Image Cover Sheet

CLASSIFICATION	SYSTEM NUMBER 52503
UNCLASSIFIED	

TITLE

PRHDEF - STRESS AND STABILITY ANALYSIS OF RING STIFFENED SUBMARINE PRESSURE HULLS

System Number:

Patron Number:

Requester:

Notes:

DSIS Use only:

Deliver to: JR

UNLIMITED DISTRIBUTION

National Defence
Research and Development Branch

Défense nationale
Bureau de recherche et développement

TECHNICAL MEMORANDUM 87/213

June 1987

PRHDEF - STRESS AND STABILITY ANALYSIS OF RING STIFFENED SUBMARINE PRESSURE HULLS

N.G. Pegg - D.R. Smith

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

Canada

DEFENCE RESEARCH ESTABLISHMENT ATLANTIC
9 GROVE STREET
P.O. BOX 1012
DARTMOUTH, N.S.
B2Y 3Z7

TELEPHONE
(902) 426-3100

CENTRE DE RECHERCHES POUR LA DÉFENSE ATLANTIQUE
9 GROVE STREET
C.P. 1012
DARTMOUTH, N.É.
B2Y 3Z7

UNLIMITED DISTRIBUTION

National Defence
Research and Development Branch

Défense nationale
Bureau de recherche et développement

PRHDEF - STRESS AND STABILITY ANALYSIS OF RING STIFFENED SUBMARINE PRESSURE HULLS

N.G. Pegg - D.R. Smith

June 1987

Approved by B.F. Peters A/Director/Technology Division

DISTRIBUTION APPROVED BY

A/D/TD

TECHNICAL MEMORANDUM 87/213

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

DEFENSIO PER SCIENTIAM

Canada

Abstract

Preliminary investigation of the structural integrity of a submarine pressure hull can be accomplished by the use of design formulae. Approximate solutions for stress and stability of uniformly stiffened cylinders subject to hydrostatic pressure have been assembled and incorporated in the computer code PRHDEF. The British pressure vessel code, BS5500, and other codes have been used where appropriate. Critical pressures are determined for yielding in the frames and shell, for interframe and overall bifurcation buckling and for collapse of the stiffened shell and endcap. The effect of out of circularity on frame failure is considered and dimension checks for stiffener tripping are made. The background and limitations of the various equations are discussed and results are compared with those obtained using the axisymmetric finite difference program BOSOR4.

The methods described in this report are particularly useful for comparison of various design alternatives on a common basis and for preliminary investigation before more complex and costly finite element or finite difference analyses are undertaken.

Résumé

L'étude préliminaire de la solidité structurale de la coque épaisse d'un sous-marin peut être effectuée à l'aide de formules théoriques. Des solutions approximatives au problème des contraintes et de la stabilité de cylindres renforcés soumis à la pression hydrostatique ont été assemblées et incorporées au code machine PRHDEF. Le code anglais pour vaisseaux sous pression BS5500 et d'autres codes ont été utilisés lorsque appropriés. Les pressions critiques sont déterminées pour ce qui est de la déformation des couples et de la coque, du flambage de bifurcation inter-couples générale ainsi que de l'affaissement de la coque et du capot renforcés. On prend en considération l'effet de la non circularité sur la défaillance des couples et on effectue des vérifications des dimensions pour mesurer le flambage des couples. On examine l'historique et les limites des diverses équations et les résultats sont comparés à ceux obtenus au moyen du programme BOSOR4 aux différences finies axi-symétriques.

Les méthodes décrites dans le présent rapport sont particulièrement utiles pour comparer diverses solutions théoriques à partir d'une base commune et pour faire une étude préliminaire avant d'entreprendre des analyses aux éléments finis ou aux différences finies plus complexes et plus coûteuses.

Contents

List of Figures

List of Tables

Notation

a	mean radius of cylinder
a_e	endcap radius
A	$\frac{a^2 A_f}{\overline{d}^2}$
A_f	area of ring stiffener
b	faying flange width
c	$\frac{\alpha}{L}\{1 - \frac{Pa^3\alpha^2}{2EhL^2}\}^{\frac{1}{2}}$
C_n	cylinder out of circularity, OOC
d	$\frac{\alpha}{L}\{1 + \frac{Pa^3\alpha^2}{2EhL^2}\}^{\frac{1}{2}}$
D	$\frac{Eh^3}{12(1-\nu^2)}$
$\overline{d}$	distance from cylinder axis to frame centroid
d_w	depth of frame web
e	$\frac{cL}{2}$
$\overline{e}$	distance from shell center to frame centroid
E	Young's modulus
f	$\frac{dL}{2}$
G	$\frac{Pa^2(1-\frac{\nu}{2})T}{Eh}$
$\overline{G}$	$-2\frac{[\sinh\frac{\alpha}{2}\cos\frac{\alpha}{2}+\cosh\frac{\alpha}{2}\sin\frac{\alpha}{2}]}{\sinh\alpha+\sin\alpha}$
h	shell thickness
H	$-\frac{4i}{L}(e\sin f\cos f + f\sinh e\cosh e)$
$\overline{H}$	$-2\frac{\{[1+\sqrt{\frac{3\nu^2}{1-\nu^2}}]\sinh\frac{\alpha}{2}\cos\frac{\alpha}{2}+[1-\sqrt{\frac{3\nu^2}{1-\nu^2}}]\cosh\frac{\alpha}{2}\sin\frac{\alpha}{2}]\}}{\sinh\alpha+\sin\alpha}$
h_e	thickness of endcap shell

h_w	thickness of frame web
h_f	thickness of standing flange
i	$\sqrt{-1}$
I_c	combined moment of inertia of stiffener and effective length of shell about axis parallel to cylinder axis
I_z	moment of inertia of stiffener about its symmetric axis
J_o	$\frac{4(e^2+f^2)(e \cosh e \sin f - f \sinh e \cos f)}{L^2(e \sin f \cos f + f \sinh e \cosh e)}$
$J_{\frac{L}{2}}$	$\frac{4(e^2+f^2)(e \sin f \cos f - f \sinh e \cosh e)}{L^2(e \sin f \cos f + f \sinh e \cosh e)}$
L	unsupported length of shell between stiffeners
L_b	length between rigid bulkhead supports
L_e	effective length of shell between stiffeners
L_f	length between stiffeners
n	buckling mode circumferential wave number
N_x	axial load on shell $\frac{Pa}{2}$
OOC	out of circularity of the cylinder
P	external applied pressure
P_B	Bresse overall buckling pressure
P_c	collapse pressure of cylinder
P_d	design depth - corresponds directly to P
P_{ec}	buckling pressure of endcap
P_{eu}	collapse pressure of endcap
P_{ey}	endcap yield pressure
P_i	pressures causing yield in shell, defined in section 6
P_{FY}	pressure causing yield in ideal circular stiffener

P_f	pressure causing yield in stiffener including OOC effect
P_n	Bryant overall buckling pressure
P_M	minimized interframe buckling pressure
P_{M1}	modified interframe buckling pressure
R	residual stress multiplication factor
$\overline{R}$	$\frac{\sinh\alpha-\sin\alpha}{\sinh\alpha+\sin\alpha}$
r_1	radius from cylinder axis to edge of standing flange
$S.F.$	safety factor applied to load
T	$\frac{A_f}{A_f+bh-\frac{a^2h^2W}{6(1-\nu^2)}}$
W	$-\frac{16ef(e^2+f^2)(\sinh^2 e+\sin^2 f)}{L^3(e\sin f\cos f+f\sinh e\cosh e)}$
w_f	thickness of faying flange of stiffener
U_x	$\frac{-4i}{L}(u_1-u_2)$
u_1	$(e\sinh e\cosh f - f\cosh e\sin f)\sinh\frac{2ex}{L}\sin\frac{2fx}{L}$
u_2	$(f\sinh d\cos f + e\cosh e\sin f)\cosh\frac{2ex}{L}\cos\frac{2fx}{L}$
z_1	distance of stiffener centroid to inner edge of shell
α^4	$\frac{3L^4(1-\nu^2)}{a^2h^2}$
ν	Poison's ratio
ω	shell radial deflection
η	$\frac{\cosh\alpha-\cos\alpha}{\sinh\alpha+\sin\alpha}$
γ	$\frac{A(1-\frac{\nu}{2})}{A+bh+\frac{2Lh\eta}{\alpha}}$
σ_y	yield stress of shell material
σ_{yf}	yield stress of stiffener material
σ_{sf}	frame stress at outer fiber
σ_{cr}	critical buckling stress of frame stiffener

1 Introduction

The structural analysis of a submarine pressure vessel consists primarily of the assessment of the pressures at which yielding and buckling occur. The shell, stiffener and bulkhead geometries and material properties can be varied to minimize the structure's weight and to avoid these failure mechanisms efficiently under a given design load. In theory, most submarine structural components are of simple geometry: cylinders, cones and hemispheres or torispheres. In reality, the submarine poses a very complex structural problem in that it deviates significantly from simplistic behaviour as a result of severe deviations from axisymmetric geometry (decks, cutouts, tanks, etc.) and fabrication effects (out of circularity, residual stresses, section connections, etc.). Today's engineers are fortunate to have access to powerful finite element programs to enable them to more accurately assess the probable behaviour of a particular design, and sophisticated test equipment to further verify analytical work. Unfortunately, these analyses come at a high price and can usually only be justified in final design stages.

Before the advent of these powerful tools, much work was concentrated on the development of manageable design formulae. These formulae have been rigorously verified through experiment and comparison to more complex analyses methods[1]. Formulae methods have proven to be useful in preliminary design and comparison work, and form the basis of most code requirements such as the British Standard Specification for Unfired Pressure Vessels BS5500[2]. Where formulae have failed to explain real structural behaviour, extensive test data have been employed to develop empirical methods of analysis.

In most cases, simplifying assumptions made to derive the design formulae limit their application. Fortunately, these limitations are generally within practical applications for submarine pressure hulls and are of a conservative nature. The use of safety factors accounts for analytical uncertainties as well as variation in material properties and load definition.

