target speeds could represent radar detection of different classes of surface vessel.

a. In Example 1, the detection probability of 64% requires a search density of 1.0. The MPA search speed is 350kt, with a detection range of 150nm, while the target speed is 10kt. The optimal search area for a single MPA is calculated to be 906nm by 906nm. The Required Time on Station Factor is 0.16 (corresponding to a revisit cycle of approximately 44 hours). To provide surveillance of a 1000nm barrier, 0.2 aircraft would be required on station. To provide year-round 24-hour surveillance of this barrier, a Force Planning Factor of 12 is applied to obtain a fleet size of 2.1 aircraft.

b. In Example 2, the detection range is decreased to 75nm. The optimal search area for a single MPA is reduced to 623nm by 623nm, and the required time on station factor is increased to 0.23 (corresponding to a revisit cycle of approximately 31 hours). The required number of aircraft on station is increased to 0.4, and the total fleet size is increased to 4.4.

c. In Example 3, the detection range remains at 75nm, while the target speed is increased from 10 kt to 15 kt. The optimal search area for a single MPA is unchanged, but the required time on station factor is increased to 0.34 (corresponding to a revisit cycle of approximately 21 hours). The required number of aircraft on station is increased to 0.6 and the total fleet size is increased to 6.6.

d. Example 4 is the same as Example 3, except that the detection probability is increased from 64% to 80%, giving a required search density of 1.6. The optimal search area for a single MPA is reduced to 497nm by 497nm. The required time on station factor is increased to 0.43 (a revisit cycle of approximately 16 hours). The resulting force levels are increased by a factor of 1.6 compared to Example 3. The required number of aircraft on station is 0.9, and the total fleet size is 10.4 .

TABLE VI: Examples of MPA Barrier Calculations

	Example 1	Example 2	Example 3	Example 4
Input				
P _D :	0.64	0.64	0.64	0.80
L ₀ (nm):	1000	1000	1000	1000
R _X (nm):	150	75	75	75
R _Y (nm):	150	75	75	75
k _X :	2.00	2.00	2.00	2.00
k _Y :	2.00	2.00	2.00	2.00
V ₀ (kt):	350	350	350	350
V _S (kt):	350	350	350	350
D (nm):	800	800	800	800
G (lb):	60,000	60,000	60,000	60,000
FAR:	0.80	0.80	0.80	0.80
FCR ₀ (lb/hr):	4100	4100	4100	4100
FCR _S (lb/hr):	4100	4100	4100	4100
V _T (kt):	10	10	15	15
YFR (hr):	1200	1200	1200	1200
Intermediate Calculations				
Φ	1.0	1.0	1.0	1.6
W _X (nm):	300	150	150	150
W _Y (nm):	300	150	150	150
A ₀ (sq.nm):	90,000	22,500	22,500	22,500
T _T (hr):	4.6	4.6	4.6	4.6
G _{0S} (lb):	29,257	29,257	29,257	29,257
T _{0S} (hr):	286	286	286	286
T _M (hr):	11.7	11.7	11.7	11.7
ETF:	0.61	0.61	0.61	0.61
APF:	7.3	7.3	7.3	7.3
T _C (hr):	7.1	7.1	7.1	7.1
L (nm):	2498	2498	2498	2498
A _T (sq.nm):	839,268	397,134	397,134	397,134
B (nm):	906	623	623	497
D _T (nm):	71	71	107	107
RTF:	0.16	0.23	0.34	0.43
N _{0S} :	0.2	0.4	0.6	0.9
FPF:	12.0	12.0	12.0	12.0
Total Force Level				
N _{FL} :	2.1	4.4	6.6	10.4

Discussion

41. Limitations of Model. The Koopman random search model is an approximation to real world search problems, but over the many years since World War II, it has proved to be a powerful analysis and predictive tool. In applying this model to the barrier problem, the original underlying assumption of target containment in a fixed area has essentially been replaced by a constraint on the rate of appearance of new targets and the disappearance of old targets. This constraint takes the form of a time limit for the search pattern.

42. Cumulative Detection Ranges. The barrier model also introduces a variation on the standard Koopman formula by defining a sweep area which consists of an initial non-zero look area, in addition to the sweep area resulting from motion of the searcher. This allows the model to be used for fixed sensors as well as mobile sensors with large sensor detection ranges. The model results for *fixed sensors* are independent of target speed and observation time, if all other parameters remain unchanged, which means that no additional credit is given for possible independent observations of a target which spends twice as much time in the detection area. This would correspond to observations which are highly correlated in the time domain. If a very slow-moving target spends several days transiting an area, it is possible that environmental parameters could fluctuate, providing changes in detection opportunity. In extreme cases, it may be necessary to change the values of k_x and k_y to reflect a change in cumulative detection ranges corresponding to a change in detection statistics for long observation periods. The effects of long-term integration in range and space must be taken into account by the model user when preparing the input data. In the case of all searchers, both fixed and mobile, it is assumed that the input sensor detection ranges represent appropriate cumulative detection ranges for the cases of interest.

