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THE GEOMETRY OF MARINE PROPELLERS

Donald R. Smith - John E. Slater

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

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Approved by L.J. Leggat Director/Technology Division

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ABSTRACT

The geometry of marine propellers is described in this report. The types of data used for blade section description are covered and methods for generating blade section leading and trailing edge forms are derived. A method for generating the blade tip from minimal data is given. The computer program BLADE developed for generating 3 dimensional propeller blade coordinates is described together with examples of its graphic output.

Sommaire

Ce rapport décrit la géométrie des hélices marines. Les auteurs traitent des types de données utilisées pour décrire le profil de la lame et le développent des méthodes de production des formes des bords d'attaque et de fuite des lames. De plus, une méthode de production de l'incidence de la lame à partir d'une très faible quantité de données est présentée. Les auteurs décrivent aussi le logiciel BLADE, développé pour le calcul des coordonnées tridimensionnelles de lame d'hélice et donnent des exemples de sorties graphiques.

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NOTATION

A_i	coefficient of cubic spline, basic form offset at point i
B_i	coefficient of cubic spline, slope at end point i
C_i	coefficient of cubic spline
c	blade section chord station as fraction of chord
D_i	coefficient of cubic spline
D	propeller diameter
CF_j	fraction of blade section chord for j^{th} offset
C_{ℓ_i}	tabulated section design lift coefficient (NACA section data) per unit width
$C_{\ell_{in}}$	required lift coefficient per unit width
C_{ℓ_r}	lift coefficient ratio
FR	fillet radius
LER	Leading edge radius (NACA section data)
P_i	pitch of blade at the i^{th} section
PMT	point of maximum section thickness as fraction of chord
PT	Points of tangency at leading edge
RF	fraction of full blade radius for a specified section
R_i	blade section radius for i^{th} section
RR	full blade radius from center of rotation to blade tip
RK	blade rake
r	leading edge radius = LER
r_t	radius of blade tip
S	chord length
S_m	modified chord length
SK	skew

T	maximum section thickness
T_f	thickness of the blade at the intersection with hub
TB	offset distance from section chord to back surface
TF	offset distance from section chord to face surface
T_s	maximum thickness to chord ratio
TER	trailing edge radius
WK	linear dimension for warp
X, Y, Z	blade global coordinates
x	blade section chord station distance from leading edge
x_m	mean line chord station distance from leading edge
x_c	blade section chord station distance as percent of chord
X_L	chordline distance to where a normal to the meanline meets the face contour of a section
X_U	chordline distance to where a normal to the meanline meets the back contour of a section
X_{ij}	j^{th} station offset for the i^{th} section
X_r	X coordinate for the center of the leading edge radius
XB	global X coordinate for blade back surface
XMB	global cartesian coordinates for measurement of blade back surface
XMF	global cartesian coordinates for measurement of blade front surface
y	basic form offset as percent of chord
y_ℓ	mean line offset at x modified by lift coefficient
y_c	mean line offset at x as percent of chord
y_m	mean line offset with lift coefficient of 1
y_{max}	maximum value of y

y_f	mean line offset as fraction of chord
y_t	basic form offset and length of normal to the mean line at x_m
$\frac{dy_c}{dx_c}$	mean line slope
Y_L	blade section offset distance to face contour
Y_U	blade section offset distance to back contour
YB	global Y coordinate for blade back surface
ZMF	global cartesian coordinates for measurement of blade front surface
ZMB	global cartesian coordinates for measurement of blade back surface
α_i	projected skew angle or warp angle of rotation about Z axis for i^{th} section
β_i	angle for generating chord stations for i^{th} section
Φ_i	pitch angle or angle of rotation about Y axis for the i^{th} section
θ	angle of the tangent to the mean line
λ	angle of rotation about X axis
ζ	transformed axis $\zeta = \sqrt{X}$
ω_{ij}	projected angle in radians of front and back offsets from Y axis

1. INTRODUCTION

The Defence Research Establishment Atlantic (DREA) is conducting research on the hydrodynamics and strength of marine propellers. Because a marine propeller is a complex three-dimensional form, the shape must be described adequately for strength determination or manufacture. The approach generally taken is to define the geometry of a single blade in detail including the intersection with the hub.

Many different blade shapes[1] have been developed (Figure 1) for reasons such as the reduction of unsteady hydrodynamic loading on the blades when the propeller is operating in a highly non-uniform wake behind the ship's hull. Thus, the method chosen for geometry definition must be able to take into account a wide variety of blade forms.