To facilitate the efficient application of these formulae, a computer program, PRHDEF, has been developed to introduce the structural parameters, change them easily and calculate the pressures associated with the various failure mechanisms. Safety factors are not included in any of the formulae with the exception of equation 22 for out of circularity described in section 2.8. They are instead calculated in relation to a given maximum design pressure. Therefore, input design pressures should not incorporate safety factors such as the 1.5 value used in BS5500. PRHDEF checks calculated safety factors against user specified values and flags output values where desired safety factors are not met. PRHDEF determines failure loads for uniform axisymmetric hydrostatic pressure only. Variations in the load pattern resulting from underwater shock or collision will produce different failure mechanisms and pressure values and require more complex analyses using finite difference or finite element methods.

It is intended that this program be used for quick comparison of various design options and as a basis for more complex analysis of submarine structure. PRHDEF should be particularly useful in preliminary comparisons of competitive design proposals. Due to variations of the design formulae and of the structural parameters used in the formulae in various codes, an independent analysis common to all designs is necessary for proper comparitive studies.

This report describes and evaluates the various formulae used in ring stiffened pressure vessel analysis and compares results obtained with those of the BOSOR4[3] axisymmetric finite difference analysis program. Examples of the use of the program PRHDEF are given in Appendices A and B, which contain input and output data respectively.

2 Theory

2.1 Description of Submarine Geometry

The majority of the formulae incorporated in this study are for uniformly ring stiffened cylindrical shells of isotropic material. Formulae for assessing the collapse pressures of endcaps are also included. Figure 1 shows the geometric information to be used for the cylinder, stiffeners and endcap.

The length of the cylinder is the distance between rigid ends. The rigid ends may be bulkheads or large frames which are much stiffer than the other frames in the section. Cylinders which terminate in endcaps can be analyzed with an extended equivalent length of 0.4 times the endcap length[2]. Sections with moderately varying stiffener spacing or dimensions can be analyzed using average values.

The stiffeners can be located on either the inner or outer surface of the cylindrical shell. For I section stiffeners the flange at the faying surface with the pressure hull may be included. For tee section stiffened shells the faying flange dimensions will be set to zero.

All material in the structure is assumed to have the same Young's modulus and Poisson's ratio; however, allowance has been made for different yield stresses in the shell and stiffener materials. The yield stress to be used is the design yield stress and the choice of this value is left to the user. There are no factors applied to the yield stress in the program as is done in the BS5500 code.

2.2 Determination of Effective Lengths of Shell Plating

In the determination of the moment of inertia of the combined stiffener and shell sections, and of the location of the neutral axis, it is necessary to determine the length of shell within one bay which acts effectively with the stiffener in circumferential bending. The effect of shear lag in the shell plating and the stress distribution in the shell resulting from longitudinal bending between stiffeners results in a variation of the shell segment contribution to stiffener bending. This variation can be compensated for by assuming a constant contribution over an effective length of shell plating.

Three options exist for determining the effective length in PRHDEF; all have been found in the literature in various submarine design formulae. The simplest method is to take 75 percent of the length between frames[4]:

$$L_e = 0.75L \tag{1}$$

where L, the length between two frames is taken to be the frame spacing minus the web thickness or the faying flange width, whichever is less:

$$L = L_f - b \;\; or = L_f - h_w \tag{2}$$

The effective length is a function of the shell geometry in its deformed state and therefore is a function of the shell radius and thickness, the frame spacing and the circumferential wave number, n, of the buckling mode of interest. Bijlaard[4] developed an expression for the effective length as a function of these parameters:

$$L_e = \frac{1.556\sqrt{ah}\eta}{[\sqrt{1 + \frac{n^4h^2}{2a^2}} + \frac{n^2h}{\sqrt{3}a}]^{\frac{1}{2}}} \tag{3}$$

An extensive set of tables (Figure 2) derived from reference 5 is used in PRHDEF which contains these tabulated data and an interpolation routine to determine effective length values as a function of the above mentioned parameters. These tables are used in the BS5500 code. A comparison of effective shell length values for various stiffened cylinder dimensions and circumferential wave numbers is given in Table 1.

2.3 Effects of Residual Stresses

Determining the total effects of residual stresses on the strength and stability behaviour of a pressure hull is an impossible task; however, it is known that the fabrication process for stiffened cylinders produces significant residual stresses in the shell and stiffeners of the order of 30 percent of the yield stress of the material. Residual stresses do not have a significant effect on the collapse loads of the shell[1] but the determination of collapse through yielding in the stiffeners may be sensitive to residual stress effects. To account for this effect, the BS5500 code has adopted factors to be applied on the design load when determining stiffener failure. These values are 1.8 for hot formed or fabricated frames and 2.0 for cold bent frames. These factors do not address the major effect of residual stresses on failure through fatigue or environmentally assisted cracking. PRHDEF allows the user to input values of $\frac{4}{3}$ for hot formed frames and $0.9(\frac{4}{3})$ for cold formed frames. These are the BS5500 values less the 1.5 general safety factor which allows determination of safety factors by PRHDEF. These values correspond to those used in reference 6.

2.4 Interframe Bifurcation Buckling Pressures

Figure 3 illustrates the mode shape associated with interframe buckling. It generally consists of half sine waves between stiffeners and several waves around the circumference. Von Mises[7] developed a solution for the buckling of a simply supported cylindrical shell section which gives reasonable and conservative predictions for interframe buckling of stiffened shells. There have been several modifications of this work, two of which have been incorporated in PRHDEF. The Von Mises expression as presented by Windenburg and Trilling[8], minimized with respect to the circumferential wave number, n, is:

$$P_M = \frac{\frac{2.42E}{(1-\nu^2)^{\frac{3}{4}}}\left[\frac{h}{2a}\right]^{\frac{5}{2}}}{\frac{L}{2a} - 0.45\left(\frac{h}{2a}\right)^{\frac{1}{2}}} \tag{4}$$

The second version of the Von Mises expression and the one used in the BS5500 code is that presented by Kendricks[9]:

$$P_{M1} = \frac{Eh}{a[n^2 - 1 + \frac{\pi^2 a^2}{2L^2}]}\left\{\frac{1}{[n^2(\frac{L}{\pi a})^2 + 1]^2} + \frac{h^2}{12a^2(1-\nu^2)}[n^2 - 1 + (\frac{\pi a}{L})^2]\right\} \tag{5}$$

To find the minimum pressure with respect to n, an iterative solution is required varying n in the range of 2 to 20 with most geometries producing minimums in the $10 \leq n \leq 16$ wave range. Equation 5 is modified to be more accurate for low wave numbers by including the n^2-1 term. A comparison of equations 4 and 5 is given in Table 2.

2.5 Determination of Stress Values

The hydrostatically loaded stiffened cylindrical section of a submarine is subject to both lateral and axial pressure. The differential equation for the lateral deflection of a cylinder subject to hydrostatic pressure is given as:

$$D\frac{\partial^4\omega}{\partial x^4} + \frac{pa}{2}\frac{\partial^2\omega}{\partial x^2} + \frac{Eh}{a^2}\omega = p(1 - \frac{\nu}{2}) \tag{6}$$

The second term arises from the inclusion of axial load and results in a nonlinear pressure vs deflection relationship. The degree of nonlinearity increases rapidly as the axisymmetric buckling pressure of the cylinder is approached. At pressures well below the axisymmetric buckling pressure the degree of nonlinearity is small. Since submarines are designed to reach their yield strength well before reaching the axisymmetric buckling pressure (see section 2.8) the pressure vs deflection relationship is nearly linear for the area of interest to submarine designers. Wilson[1] took advantage of this fact and derived expressions for stresses in the critical regions of the structure neglecting the second term of equation 6.

The regions where stresses are of concern are:

- σ_3 - the maximum circumferential stress in the outer fiber of the shell at midbay,
- σ_5 - the mean circumferential stress in the shell at midbay,
- σ_7 - the longitudinal stress in the shell on the inside surface at the stiffener connection,
- σ_{fy} - the circumferential stress in the standing flange of the stiffener.

The expressions presented by Wilson for these stresses, rearranged to give external pressures which cause yielding to occur, are:

$$P_3 = \frac{h\sigma_y}{a}\left(\frac{1}{1+\gamma\overline{H}}\right) \tag{7}$$

$$P_5 = \frac{h\sigma_y}{a}\left(\frac{1}{1+\gamma\overline{G}}\right) \tag{8}$$

$$P_7 = \frac{2h\sigma_y}{a}\left(\frac{1}{[1+(\frac{12}{1-\nu^2})^{\frac{1}{2}}]\gamma\overline{R}}\right) \tag{9}$$

$$P_{FY} = \frac{h\sigma_{yf}A_f}{a^2(1-\frac{\nu}{2})}\left[1+\frac{A}{bh+\frac{2\eta Lh}{\alpha}}\right] \tag{10}$$

where A,$\gamma, \overline{H}, \overline{G}, \overline{R}, \eta$ and α are defined in the notation.

A more realistic determination of the pressure causing yield in the midplane of the shell at midbay is found using the Hencky-Von Mises yield criteria. This is determined by:

$$P_6 = \frac{h\sigma_y}{a}[\gamma^2\overline{G}^2 + 1.5\gamma\overline{G} + 0.75]^{-\frac{1}{2}} \tag{11}$$

This results in higher allowable pressures than consideration of the circumferential stress value P_{C5}.

Salerno and Pulos[10] developed solutions for the stress including the effect of axial pressure (ie. not neglecting the second term of equation 6). These solutions are nonlinear functions of the pressure, P, and therefore an iterative solution is required. Again the PRHDEF code solves for the pressures to cause yielding at the critical locations in the shell:

$$P_{3A} = \sigma_y[-\frac{h}{a} + \frac{hH}{a(1-\frac{\nu}{2})TU_0} - (\frac{2(1-\nu^2)L^2}{\nu a^2(1-\frac{\nu}{2})})\frac{1}{TJ_0L^2}] \tag{12}$$

$$P_{5A} = \sigma_y[-\frac{h}{a} + \frac{hH}{a(1-\frac{\nu}{2})TU}] \tag{13}$$

$$P_{7A} = \sigma_y[\frac{2h}{a} + (\frac{2(1-\nu^2)L^2}{a^2(1-\frac{\nu}{2})})\frac{1}{TJ_{\frac{L}{2}}L^2}] \tag{14}$$

$$P_{FYA} = \frac{Eh}{a^2(1-\frac{\nu}{2})}[\frac{GU}{H} - \frac{\sigma_y f\bar{d}}{E}] \tag{15}$$

where T,J_0, $J_{\frac{L}{2}}$,U,G and H are defined in the notation.