Summary

43. This report documents the barrier surveillance model used in DMOR Project 21279-13 "Atlantic/ Pacific Undersea and Surface Surveillance Options". The model is an adaptation of the classic Koopman random search model. The basic model computes the force level requirements for surveillance cueing barriers consisting of patrol ships or fixed sensors. A special variant of the model is presented for Maritime Patrol Aircraft.

44. The basic surveillance model and the MPA variant are relatively simple and straightforward. They are easily implemented in spread sheet calculations, as shown in the tables contained in this report. The time required by the author to create these typical spreadsheets was on the order of 15 minutes.

References

1. 3240-10 (DGMD) dated 15 September 1992; DMFD Staff Report: "Canadian Surveillance Strategy: The Maritime Surface and Air Surveillance Requirements (U)"; SECRET.
2. The Maritime Defence Strategy, May 1992, NDHQ.
3. Koopman, Bernard Osgood; Search and Screening; Pergammon Press, New York, 1980.
4. Reference 3, pp 71-74
5. Reference 3, page 79 and Ch. 8
6. Reference 3, bottom of page 72
7. Reference 3, page 65, equation 27
8. Reference 3, pp 21, 73, 85
9. Reference 3, page 39
10. 11900-55/DMOR (DMES 3-4-4); 17 January 1994.
11. Informal communication, Maj. J. H. Jerry, DARMR 2-5; February 1994.
12. Gradshteyn, I. S. and Ryzhik, I. M.; Table of Integrals, Series and Products; Academic Press, New York, 1980.

Annex A.

Optimal barrier width for ship barrier tactic.

1. In this annex it is shown that the optimal width of a ship surveillance barrier is equal to the sweep width of the search platform, provided no secondary credit is given for barrier width, and the only objective is to achieve an initial alert. Equations [12] to [20] are revisited, but allowing an arbitrary barrier width $W_0 = kW_Y$, where $k \geq 1$.

2. As before, the effective searcher sweep widths along and across the searcher path are given respectively by:

$$W_X = k_X \cdot R_X \quad [A1]$$

$$W_Y = k_Y \cdot R_Y \quad [A2]$$

3. The width of the surveillance barrier is allowed to vary, as defined by the non-zero scaling factor, k:

$$W_0 = k \cdot W_Y \quad [A3]$$

4. Equation [A3] is substituted into equation [4] to obtain the target area:

$$\begin{aligned} A &= L_0 \cdot W_0 \\ &= L_0 \cdot k \cdot W_Y \end{aligned} \quad [A4]$$

5. As before, from equation [10] the initial sweep area is given by:

$$A_0 = W_X \cdot W_Y \quad [A5]$$

6. Equation [A3] is substituted into equation [15] to obtain the search pattern cutoff time:

$$\begin{aligned} T_C &= \min (0.5 W_0 / V_T, T_{OS}) \\ &= \min (0.5 k W_Y / V_T, T_{OS}) \end{aligned} \quad [A6]$$

7. Because of the typical long time on station (T_{OS}) capability of ships, the search pattern cutoff time T_C is usually less than the ship time on station T_{OS} . The search pattern is continuous and repetitive and therefore the required time on station factor, RTF, is unity, independent of the value of k , for reasonable values of k :

$$\text{If: } 1 \leq k \leq 2 T_{OS} \cdot V_T / W_Y \quad [A7]$$

then:

$$T_C = 0.5 k W_Y / V_T \quad [A6a]$$

8. In this case, substituting equation [A6a] into equations [16] and [17], one obtains the distance travelled by the target during time interval T_C , and the required time on station factor RTF, as functions of k :

$$\begin{aligned} D_T &= V_T \cdot T_C \\ &= 0.5 k \cdot W_Y \end{aligned} \quad [A8]$$

$$\begin{aligned} \text{RTF} &= D_T / (0.5 W_0) \\ &= (0.5 k W_Y) / (0.5 k W_Y) \\ &= 1 \end{aligned} \quad [A9]$$

9. Further substitution into equations [9] and [11] gives the searcher path length L and swept area A_T during the time interval T_C as functions of k :

$$\begin{aligned} L &= V_S \cdot T_C \\ &= V_S \cdot 0.5 k W_Y / V_T \end{aligned} \quad [A10]$$

$$\begin{aligned} A_T &= A_0 + W_Y \cdot L \\ &= W_X \cdot W_Y + W_Y \cdot V_S \cdot 0.5 k W_Y / V_T \end{aligned} \quad [A11]$$

10. If the problem parameters are sufficiently unusual that the maximum value of k would be less than unity, i.e.:

$$\text{If } k \leq 2 T_{OS} \cdot V_T / W_Y < 1 \quad [A12]$$

then the ship barrier tactic may not be optimal, and consideration should be given to the MPA barrier tactic. An alternative may be to consider a lower effective value of W_Y .