It is expected that for most realistic submarine geometries, the two methods of determining stress will give comparable results. Table 3 gives a comparison of the two methods for models A, B and C.

2.6 BS5500 Collapse Curve for Cylinders

The determination of the elastic interframe buckling pressure (section 2.4) and of the pressure at which yield is reached (section 2.5) is for ideal stiffened cylinders. In experimental studies[1], interaction between plastic collapse and elastic buckling has given significant scatter to collapse pressure data. Upper and lower bound curves for all available experimental data established a relationship between $\frac{P_{M1}}{P_5}$ and $\frac{P_c}{P_5}$ used in BS5500 and reproduced in Figure 4. P_{M1} is defined in equation 5, P_5 is defined in equation 8 and P_c is the collapse pressure of the cylinder. A simplified expression for the lower bound curve of Figure 4 has been coded into PRHDEF[4]:

$$\frac{P}{P_5} \simeq 1 - \frac{P_5}{2P_{M1}} \qquad \frac{P_{M1}}{P_5} \geq 1.0 \tag{16}$$

$$\frac{P_c}{P_5} \simeq \frac{P_{M1}}{2P_5} \qquad \frac{P_{M1}}{P_5} < 1.0$$

This is accurate to within 1% of the lower bound curve of Figure 4.

The BS5500 code uses this curve by applying a safety factor of 1.5 to the lower bound curve. The 1.5 safety factor has not been included in the PRHDEF calculation so that a safety factor can be calculated for the analyst to use in his own criteria.

The curve of Figure 4 is limited to cylinders with a maximum out of circularity of 5% on radius and for cylinders of $5.9 < \frac{a}{h} < 250$ and $0.04 < \frac{L}{a} < 50$[1]. These limits result from the range of experimental data surveyed in forming the curve and are applicable to most submarine dimensions. PRHDEF checks input data for these dimension limits and flags violations.

2.7 Overall Buckling Pressure

Figure 5 shows the mode shape for overall buckling of a ring stiffened cylinder. It consists of a half sine wave between rigid ends and usually 2 to 6 waves circumferentially. The circumferential wave number depends on the length to radius ratio of the cylinder.

Two expressions for overall buckling have been incorporated in PRHDEF. Both of these are dependent on the wave number, n, and are presented for wave numbers 2 to 6.

Bresse[7] developed an expression for an infinitely long stiffened shell:

$$P_B(n) = \frac{(n^2-1)EI_c}{a^3 L} \tag{17}$$

where I_c is the moment inertia of a combined shell and stiffener section and is therefore sensitive to the effective length of shell chosen (section 2.3). This formula greatly underestimates overall buckling loads for finite lengths of shell supported by rigid bulkheads. The membrane shear stresses that occur in a realistic stiffened shell are not accounted for by equation 17. This inaccuracy decreases with increasing length of shell and circumferential wave number.

Bryant[11] developed an approximate equation for the overall buckling load by combining equation 5 for the buckling of the shell between rigid ends and equation 17 to incorporate the effect of the stiffeners:

$$P_n(n) = \frac{Eh}{a}\frac{1}{[n^2-1+\frac{1}{2}(\frac{\pi a}{L_b})^2][(\frac{nL_b}{\pi a})^2+1]^2} + \frac{(n^2-1)EI_c}{a^3 L} \tag{18}$$

It has been shown that this formula gives good results in comparison with more complex theories; however, it gives unconservative results for cylinders which fail with circumferential wave numbers greater than 3, as would be the case for short stubby sections. Table 4 compares formulae 17 and 18 with numerical results from BOSOR4.

A formula for the axisymmetric collapse load of the stiffened cylinder is included in PRHDEF. This mode of failure usually occurs at much higher loads than non-axisymmetric modes (equations 17 and 18), but is useful in determining the degree of nonlinearity present in the yield stress calculations (section 2.6). The axisymmetric buckling load equation is[6]:

$$P_e = \frac{8D\pi^2}{aL^2} + \frac{hEL^2}{2\pi^2 a^3}[\frac{Lh+3(A+bh)}{Lh+A+bh}] \tag{19}$$

2.8 Out of Circularity and Frame Collapse

Equations 17 and 18 give overall buckling loads for ideal stiffened cylinders. In practice, these values are difficult to attain as the structures or experimental models are not ideal. Out of circularity (OOC) of the cylinder causes reduction of the ideal buckling loads particularly if the OOC is in the form of the critical buckling mode shape. The bending stress induced in the stiffener as a result of OOC greatly reduces the pressure at which it reaches yield. The effects of OOC are incorporated in design by determining the failure of an out of round stiffener and assumming that stiffener failure precipitates overall buckling collapse. The stress in a stiffener of a shape corresponding to the critical overall buckling mode as a result of compression and bending is[9]:

$$\sigma_{sf} = \frac{RP\sigma_{yf}}{P_{FY}} + \frac{E\bar{e}C_n(n^2-1)RP}{a^2(P_n - RP)} \quad (20)$$

where C_n is the out of circularity.

To determine the pressure P_f, at which the stiffener will reach yield including OOC effects, equation 20 must be rewritten in the form:

$$R^2\sigma_{yf}a^2P_f^2 - [P_{FY}a^2R\sigma_{yf} + R\sigma_{yf}a^2P_n + P_{FY}E\bar{e}C_n(n^2-1)R]P_f - \sigma_{yf}a^2P_n = 0 \quad (21)$$

This quadratic is then solved for P_f as a function of n.

The BS5500 code uses an allowable OOC of 5% of the radius. C_n can be replaced by .005a in equations 20 and 21 and the residual stress factor, R set equal to 2 (which includes the 1.5 safety factor in addition to the $\frac{4}{3}$ residual stress factor) to get the form found in BS5500.

Since σ_{sf} is a function of the wave number, n, an alternative approach is to determine the OOC which will cause yield ($\sigma_{sf} = \sigma_{yf}$) in the stiffener for a given n. The determination of C_n will not result in the calculation of a safety factor since it is a nonlinear function of the applied load, P. Therefore, in this case the applied load has been multiplied by 1.5 to give C_n including a safety factor. Rewriting equation 20 to solve for C_n as a function of n, gives:

$$C_n(n) = \frac{\sigma_{yf}a^2(P_n - 1.5RP)}{E\bar{e}(n^2-1)1.5RP}[1 - \frac{1.5RP}{P_{FY}}] \quad (22)$$

PRHDEF determines C_n for wave numbers 2 to 6 using equation 22. Frame failure loads are calculated using equation 21 with the value of C_n being derived from equation 22, given a value of 0.005a from BS5500, or given a user specified OOC value. To determine C_n for a different safety factor than 1.5, R can be input such that R $= \frac{S.F.}{1.5}(\frac{4}{3})$. It should also be noted that the frame failure load P_f predicted by PRHDEF using C_n from equation 22 will be equal to the input design pressure times the 1.5 safety factor.

2.9 Stiffener Buckling

Two types of buckling failure have been discussed in previous sections: interframe and overall buckling. A third possible mode of buckling failure can occur in the ring stiffeners from lateral torsional buckling (tripping) or local buckling of the web or flange. Interaction of the stiffener with the shell plating requires more sophisticated analyses such as finite element or finite difference for the determination of the loads causing this mode of failure. Since stiffener buckling is easy to avoid by proper dimensioning of the stiffeners, simple conservative formulae have been introduced in BS5500 to check stiffener dimensions.

A formula for determining the torsional buckling of a T stiffener which is pinned at its connection to the shell is given as[1]:

$$\sigma_{cr} = \frac{EI_z}{A_f r_1 z_1} < \sigma_{yf} \tag{23}$$

Dimensions for the web and flange are checked respectively by[2]:

$$\frac{d_w}{h_w} \not> 1.1\sqrt{\frac{E}{\sigma_{yf}}} \tag{24}$$

$$\frac{w_f}{h_f} \not> 0.5\sqrt{\frac{E}{\sigma_{yf}}} \tag{25}$$

2.10 Endcap Collapse

Integrity of endcap structure is checked in a similar manner to cylinders using an experimentally derived collapse curve (Figure 4). Hemispheres, spheres and torispheres (using the outer radius) are analyzed by the same method. The pressure causing yield in the shell is determined from:

$$P_{ey} = 2\frac{h_e}{a_e}\sigma_y \tag{26}$$

The elastic critical buckling pressure for a perfect sphere is determined from[1]:

$$P_{ec} = \frac{2Eh_e^2}{a_e^2\sqrt{3(1-\nu^2)}} \tag{27}$$

The ratio $\frac{P_{ec}}{P_{ey}}$ is then used to determine $\frac{P_{cu}}{P_{cy}}$ from Figure 4. PRHDEF uses a tabulated form of Figure 4 to determine P_{eu}. This experimental curve is limited to endcaps which do not vary in radius by more than 1 percent.

3 Discussion

Classic formulae used to check the integrity of a stiffened pressure vessel design against various failure mechanisms have been described. The methods of the BS5500 code for externally loaded pressure vessels include most of these formulae. Design curves incorporating experimental data to derive the collapse load have also been given. The results obtained by these methods are subject to the geometry of the structure. The computer code PRHDEF has been written to incorporate the formulae described in the previous section and enable easy input of structural data. Appendix A contains a sample terminal session with PRHDEF and Appendix B contains samples of PRHDEF analysis for three models. Original geometry data are stored on a file which can be used by PRHDEF in subsequent runs to study the effects of varying one or several structural parameters. PRHDEF is a self explanatory interactive Fortran computer code operational on DEC-20, VAX 11/750 and IBM PC computers.

Three options of determining the effective length of shell segment to be included in determining equivelent moments of inertia of the stiffener for circumferential bending have been given. Table 1 compares these options for three models. As would be expected, the larger the stiffener spacing and the thinner the shell, the smaller the effective length. The effective length of $0.75L_f$ was not very satisfactory for models B or C when compared to the other methods. The second two options which include the effect of the circumferential wave number n, show decreasing L_e with increasing n. Both of these methods give comparable results, with the tables used in the BS5500 code (derived from reference 4) being most conservative.

Table 2 gives results of interframe buckling pressures for the three models described in Table 1. The minimum values from equation 5 agree well with those obtained from equation 4. All of the minimum values from the formulae are less than the numerical results for the three models. This is expected as the formulae consider only one bay of shell with a simply supported boundary condition at each end. The full structure of several bays provides some rotational restraint at the frame supports which results in a stiffer structure and thus higher buckling loads. This is particularly evident in model A which has a thicker shell. For practical purposes, the minimized Von Mises expression of equation 4 appears to give adequate results.

Table 3 compares results obtained for the three models for the linear and nonlinear stress values. The value of the axisymmetric buckling pressure is also included in the table as it indicates the degree of nonlinearity expected in the pressure - stress relationship. Model B shows yield pressures which approach half of the axisymmetric buckling load and some difference between the linear and nonlinear values is present. The differences agree with the trend indicated in Figure 6[1]; ie. nonlinear decrease in shell stress and nonlinear increase in stiffener stress as the pressure approaches the axisymmetric buckling pressure. These stress values agree well with the BOSOR4 results.

Table 4 compares the overall buckling loads obtained by equations 17 and 18 and numerical results with BOSOR4 for the three models. The results of equation 17 for a section of shell and frame alone (infinite cylinder) are too low for small wave numbers. This is a result of the increased stiffness of the cylinder from localized membrane shear stresses at the boundaries. This effect decreases with increasing length to radius ratios. Equation 18 neglects membrane shear stiffness from interframe deformation which results in buckling loads which approximate those of a stiffer cylinder. This effect decreases as the length increases and as the minimum wave number approaches 2. The numerical results should converge to a better approximation of the true buckling mode for stubby ring stiffened cylinders. The BOSOR4 models had simply supported boundary conditions and were fixed axially at one end. Variation of these boundary conditions drastically alter the buckling loads and mode shapes.

The BS5500 code method of determining frame failure is very strict. All models had this as their lowest failure load. This approach is generally viewed to be pessimistic; however, it has proven to be the most practical approach to incorporating the OOC effects on the overall buckling load collapse mechanism.

The stiffener tripping criteria are also recognized as being pessimistic. Model C which takes stiffener dimensions from an actual submarine fails this criteria. In this event more complex analysis with a program such as BOSOR4 which will take into account the interaction between the stiffener and the shell may prove that the stiffener is more than adequate.

4 Conclusions

The use of formulae and the BS5500 pressure vessel design code to evaluate hydrostatically loaded, uniformly stiffened cylinders has been presented. This approach relies on the structure resisting three mechanisms of failure. These are: interframe collapse which depends on the interframe bifurcation buckling pressure and the shell yield strength; overall collapse which depends on the stiffener yield strength and the overall bifurcation buckling pressure including the effects of out of circularity; and localized stiffener failure. The loads associated with these failure mechanisms can be quickly determined for a variety of geometries with the computer code PRHDEF which accompanies this work.

The simplified formulae compare well with BOSOR4 numerical calculations for the three models considered with the exception of the overall buckling formula (equation 18) which significantly overestimates the buckling load for short cylindrical sections. Better approximations to the stress, interframe and overall bifurcation buckling loads and local stiffener failure may be obtained by using finite element or finite difference methods, the results of which may be applied to the design curve (section 2.7) and equation 21 to determine overall collapse. Geometries which differ significantly from uniformly stiffened cylinders should be analyzed with numerical methods. For preliminary design or comparitive studies, the formulae presented here and incorporated in PRHDEF should be adequate for most cylindrical submarine pressure vessel sections.

TABLE 1: Comparison of Effective Lengths(mm) for Various Formulae

Wave Number	Model A[a]			Model B[b]			Model C[c]		
	$0.75L_f$	Eqn 3	BS5500	$0.75L_f$	Eqn 3	BS5500	$0.75L_f$	Eqn 3	BS5500
2	375	421	413	600	472	459	570	442	429
3	375	409	393	600	468	453	570	437	419
4	375	392	371	600	464	444	570	430	406
5	375	371	346	600	457	430	570	422	389
6	375	347	319	600	450	415	570	411	371

[a] Model A: $h = 40mm$ $L_f = 500mm$ $d_w = 150mm$ $h_w = 15mm$ $w_f = 150mm$
$h_f = 15mm$ $L_b = 10000mm$ $a = 2000mm$ $\sigma_y = 500MPa$ $\sigma_{yf} = 450MPa$
$\nu = 0.3$ $E = 207000MPa$ $R = 1.2$ $P_d = 500m$
$internal framing$

[b] Model B: $h = 20mm$ $L_f = 800mm$ $d_w = 150mm$ $h_w = 15mm$ $w_f = 150mm$
$h_f = 15mm$ $L_b = 8000mm$ $a = 4000mm$ $\sigma_y = 600MPa$ $\sigma_{yf} = 450MPa$
$\nu = 0.3$ $E = 207000MPa$ $R = 1.2$ $P_d = 125m$
$internal framing$

[c] Model C: $h = 23mm$ $L_f = 760mm$ $d_w = 203mm$ $h_w = 6.1mm$ $w_f = 101mm$
$h_f = 8.4mm$ $L_b = 13680mm$ $a = 3086mm$ $\sigma_y = 450MPa$ $\sigma_{yf} = 450MPa$
$\nu = 0.3$ $E = 207000MPa$ $R = 1.0$ $P_d = 200m$
$internal framing$

TABLE 2: Comparison of Interframe Buckling Pressures (MPa)

Wave Number	Model A			Model B			Model C		
	P_M^a	P_{M1}^b	BOSOR4[c]	P_M	P_{M1}	BOSOR4	P_M	P_{M1}	BOSOR4
6		76.1	74.2		6.1	3.8		11.5	7.2
7		72.4	74.3		5.4	3.6		9.8	6.7
8		69.8	75.8		4.7	3.3		8.3	6.2
9		68.3	78.5		4.2	3.1		7.2	5.8
10	70.5	67.9	83.2		3.7	3.0		6.3	5.5
11		68.5	88.9		3.3	2.9		5.7	5.2
12		69.9	93.7		3.0	2.8		5.3	5.1
13		72.0	98.8		2.7	2.7		5.0	5.0
14		74.7	104.2		2.5	2.6		4.8	5.1
15		78.1	109.8		2.4	2.5	4.8	4.8	5.1
16		81.9	115.5		2.3	2.5		4.8	5.3
17		86.2	121.4		2.23	2.50		4.9	5.4
18		90.9	127.4		2.21	2.51		5.0	5.6
19		96.1	133.6	2.22	2.20	2.53		5.2	5.9
20		101.6	140.0		2.23	2.6		5.5	6.1

[a] equation 4, [b] equation 5, [c] reference 3

TABLE 3: Comparison of Linear and Nonlinear Yield Pressures (MPa)

Model	P_3^a	P_{3A}^b	P_5^c	P_{5A}^d	P_7^e	P_{7A}^f	P_{FY}^g	P_{FYA}^h	P_e^i
A	11.4	11.3	11.9	11.8	13.4	13.2	12.5	12.5	221.1
B	3.0	2.7	3.3	3.0	2.9	2.5	3.8	5.2	7.8
C	3.4	3.3	3.5	3.4	4.2	4.0	4.5	4.6	16.1

[a] Wilson equation 7, [b] Salerno and Pulos equation 12, [c] Wilson equation 8
[d] Salerno and Pulos equation 13, [e] Wilson equation 9, [f] Salerno and Pulos equation 14
[g] Wilson equation 10, [h] Salerno and Pulos equation 15, [i] Axisymmetric buckling load equation 19

TABLE 4: Comparison of Overall Buckling Pressures (MPa)

Wave Number	Model A			Model B			Model C		
	P_n^a	P_B^b	BOSOR4[c]	P_n	P_B	BOSOR4	P_n	P_B	BOSOR4
2	22.8	12.4	25.0	36.3	0.7	3.9	7.4	1.5	5.2
3	33.6	32.7	36.7	7.1	1.9	3.1	4.4	3.9	4.9
4	60.7	60.5	54.2	4.8	3.6	3.5	7.3	7.2	6.8
5	95.4	95.4	67.9	6.1	5.8	3.9	11.5	11.4	7.4
6	136.6	136.6	73.2	8.4	8.3	3.9	16.6	16.6	7.2

[a] Bryant equation 18, [b] Bresse equation 17, [c] reference 3

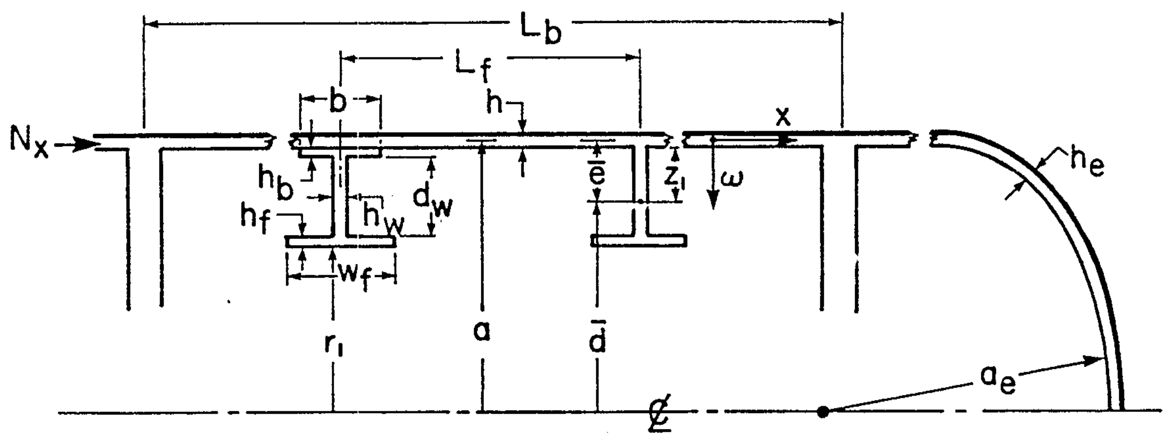

Figure 1: Geometry of Cylinder and Endcap Section

$\frac{e^2}{12R^2} \geqslant 10^{-4}$

L_e / L_s

$\frac{L_s}{2\pi R}$ \ n	2	3	4	5	6
0	1.0980	1.0980	1.0980	1.0980	1.0980
0.01	1.0823	1.0823	1.0663	1.0663	1.0504
0.02	1.0663	1.0504	1.0265	0.9947	0.9629
0.03	1.0504	1.0027	0.9549	0.9019	0.8435
0.04	0.9907	0.9231	0.8515	0.7838	0.7082
0.05	0.8976	0.8276	0.7512	0.6716	0.5952
0.06	0.7921	0.7298	0.6609	0.5871	0.5143
0.07	0.6866	0.6321	0.5707	0.5025	0.4343
0.08	0.6111	0.5630	0.5088	0.4480	0.3877
0.09	0.5355	0.4940	0.4470	0.3935	0.3410
0.1	0.4600	0.4249	0.3852	0.3390	0.2944