11. Equations [A4], [A8] and [A9] are substituted into equations [19] and [20], and simplified to obtain the revised estimate of the number of units required on station, N_{OS} , as a function of k :

$$N(k) = \Phi \cdot A / A_T \quad [A13]$$

$$\begin{aligned} N_{OS}(k) &= N(k) \cdot RTF \\ &= \Phi \cdot (A / A_T) \cdot RTF \\ &= \Phi \cdot L_0 \cdot k \cdot W_Y / (W_X \cdot W_Y + W_Y \cdot V_S \cdot 0.5 k W_Y / V_T) \cdot RTF \\ &= \Phi \cdot L_0 \cdot k / (W_X + V_S \cdot 0.5 k W_Y / V_T) \cdot RTF \\ &= \Phi \cdot L_0 / ((W_X + V_S \cdot 0.5 W_Y / V_T)) \cdot RTF \cdot H(k) \\ &= N_{OS}(k=1) \cdot H(k) \end{aligned} \quad [A14]$$

where

$$H(k) = k \cdot [(W_X + V_S \cdot 0.5 W_Y / V_T) / (W_X + V_S \cdot 0.5 k W_Y / V_T)] \quad [A15]$$

12. From inspection of the force level factor $H(k)$ given by equation [A15] it is evident that the required number of units on station N_{OS} is independent of k , if $W_X=0$. If $W_X \neq 0$, then N_{OS} increases as k increases. For small values of W_X , there is a very weak increase in N_{OS} as k is increased. When W_X is large, N_{OS} increases linearly with k . The optimal value of k , to minimize the value of N_{OS} , is therefore unity.

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Annex B

Calculation of average speed of encounters for open ocean scenarios

1. This annex presents a variation on the basic surveillance model, which would be of interest in the special case of open ocean transit. In this variant, the searchers and targets move at constant speed in straight lines. There are no maneuvers, no changes in heading or speed by any of the players. This type of scenario might be of interest if the targets were ocean liners and the searcher detection ranges were very long, so that the area under surveillance is quite large. It would also be of interest if the targets were actually confined to a fixed area, but moving in a nonrandom way. The variant can be implemented by computing a revised value for the searcher speed, and substituting this value into the equations for the basic surveillance model. This variant has not been used in DMOR Project 21279-13.

2. For open ocean transit, or a case in which *the searcher and the targets move only in straight lines*, at steady speed with no course changes, the relative average speed of encounters V_E can be expressed analytically [Ref. 9].

3. This annex is used to compute and plot V_E to show that it lies on the interval:

$$\max(V_S, V_T) \leq V_E \leq 1.273 \max(V_S, V_T) . \quad [B1]$$

4. The relevant equations are based on Ref. 9:

$$V_E = (2/\pi) (V_S + V_T) E(\sin \sigma) \quad [B2]$$

$$\sin \sigma = 2 (V_S \cdot V_T)^{0.5} / (V_S + V_T) \leq 1 \quad [B3]$$

where E is the complete elliptical integral of the second kind [Ref.12]:

$$E(\sin \sigma) = \int_0^{\pi/2} [1 - \sin^2 \sigma \sin^2 x]^{1/2} dx \quad [B4]$$

5. To simplify the equations, the following definitions are introduced:

$$V_{\text{MIN}} = \min (V_S, V_T) \quad [\text{B5}]$$

$$V_{\text{MAX}} = \max (V_S, V_T) \quad [\text{B6}]$$

$$u = V_{\text{MIN}} / V_{\text{MAX}} \quad [\text{B7}]$$

$$r = V_E / V_{\text{MAX}} \quad [\text{B8}]$$

6. Using the definitions [B5] to [B8], the original equations can be simplified as follows:

$$\begin{aligned} V_E &= (2/\pi) (V_S + V_T) E(\sin \sigma) \\ &= (2/\pi) V_{\text{MAX}} (1+u) E(\sin \sigma) \\ &= r \cdot V_{\text{MAX}} \end{aligned} \quad [\text{B9}]$$