$\frac{e^2}{12R^2} = 10^{-7}$

L_e / L_s

$\frac{L_s}{2\pi R}$ \ n	2	3	4	5	6
0	1.0980	1.0980	1.0980	1.0980	1.0980
0.01	0.9072	0.9072	0.8913	0.8913	0.8913
0.02	0.4297	0.4297	0.4218	0.4218	0.4218
0.03	0.2759	0.2759	0.2759	0.2759	0.2759
0.04	0.2207	0.2207	0.2207	0.2191	0.2191
0.05	0.1655	0.1655	0.1655	0.1623	0.1623
0.06	0.1490	0.1487	0.1487	0.1461	0.1461
0.07	0.1324	0.1318	0.1318	0.1299	0.1299
0.08	0.1159	0.1149	0.1149	0.1136	0.1136
0.09	0.0993	0.0980	0.0980	0.0974	0.0974
0.1	0.0828	0.0812	0.0812	0.0812	0.0812

$\overline{12R^2} = 10^{-5}$

L_e / L_s

$\frac{L_s}{2\pi R}$ \ n	2	3	4	5	6
0	1.0980	1.0980	1.0980	1.0980	1.0980
0.01	1.0823	1.0823	1.0663	1.0663	1.0504
0.02	1.0345	1.0186	0.9947	0.9629	0.9311
0.03	0.9019	0.8807	0.8541	0.8117	0.7639
0.04	0.7242	0.7003	0.6724	0.6326	0.5929
0.05	0.5602	0.5411	0.5220	0.4934	0.4647
0.06	0.4483	0.4350	0.4218	0.4005	0.3793
0.07	0.3752	0.3661	0.3547	0.3388	0.3206
0.08	0.3263	0.3163	0.3084	0.2964	0.2805
0.09	0.2920	0.2847	0.2775	0.2660	0.2525
0.1	0.2578	0.2531	0.2467	0.2355	0.2244

$\frac{e^2}{12R^2} = 10^{-6}$

L_e / L_s

$\frac{L_s}{2\pi R}$ \ n	2	3	4	5	6
0	1.0980	1.0980	1.0980	1.0980	1.0980
0.01	1.0663	1.0504	1.0504	1.0504	1.0345
0.02	0.8276	0.8196	0.8037	0.7878	0.7719
0.03	0.5252	0.5199	0.5146	0.5040	0.4934
0.04	0.3740	0.3700	0.3661	0.3621	0.3541
0.05	0.2960	0.2928	0.2897	0.2865	0.2801
0.06	0.2661	0.2632	0.2604	0.2575	0.2521
0.07	0.2362	0.2336	0.2311	0.2285	0.2241
0.08	0.2063	0.2040	0.2018	0.1996	0.1961
0.09	0.1763	0.1744	0.1725	0.1706	0.1681
0.1	0.1464	0.1448	0.1432	0.1416	0.1401

Figure 2: Effective Length Tables[2]

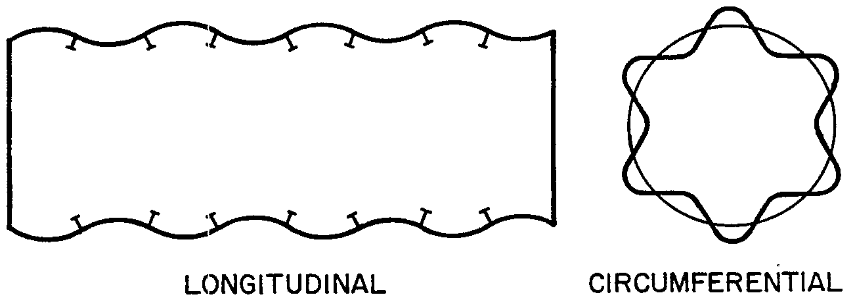

Figure 3: Interframe Buckling Mode[2]

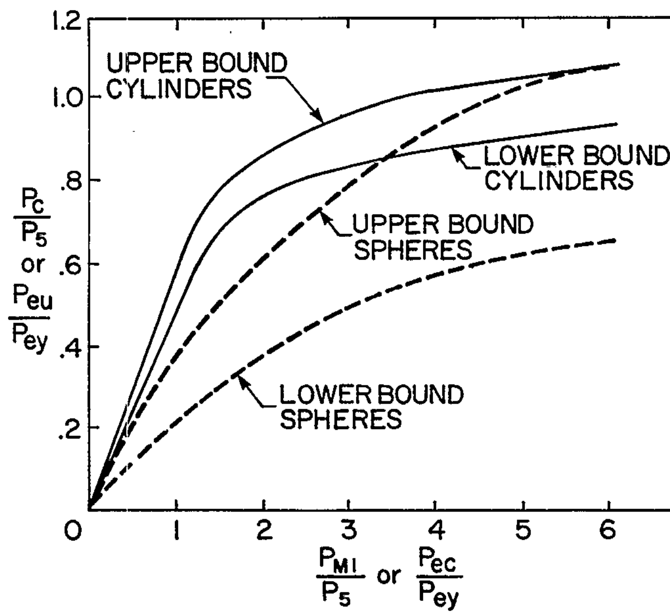

Figure 4: Collapse Pressure Design Curve[3]

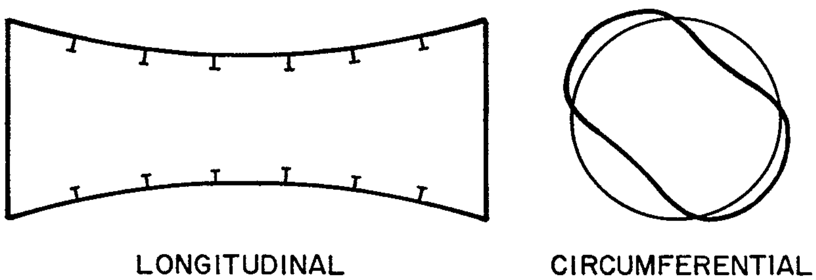

Figure 5: Overall Buckling Mode Shape

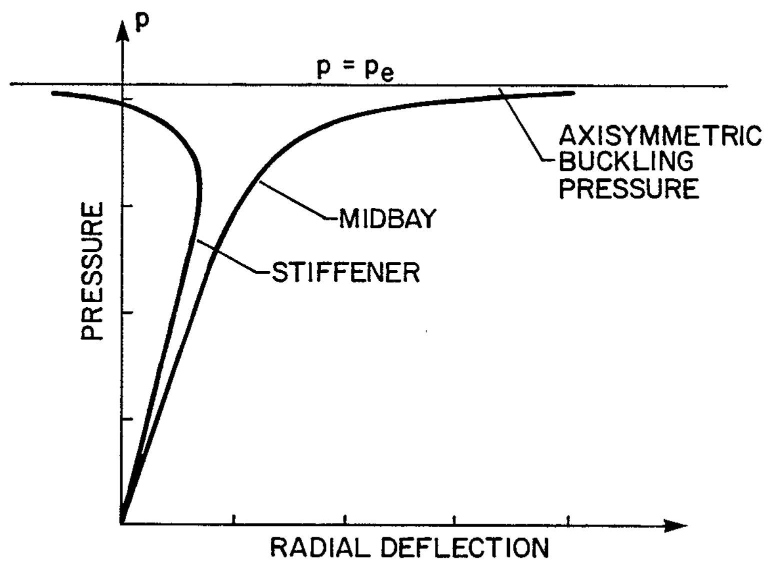

Figure 6: Stress vs Pressure Relationship[1]

Appendix A: Sample Terminal Input Session of PRHDEF

```
COMPILE SOURCE IF NECESSARY
$ FOR PRHDEF

LINK .OBJ FILE TO GET .EXE FILE
$ LINK PRHDEF

$ RUN PRHDEF

PROGRAM PREDICTS THE STRENGTH OF FRAME STIFFENED CYLINDERS AND ENDCAPS

TO RUN EXISTING DATA FILE ENTER 1
TO GENERATE A NEW MODEL ENTER  0
0

ENTER A FIVE CHARACTER PREFIX NAME TO IDENTIFY NEW OR OLD DATA FILE
MODLA

FOR OUTPUT TO SCREEN ENTER 5
FOR OUTPUT TO FILE PREFX.OUT ENTER 3
3


INPUT TITLE BLOCK - 70 CHARACTERS MAXIMUM

MODLA TERMINAL SESSION
ENTER ALL DATA IN CONSISTENT UNITS

IN.=25.4MM.
1 PSI.= 0.00689 MPa
1 METER=39.3701IN.
MOMENT OF INERTI
IN**4 = 41.623CM**4
IN**4 = 416231MM**4
 YOUNGS MODULUS
STEEL = 30000000 PSI    207000 MPa
ALUM = 10300000 PSI  70000 MPa
 YIELD STRESS
STEEL = 45000 PSI   310 MPa
ALUM = 39000 PSI   270 MPa

INS. OR MM ARE THE MOST FAVOURED UNITS
CHOOSE FROM FOLLOWING UNITS FOR THE ANALYSIS
INCHES = 1
MILIMETERS = 2
FEET = 3
METERS = 4
2


ENTER THICKNESS OF CYLINDER SHELL PLATING
40
ENTER FRAME SPACING
500
ENTER WIDTH OF FAYING FLANGE IN CONTACT WITH PLATING
0
ENTER THICKNESS OF FAYING FLANGE
0.0 FOR WELDED TEE BAR
0
ENTER DEPTH OF FRAME WEB
150
```

```
ENTER THICKNESS OF FRAME WEB
15
ENTER WIDTH OF FRAME FLANGE
150
ENTER THICKNESS OF FRAME FLANGE
15

                ** INPUT DATA **

     1 THICKNESS OF PLATING 40.0000 MM.

     2 FRAME SPACING  500.0000 MM.

     3 WIDTH OF FAYING FLANGE IN CONTACT WITH PLATING  0.0000 MM.

     4 THICKNESS OF FAYING FLANGE  0.0000 MM.

     5 DEPTH OF FRAME WEB  150.0000 MM.

     6 THICKNESS OF FRAME WEB 15.0000 MM.

     7 WIDTH OF FRAME INNER FLANGE  150.0000 MM.

     8 THICKNESS OF FRAME FLANGE   15.0000 MM.

TO CORRECT A DATA ERROR ENTER LINE NO
TO CONTINUE ENTER 0
0

ENTER RADIUS OF MEAN SURFACE OF CYLINDRICAL SHELL PLATING
2000
ENTER DISTANCE BETWEEN RIGID ENDS
8000
ENTER YIELD STRESS OF SHELL PLATING
500
ENTER YIELD STRESS OF FRAME FLANGE
450
ENTER POISSONS RATIO
.3
ENTER YOUNGS MODULUS
207000
ENTER 1.0 FOR INTERNAL FRAME OR -1.0 FOR EXTERNAL FRAMING
1
ENTER OUT OF CIRCULARITY - A MINUS VALUE WILL USE BS5500 CODE
-1
CHOOSE MULTIPLIER FOR RESIDUAL STRESS IN FRAMES
NO ALLOWANCE FOR RESIDUAL STRESS = 1
MULTIPLIER FOR COLD  BENT FRAMES= 1.33
MULTIPLIER FOR HOT FORMED FRAMES = 1.2
1.2

     1 RADIUS OF MEAN SURFACE OF SHELL PLATING   2000.00 MM.

     2 DISTANCE BETWEEN RIGID ENDS    8000.0 MM.

     3 YIELD STRESS OF SHELL PLATING       500. MPa

     4 YIELD STRESS OF FRAME FLANGE       450. MPa

     5 POISSONS RATIO    0.300
```

6 YOUNGS MODULUS 207000. MPa

7 FRAMING TYPE 1. INTERNAL FRAMING

8 OUT OF CIRCULARITY -1.000 MM.

9 MULTIPLIER FOR RESIDUAL FRAME STRESS 1.20

RADIUS THICKNESS RATIO a/h 50.00

LENGTH RADIUS RATIO L/a 4.000

```
TO CORRECT A DATA ERROR ENTER LINE NO
TO CONTINUE ENTER 0
2
ENTER DISTANCE BETWEEN RIGID ENDS
10000
```

1 RADIUS OF MEAN SURFACE OF SHELL PLATING 2000.00 MM.

2 DISTANCE BETWEEN RIGID ENDS 10000.0 MM.

3 YIELD STRESS OF SHELL PLATING 500. MPa

4 YIELD STRESS OF FRAME FLANGE 450. MPa

5 POISSONS RATIO 0.300

6 YOUNGS MODULUS 207000. MPa

7 FRAMING TYPE 1. INTERNAL FRAMING

8 OUT OF CIRCULARITY -1.000 MM.

9 MULTIPLIER FOR RESIDUAL FRAME STRESS 1.20

RADIUS THICKNESS RATIO a/h 50.00

LENGTH RADIUS RATIO L/a 5.000

```
TO CORRECT A DATA ERROR ENTER LINE NO
TO CONTINUE ENTER 0
0

ENTER ENDCAP THICKNESS - ENTER 0 FOR NO ENDCAP
40
ENTER ENDCAP RADIUS - OUTER END RADIUS FOR TORISPHERE
2000
```

1 THICKNESS OF ENDCAP 40.0000 MM.

2 RADIUS OF ENDCAP 2000.0000 MM.

```
TO CORRECT A DATA ERROR ENTER LINE NO
TO CONTINUE ENTER 0
0
ARE METRIC UNITS USED
YES = 1   NO = 0
```

```
1
ENTER MAXIMUM -SURVIVABLE- DESIGN DEPTH IN METERS
500
ENTER SAFETY FACTORS FOR YIELDING -  INTERFRAME BUCKLING
 AND OVERALL BUCKLING - VALUES OF 1.5 2.5 AND 3.5 ARE TYPICAL
 THESE VALUES ARE USED FOR COMPARITIVE PURPOSES ONLY AND
 NOT IN CALCULATION
1.0 2.5 3.5

      1 SPECIFIED MAXIMUM DEPTH      500.0 M.

      MAXIMUM DESIGN PRESSURE      5.000 MPa

      2 SAFETY FACTOR FOR STRESS COMPARISONS - 1.50
      SAFETY FACTOR FOR INTERFRAME BUCKLING COMPARISONS - 2.50
      SAFETY FACTOR FOR OVERALL BUCKLING COMPARISONS - 3.50

TO CORRECT A DATA ERROR ENTER LINE NO
TO CONTINUE ENTER 0
0

ENTER FORM OF EFFECTIVE LENGTH OF SHELL BETWEEN
STIFFENERS FOR USE IN CALCULATIONS --
1=0.75XFRAME SPACING
2=BIJLAARDS FORMULA - SEE REPORT ON PRHDEF
3=BS5500 TABLES FOR EFFECTIVE LENGTH
3
CALCULATION FINISHED
```

Appendix B: Sample Output Analysis of PRHDEF for Three Models

MODEL A

** INPUT DATA **

1 THICKNESS OF PLATING 40.0000 MM.

2 FRAME SPACING 500.0000 MM.

3 WIDTH OF FAYING FLANGE IN CONTACT WITH PLATING 0.0000 MM.

4 THICKNESS OF FAYING FLANGE 0.0000 MM.

5 DEPTH OF FRAME WEB 150.0000 MM.

6 THICKNESS OF FRAME WEB 15.0000 MM.

7 WIDTH OF FRAME INNER FLANGE 150.0000 MM.

8 THICKNESS OF FRAME FLANGE 15.0000 MM.

1 RADIUS OF MEAN SURFACE OF SHELL PLATING 2000.00 MM.

2 DISTANCE BETWEEN RIGID ENDS 10000.0 MM.

3 YIELD STRESS OF SHELL PLATING 500. MPa

4 YIELD STRESS OF FRAME FLANGE 450. MPa

5 POISSONS RATIO 0.300

6 YOUNGS MODULUS 207000. MPa

7 FRAMING TYPE 1. INTERNAL FRAMING

8 OUT OF CIRCULARITY -1.000 MM.

9 MULTIPLIER FOR RESIDUAL FRAME STRESS 1.20

RADIUS THICKNESS RATIO a/h 50.00

LENGTH RADIUS RATIO L/a 5.000

1 THICKNESS OF ENDCAP 40.0000 MM.

2 RADIUS OF ENDCAP 2000.0000 MM.

1 SPECIFIED MAXIMUM DEPTH 500.0 M.

MAXIMUM DESIGN PRESSURE 5.000 MPa

2 SAFETY FACTOR FOR STRESS COMPARISONS - 1.50
SAFETY FACTOR FOR INTERFRAME BUCKLING COMPARISONS - 2.50
SAFETY FACTOR FOR OVERALL BUCKLING COMPARISONS - 3.50

EFFECTIVE LENGTH OF SHELL IS FROM BS5500 TABLES

******CALCULATED OUTPUT******

	PRESSURE	SAFETY FACTOR
INTERFRAME COLLAPSE		
VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE PM WINDENBURG AND TRILLING	70.512 MPa	14.1025
KENDRICKS MINIMIZED MODIFIED VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE AT WAVE NUMBER= 10	67.916 MPa	13.5833
BS5500 LOWER BOUND COLLAPSE CURVE GIVES PM1/PY=5.73 PC/PY=0.91 AND COLLAPSE PRESSURE =	10.812 MPa	2.1625
***** SHELL YIELDING**		
HOOP STRESS FOR AN UNSTIFFENED CYLINDER PB	10.000 MPa	2.0000
PRESSURE WHERE CIRCUMFERENTIAL STRESS EQUALS YIELD STRESS IN PLATING P3 - WILSON LINEAR FORMULATION	11.353 MPa	2.2706
PRESSURE WHERE MAX. LONGITUDINAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P7 - WILSON	13.376 MPa	2.6751
PRESSURE WHERE MEAN MIDBAY CIRCUMFERENTIAL PLATING STRESS EQUALS YIELD STRESS P5 - WILSON	11.845 MPa	2.3691
PRESSURE WHERE MEAN MIDBAY STRESS IN PLATING WITH HENCKY-VON MISES YIELD CRITERION EQUALS YIELD	13.601 MPa	2.7202
***** FRAME YIELDING**		
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN THE STANDING FLANGE EQUALS YIELD PFY - WILSON	12.494 MPa	2.4988
***** MORE ACCURATE ITERATED SOLUTION FOR STRESS SALERNO AND PULOS NONLINEAR FORMULATION - SHELL**		
PRESSURE AT WHICH THE MAXIMUM CIRCUMFERENTIAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P3A	11.317 MPa	2.2634
PRESSURE AT WHICH THE LONGITUDINAL STRESS IN THE IN THE PLATING EQUALS THE YIELD STRESS P7A	13.199 MPa	2.6397
PRESSURE AT WHICH THE CIRCUMFERENTIAL STRESS IN PLATING AT MIDBAY EQUALS THE YIELD STRESS P5A	11.826 MPa	2.3653
*****FRAME STRESS**		
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN STANDING FLANGE OF THE FRAME EQUALS YIELD - PFYA	12.513 MPa	2.5025

***BS5500 ENDCAP RESULTS

PRESSURE AT WHICH ENDCAP REACHES YIELD	20.000 MPa	4.0000
CRITICAL BUCKLING PRESSURE OF ENDCAP	100.188 MPa	20.0376
ULTIMATE COLLAPSE PRESSURE FROM BS5500 P/PY CURVE THIS IS THE LOWER BOUND CURVE OF EXPERIMENTAL DATA	9.989 MPa	1.9977

***OVERALL COLLAPSE

SYMMETRIC BUCKLING PRESSURE OF THE RING-SHELL COMBINATION - WAVE NUMBER 0 MODE	221.147 MPa	44.2294

BRYANT OVERALL BUCKLING WITH EFFECTIVE WIDTH
BRESSE SOLUTION IS FOR A SINGLE STIFFENER AND SHELL
SECTION ASSUMMING SHELL FULLY EFFECTIVE

MODE	EFFECTIVE LENGTH OF SHELL	COLLAPSE PRESSURE BRESSE	BRYANT	
2	412.894	12.384	22.828 MPa	4.5655
3	392.893	32.688	33.579 MPa	6.7158
4	370.895	60.542	60.700 MPa	12.1399
5	345.871	95.383	95.424 MPa	19.0848
6	319.448	136.573	136.587 MPa	27.3173

***CHECK ON STIFFENER PROPORTIONS FROM BS5500

CRITICAL TRIPPING STRESS OF STIFFENER = 851.540 MPa
THIS IS 1.8923 TIMES YIELD

GENERAL STIFFENER PROPORTIONS 0.00411
WITHIN CODE RECOMMENDATIONS 0.00217

WEB DEPTH TO THICKNESS RATIO 10.000
IS WITHIN CODE RECOMMENDATIONS 23.592

HALF FLANGE WIDTH TO THICKNESS RATIO 5.000
IS WITHIN CODE RECOMMENDATIONS 10.724

***STIFFENER FLANGE FAILURE

TWO VALUES OF OUT OF CIRCULARITY ARE USED TO DETERMINE THE FAILURE PRESSURE OF THE STIFFENER - KENDRICKS FORMULA WHICH IS A FUNCTION OF WAVE NUMBER AND OVERALL BUCKLING LOAD AND EITHER THE BS5500 VALUE OR A GIVEN OOC VALUE, BOTH OF WHICH ARE CONSTANT WITH WAVE NUMBER.