where

$$r = (2/\pi) (1+u) E(\sin \sigma) \quad [\text{B10}]$$

and

$$\begin{aligned} \sin \sigma &= 2 (V_S \cdot V_T)^{0.5} / (V_S + V_T) \\ &= 2 V_{\text{MAX}} u^{0.5} / (V_{\text{MAX}} (1+u)) \\ &= 2 u^{0.5} / (1+u) \end{aligned} \quad [\text{B11}]$$

$$\sigma = \sin^{-1} [2 u^{0.5} / (1+u)] \quad [\text{B12}]$$

7. Figure B1 shows a plot of r (the ratio of V_E to V_{MAX}) as a function of u (the ratio of V_{MIN} to V_{MAX}). The maximum value of r is $(4/\pi) \approx 1.273$, which occurs when $u=1$, i.e. when the searcher and target speeds are equal. An empirical fit to this curve is given by:

$$r \approx 1 + 0.273 u^2 . \quad [\text{B13}]$$

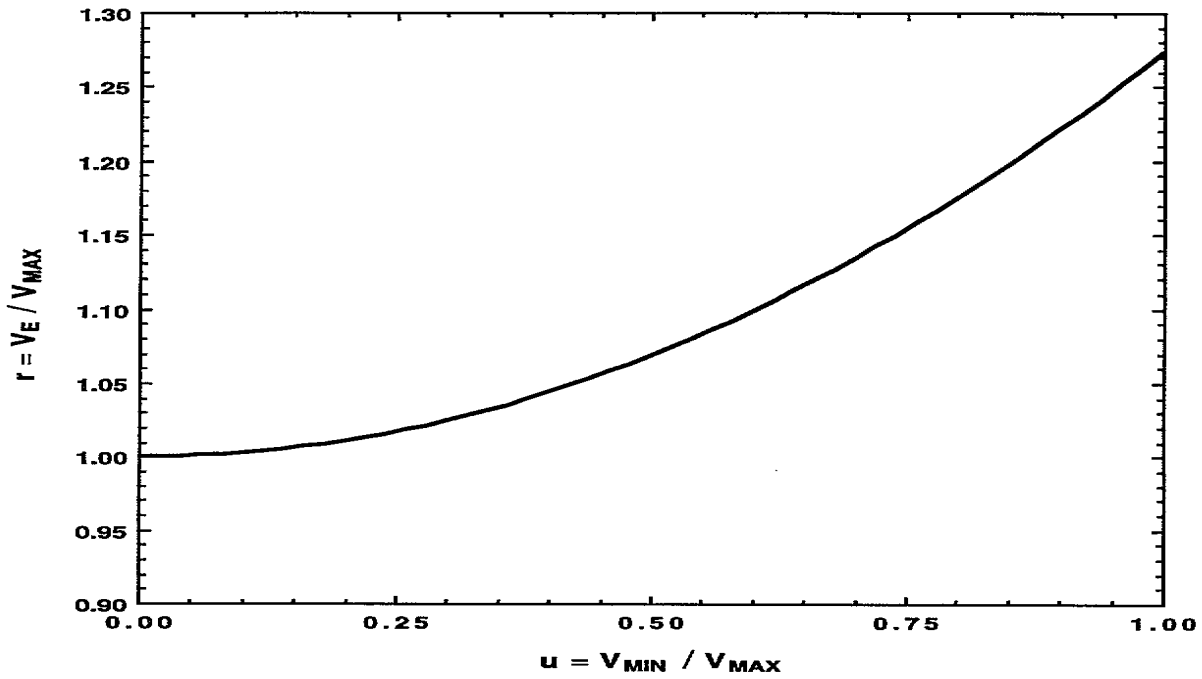


Fig. B1. A plot of the ratio $r = V_E / V_{MAX}$, as a function of the ratio $u = V_{MIN} / V_{MAX}$. The average speed of encounter is equal to the maximum of the searcher and target speeds, if they are significantly different; it approaches 1.273 times the maximum of the two speeds, if the searcher and target speeds are equal.

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This report documents the barrier surveillance model used in DMOR Project 21279-13 "Atlantic/ Pacific Undersea and Surface Surveillance Options". In this model surveillance is limited to the initial detection and cueing function. When very large areas are involved a less costly cueing solution is to provide one or more layered barriers rather than full area coverage. The classic Koopman random search model is adapted to the problem of barrier surveillance and applied to fixed systems as well as mobile search systems. Barrier surveillance is modelled as area search subject to a time constraint which depends on the minimum time for a target to transit the barrier. A special variant of the model is developed to meet the unique requirements of maritime patrol aircraft. The depth of a barrier depends on the capability of the surveillance systems. For some of the systems considered, the barriers may be sufficiently wide that significant area coverage is achieved.

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