EFFECTIVE LENGTH OF SHELL	MODE	OUT OF ROUND ALLOWABLE	FAILURE PRESSURE	OUT OF ROUND BS5500 CODE	FAILURE PRESSURE
412.894 MM.	2	7.993 MM.	7.500 MPa	10.000 MM.	7.088 MPa
392.893 MM.	3	5.367 MM.	7.500 MPa	10.000 MM.	6.197 MPa
370.895 MM.	4	6.075 MM.	7.500 MPa	10.000 MM.	6.414 MPa
345.871 MM.	5	6.420 MM.	7.500 MPa	10.000 MM.	6.521 MPa
319.448 MM.	6	6.588 MM.	7.500 MPa	10.000 MM.	6.572 MPa

** DATA STORED ON FILE MODLA.SBD **

MODEL B

** INPUT DATA **

1 THICKNESS OF PLATING 20.0000 MM.

2 FRAME SPACING 800.0000 MM.

3 WIDTH OF FAYING FLANGE IN CONTACT WITH PLATING 0.0000 MM.

4 THICKNESS OF FAYING FLANGE 0.0000 MM.

5 DEPTH OF FRAME WEB 150.0000 MM.

6 THICKNESS OF FRAME WEB 15.0000 MM.

7 WIDTH OF FRAME INNER FLANGE 150.0000 MM.

8 THICKNESS OF FRAME FLANGE 15.0000 MM.

1 RADIUS OF MEAN SURFACE OF SHELL PLATING 4000.00 MM.

2 DISTANCE BETWEEN RIGID ENDS 8000.0 MM.

3 YIELD STRESS OF SHELL PLATING 600. MPa

4 YIELD STRESS OF FRAME FLANGE 450. MPa

5 POISSONS RATIO 0.300

6 YOUNGS MODULUS 207000. MPa

7 FRAMING TYPE 1. INTERNAL FRAMING

8 OUT OF CIRCULARITY -1.000 MM.

9 MULTIPLIER FOR RESIDUAL FRAME STRESS 1.20

RADIUS THICKNESS RATIO a/h 200.00

LENGTH RADIUS RATIO L/a 2.000

1 THICKNESS OF ENDCAP 30.0000 MM.

2 RADIUS OF ENDCAP 4000.0000 MM.

1 SPECIFIED MAXIMUM DEPTH 125.0 M.

MAXIMUM DESIGN PRESSURE 1.250 MPa

2 SAFETY FACTOR FOR STRESS COMPARISONS - 1.50
SAFETY FACTOR FOR INTERFRAME BUCKLING COMPARISONS - 2.50
SAFETY FACTOR FOR OVERALL BUCKLING COMPARISONS - 3.50

EFFECTIVE LENGTH OF SHELL IS FROM BS5500 TABLES

******CALCULATED OUTPUT******

	PRESSURE	SAFETY FACTOR	
INTERFRAME COLLAPSE			
VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE PM WINDENBURG AND TRILLING	2.222 MPa	1.7774	CHECK
KENDRICKS MINIMIZED MODIFIED VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE AT WAVE NUMBER= 19	2.209 MPa	1.7672	CHECK
BS5500 LOWER BOUND COLLAPSE CURVE GIVES PM1/PY=0.68 PC/PY=0.34 AND COLLAPSE PRESSURE =	1.104 MPa	0.8836	CHECK
***** SHELL YIELDING**			
HOOP STRESS FOR AN UNSTIFFENED CYLINDER PB	3.000 MPa	2.4000	
PRESSURE WHERE CIRCUMFERENTIAL STRESS EQUALS YIELD STRESS IN PLATING P3 - WILSON LINEAR FORMULATION	3.042 MPa	2.4333	
PRESSURE WHERE MAX. LONGITUDINAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P7 - WILSON	2.876 MPa	2.3010	
PRESSURE WHERE MEAN MIDBAY CIRCUMFERENTIAL PLATING STRESS EQUALS YIELD STRESS P5 - WILSON	3.256 MPa	2.6047	
PRESSURE WHERE MEAN MIDBAY STRESS IN PLATING WITH HENCKY-VON MISES YIELD CRITERION EQUALS YIELD	3.755 MPa	3.0040	
***** FRAME YIELDING**			
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN THE STANDING FLANGE EQUALS YIELD PFY - WILSON	3.811 MPa	3.0488	
***** MORE ACCURATE ITERATED SOLUTION FOR STRESS SALERNO AND PULOS NONLINEAR FORMULATION - SHELL**			
PRESSURE AT WHICH THE MAXIMUM CIRCUMFERENTIAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P3A	2.736 MPa	2.1886	
PRESSURE AT WHICH THE LONGITUDINAL STRESS IN THE IN THE PLATING EQUALS THE YIELD STRESS P7A	2.542 MPa	2.0332	
PRESSURE AT WHICH THE CIRCUMFERENTIAL STRESS IN PLATING AT MIDBAY EQUALS THE YIELD STRESS P5A	3.004 MPa	2.4032	
*****FRAME STRESS**			
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN STANDING FLANGE OF THE FRAME EQUALS YIELD - PFYA	6.569 MPa	5.2553	

***BS5500 ENDCAP RESULTS

PRESSURE AT WHICH ENDCAP REACHES YIELD	9.000 MPa	7.2000
CRITICAL BUCKLING PRESSURE OF ENDCAP	14.089 MPa	11.2712
ULTIMATE COLLAPSE PRESSURE FROM BS5500 P/PY CURVE THIS IS THE LOWER BOUND CURVE OF EXPERIMENTAL DATA	2.196 MPa	1.7571

***OVERALL COLLAPSE

SYMMETRIC BUCKLING PRESSURE OF THE RING-SHELL COMBINATION - WAVE NUMBER 0 MODE	7.823 MPa	6.2580

BRYANT OVERALL BUCKLING WITH EFFECTIVE WIDTH
BRESSE SOLUTION IS FOR A SINGLE STIFFENER AND SHELL
SECTION ASSUMMING SHELL FULLY EFFECTIVE

		COLLAPSE PRESSURE		
MODE	EFFECTIVE LENGTH OF SHELL	BRESSE	BRYANT	
2	459.645	0.732	36.306 MPa	29.0447
3	452.826	1.945	7.132 MPa	5.7059
4	444.351	3.628	4.766 MPa	3.8128
5	430.460	5.755	6.086 MPa	4.8685
6	415.269	8.308	8.426 MPa	6.7407

***CHECK ON STIFFENER PROPORTIONS FROM BS5500

CRITICAL TRIPPING STRESS OF STIFFENER = 422.569 MPa
THIS IS 0.9390 TIMES YIELD

GENERAL STIFFENER PROPORTIONS 0.00204
LESS THAN CODE RECOMMENDATIONS 0.00217

WEB DEPTH TO THICKNESS RATIO 10.000
IS WITHIN CODE RECOMMENDATIONS 23.592

HALF FLANGE WIDTH TO THICKNESS RATIO 5.000
IS WITHIN CODE RECOMMENDATIONS 10.724

***STIFFENER FLANGE FAILURE

TWO VALUES OF OUT OF CIRCULARITY ARE USED TO DETERMINE THE FAILURE PRESSURE OF THE STIFFENER - KENDRICKS FORMULA WHICH IS A FUNCTION OF WAVE NUMBER AND OVERALL BUCKLING LOAD AND EITHER THE BS5500 VALUE OR A GIVEN OOC VALUE, BOTH OF WHICH ARE CONSTANT WITH WAVE NUMBER.

EFFECTIVE LENGTH OF SHELL	MODE	OUT OF ROUND ALLOWABLE	FAILURE PRESSURE	OUT OF ROUND BS5500 CODE	FAILURE PRESSURE
459.645 MM.	2	538.402 MM.	1.875 MPa	20.000 MM.	3.093 MPa
452.826 MM.	3	29.036 MM.	1.875 MPa	20.000 MM.	2.108 MPa
444.351 MM.	4	8.012 MM.	1.875 MPa	20.000 MM.	1.336 MPa
430.460 MM.	5	7.686 MM.	1.875 MPa	20.000 MM.	1.263 MPa
415.269 MM.	6	8.553 MM.	1.875 MPa	20.000 MM.	1.292 MPa

** DATA STORED ON FILE modlb.SBD **

MODEL C

** INPUT DATA **

1 THICKNESS OF PLATING 23.0000 MM.

2 FRAME SPACING 760.0000 MM.

3 WIDTH OF FAYING FLANGE IN CONTACT WITH PLATING 0.0000 MM.

4 THICKNESS OF FAYING FLANGE 0.0000 MM.

5 DEPTH OF FRAME WEB 203.0000 MM.

6 THICKNESS OF FRAME WEB 6.1000 MM.

7 WIDTH OF FRAME INNER FLANGE 101.0000 MM.

8 THICKNESS OF FRAME FLANGE 8.4000 MM.

1 RADIUS OF MEAN SURFACE OF SHELL PLATING 3086.00 MM.

2 DISTANCE BETWEEN RIGID ENDS 13680.0 MM.

3 YIELD STRESS OF SHELL PLATING 450. MPa

4 YIELD STRESS OF FRAME FLANGE 450. MPa

5 POISSONS RATIO 0.300

6 YOUNGS MODULUS 207000. MPa

7 FRAMING TYPE 1. INTERNAL FRAMING

8 OUT OF CIRCULARITY -1.000 MM.

9 MULTIPLIER FOR RESIDUAL FRAME STRESS 1.00

RADIUS THICKNESS RATIO a/h 134.17

LENGTH RADIUS RATIO L/a 4.433

1 THICKNESS OF ENDCAP 0.0000 MM.

2 RADIUS OF ENDCAP 0.0000 MM.

1 SPECIFIED MAXIMUM DEPTH 200.0 M.

MAXIMUM DESIGN PRESSURE 2.000 MPa

2 SAFETY FACTOR FOR STRESS COMPARISONS - 1.50
SAFETY FACTOR FOR INTERFRAME BUCKLING COMPARISONS - 2.50
SAFETY FACTOR FOR OVERALL BUCKLING COMPARISONS - 3.50

EFFECTIVE LENGTH OF SHELL IS FROM BS5500 TABLES

******CALCULATED OUTPUT******

	PRESSURE	SAFETY FACTOR	
INTERFRAME COLLAPSE			
VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE PM WINDENBURG AND TRILLING	4.821 MPa	2.4107	CHECK
KENDRICKS MINIMIZED MODIFIED VON MISES ELASTIC INTERFRAME BUCKLING PRESSURE AT WAVE NUMBER= 15	4.790 MPa	2.3948	CHECK
BS5500 LOWER BOUND COLLAPSE CURVE GIVES PM1/PY=1.37 PC/PY=0.64 AND COLLAPSE PRESSURE =	2.219 MPa	1.1096	CHECK
***** SHELL YIELDING**			
HOOP STRESS FOR AN UNSTIFFENED CYLINDER PB	3.354 MPa	1.6769	
PRESSURE WHERE CIRCUMFERENTIAL STRESS EQUALS YIELD STRESS IN PLATING P3 — WILSON LINEAR FORMULATION	3.369 MPa	1.6846	
PRESSURE WHERE MAX. LONGITUDINAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P7 — WILSON	4.198 MPa	2.0990	
PRESSURE WHERE MEAN MIDBAY CIRCUMFERENTIAL PLATING STRESS EQUALS YIELD STRESS P5 — WILSON	3.492 MPa	1.7462	
PRESSURE WHERE MEAN MIDBAY STRESS IN PLATING WITH HENCKY-VON MISES YIELD CRITERION EQUALS YIELD	4.031 MPa	2.0157	
***** FRAME YIELDING**			
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN THE STANDING FLANGE EQUALS YIELD PFY — WILSON	4.488 MPa	2.2438	
***** MORE ACCURATE ITERATED SOLUTION FOR STRESS SALERNO AND PULOS NONLINEAR FORMULATION — SHELL**			
PRESSURE AT WHICH THE MAXIMUM CIRCUMFERENTIAL STRESS IN THE PLATING EQUALS THE YIELD STRESS P3A	3.280 MPa	1.6402	
PRESSURE AT WHICH THE LONGITUDINAL STRESS IN THE IN THE PLATING EQUALS THE YIELD STRESS P7A	3.934 MPa	1.9670	
PRESSURE AT WHICH THE CIRCUMFERENTIAL STRESS IN PLATING AT MIDBAY EQUALS THE YIELD STRESS P5A	3.432 MPa	1.7159	
*****FRAME STRESS**			
PRESSURE WHERE CIRCUMFERENTIAL STRESS IN STANDING FLANGE OF THE FRAME EQUALS YIELD — PFYA	4.625 MPa	2.3126	

***OVERALL COLLAPSE

SYMMETRIC BUCKLING PRESSURE OF THE RING-SHELL
COMBINATION - WAVE NUMBER 0 MODE 16.158 MPa 8.0790

BRYANT OVERALL BUCKLING WITH EFFECTIVE WIDTH
BRESSE SOLUTION IS FOR A SINGLE STIFFENER AND SHELL
SECTION ASSUMING SHELL FULLY EFFECTIVE

MODE	EFFECTIVE LENGTH OF SHELL	COLLAPSE PRESSURE BRESSE	COLLAPSE PRESSURE BRYANT		
2	429.490	1.452	7.355 MPa	3.6774	
3	418.590	3.858	4.380 MPa	2.1898	CHECK
4	406.256	7.200	7.294 MPa	3.6468	
5	388.931	11.441	11.466 MPa	5.7331	
6	370.565	16.555	16.564 MPa	8.2819	

***CHECK ON STIFFENER PROPORTIONS FROM BS5500

CRITICAL TRIPPING STRESS OF STIFFENER = 161.924 MPa
THIS IS 0.3598 TIMES YIELD

GENERAL STIFFENER PROPORTIONS 0.00078
LESS THAN CODE RECOMMENDATIONS 0.00217

WEB DEPTH TO THICKNESS RATIO 33.279
GREATER THAN CODE RECOMMENDATIONS 23.592

HALF FLANGE WIDTH TO THICKNESS RATIO 6.012
IS WITHIN CODE RECOMMENDATIONS 10.724

***STIFFENER FLANGE FAILURE

TWO VALUES OF OUT OF CIRCULARITY ARE USED TO DETERMINE THE
FAILURE PRESSURE OF THE STIFFENER - KENDRICKS FORMULA WHICH
IS A FUNCTION OF WAVE NUMBER AND OVERALL BUCKLING LOAD AND
EITHER THE BS5500 VALUE OR A GIVEN OOC VALUE,
BOTH OF WHICH ARE CONSTANT WITH WAVE NUMBER.

EFFECTIVE LENGTH OF SHELL	MODE	OUT OF ROUND ALLOWABLE	FAILURE PRESSURE	OUT OF ROUND BS5500 CODE	FAILURE PRESSURE
429.490 MM.	2	16.968 MM.	3.000 MPa	15.430 MM.	3.076 MPa
418.590 MM.	3	2.022 MM.	3.000 MPa	15.430 MM.	1.570 MPa
406.256 MM.	4	3.368 MM.	3.000 MPa	15.430 MM.	1.646 MPa
388.931 MM.	5	4.173 MM.	3.000 MPa	15.430 MM.	1.730 MPa
370.565 MM.	6	4.612 MM.	3.000 MPa	15.430 MM.	1.779 MPa

** DATA STORED ON FILE modlc.SBD **

References

1. Gill S.S., 'The Stress Analysis of Pressure Vessels and Pressure Vessel Components', Pergamon Press, Toronto, 1970.

2. BS5500, British Standard Specification for Unfired Fusion Welded Pressure Vessels, Issue 5, April 1980.

3. Bushnell D., 'BOSOR4: Program for Stress, Buckling and Vibration of Complex Shells of Revolution', Structural Mechanics Laboratory, Lockheed Missiles and Space Co. Inc., Palo Alto, California.

4. Faulkner D., 'The Collapse Strength and Design of Submarines', RINA Symposium on Naval Submarines, London, 1983.

5. Biezeno C.B., Grammel R.,'Engineering Dynamics Vol II', Blackie and Sons Ltd., Glasgow, 1956, pp 348-357.

6. European Recommendations for Steel Construction: Buckling of Shells, ECCS - Technical Committee 8, Third Edition, Brussels, 1984.

7. Timoshenko S.P., Gere J.M.,'Theory of Elastic Stability', McGraw-Hill Inc., New York, 1961.

8. Windenburg D.F., Trilling C., 'Collapse by Instability of Thin Cylindrical Shells under External Pressure', Trans ASME, Vol 56, 1934.

9. Kendrick S.,'The Buckling Under External Pressure of Ring Stiffened Circular Cylinders', Trans RINA, Vol 107, 1965.

10. Salerno V.L., Pulos J.G.,'Stress Distribution in a Circular Cylindrical Shell Under Hydrostatic Pressure Supported by Equally Spaced Circular Ring Frames - Part 1 Theory', Polytechnic Institute of Brooklyn, June, 1951.

11. Bryant A.R., 'Hydrostatic Pressure Buckling of a Ring Stiffened Tube', Naval Construction Research Establishment Report 306, October, 1954.

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 12.) Defence Research Establishment Atlantic	2. DOCUMENT SECURITY CLASSIFICATION (overall security classification of the document, including special warning terms if applicable) Unclassified

3. DOCUMENT TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C,R or U) in parentheses after the title.)

PRHDEF - STRESS AND STABILITY ANALYSIS OF RING STIFFENED SUBMARINE PRESSURE HULLS

4. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

Technical Memorandum

5. AUTHOR(S) (Last name, first name, middle initial. If military, show rank, e.g. Doe, Maj. John E.)

Pegg, N.G. and Smith, D.R.

6. DOCUMENT DATE (month and year of publication of document) JUNE 1987	7a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.) 45	7b. NO. OF REFS (total cited in document) 11

8a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant)	8b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)
9a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.) DREA TECHNICAL MEMORANDUM 87/213	9b. OTHER DOCUMENT NO.(S) (Any other numbers which may be assigned this document either by the originator or by the sponsor)

10. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)

(X) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; further distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other

11. ANNOUNCEMENT AVAILABILITY (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availabilty (10). However, where further distribution (beyond the audience specified in 10) is possible, a wider announcement audience may be selected.)

12. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.)

Defence Research Establishment Atlantic

SAMPLE PRINT OF FORM : DCD03

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), (R), or (U). It is not necessary to include here abstracts in both offical languages unless the text is bilingual).

Preliminary investigation of the structural integrity of a submarine pressure hull can be accomplished by the use of design formulae. Approximate solutions for stress and stability of uniformly stiffened cylinders subject to hydrostatic pressure have been assembled and incorporated in the computer code PRHDEF. The British pressure vessel code, BS5500, and other codes have been used where appropriate. Critical pressures are determined for yielding in the frames and shell, for interframe and overall bifurcation buckling and for collapse of the stiffened shell and endcap. The effect of out of circularity on frame failure is considered and dimension checks for stiffener tripping are made. The background and limitations of the various equations are discussed and results are compared with those obtained using the axisymmetric finite difference program BOSOR4.

The methods described in this report are particularly useful for comparison of various design alternatives on a common basis and for preliminary investigation before more complex and costly finite element or finite difference analyses are undertaken.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible, keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Buckling
Stability
Pressure hull
Submarine
Pressure vessel
Structural design
Ring stiffened

D
R
E
A

DEFENSIO PER SCIENTIAM

C
R
D
A