Propeller blade geometry is most often supplied as two-dimensional data in the form of blade sections from which three-dimensional spatial coordinates must be generated. Specification of the blade shape can be complicated by the fact that no one universal format and axis system has been established by blade designers for presenting the geometry data. Therefore, the method developed to generate the spatial coordinates required for structural analysis, manufacture and measurement must be easily adapted to the various forms in which the data are presented.

This report describes in detail the geometry of propellers and an exact method for converting the two-dimensional data into three-dimensional coordinates. The method can handle blade geometry in the form of NACA section thickness and camberline data or in the form of individual section coordinates. In regions such as the leading edge and the blade tip, where coordinate information is frequently insufficient, the procedure has been further developed to fit acceptable faired curves.

To reduce the effort required to produce propeller geometry data, a computer program has been developed based on the method of geometry definition described in this report. It can be used to generate three-dimensional blade coordinates and to display graphically the geometry data and the propeller blade shape according to naval architectural convention.

2. PROPELLER GEOMETRY

2.1 Blade Description

Marine propellers are best described by defining their parts. The four-bladed variable pitch propeller shown in Figure 2 illustrates the major propeller parts. Variable pitch propellers can rotate each blade around its spindle axis; otherwise, they are similar in geometry to fixed pitch propellers except at the blade hub intersection. This intersection portion of a variable pitch propeller is referred to as the blade palm, which is a circular base centered on the spindle axis. The palm frequently, though not always, forms a portion of the spherical shaped hub. A sphere is used to ensure a good hydrodynamic fairing at the blade hub intersection regardless of the angle at which the blade is set.

Fixed pitched propeller blades, on the other hand, (Figure 3) intersect with a hub that can vary in shape from spherical to cylindrical and from barrel to conical. In this case, the blades are cast integral with the hub or they are bolted to it.

A propeller blade has two main hydrodynamic surfaces. The surface of the blade which faces aft is referred to as the face of the blade. The surface which faces forward is called the back of the blade. A propeller which normally rotates clockwise when viewed from astern is called right handed and the leading edge is that edge to the right formed by the intersection of the face and back surfaces of an upward oriented blade. The trailing edge is the edge to the left. The reverse is true for counterclockwise rotating propellers generally referred to as left handed propellers. The convention used in this report assumes a right handed propeller; therefore, the illustrations of the geometry are shown with the leading edge to the right. The exceptions are in the derivation of section coordinates where, for more mathematical convenience, the sections are shown with the leading edge to the left in accordance with airfoil convention.

The tip of the blade joins the leading edge to the trailing edge at the intersection of the face and back surfaces at the maximum radius from the center of rotation from which the blade extends.

To make a smooth transition from the blade to the hub or palm, a fillet is used (Figure 4). The fillet is most easily formed by a series of single radius curves, the radii varying between a maximum near mid chord and a minimum at the leading and trailing edges. Compound curve- are often used in attempts to produce constant stress fillets. These are described in more detail later in the report.

The assembly of blades forming a propeller has often been referred to as a screw, as in the case of machine screw, because they are similar in concept. A propeller blade is generated from a series of fully developed sections such as the one shown in Figure 5. Each of the sections forming the blade is located at a specified radius R_i from the center of rotation. Each section is then rotated about its vertical or spindle axis to a prescribed angle ϕ_i and then wrapped or curved so as to fit on a fictitious cylinder with a radius equivalent to the specified radius for the section. The chord line of the section thus forms part of a helix on the cylinder in the same manner as a screw thread. By treating each of the sections in this manner and filling and blending between the sections, two smooth and continuous surfaces are produced forming the face and back of the propeller blade.

2.2 Blade Coordinate Systems

The coordinate system for a propeller blade in most cases is specified in the form of right handed XYZ cartesan coordinates. Unfortunately, there is no uniform standard for propeller coordinate systems, as shown in Figures 6a to 6d. The Defence Research Establishment Atlantic (DREA) axis system (Figure 6a) has the Z axis positive aft coinciding with the axis of propeller shaft rotation. The Y axis is the vertical reference referred to as the pitch or spindle axis, about which a controllable pitch blade is rotated to change pitch. The Y axis is positive up and thus the X axis is positive to starboard.

This axis system was chosen because it maintains the XY plane as the major projection plane when looking forward thus placing the principal transverse view in the most widely used XY coordinate plane. Blade cross sections are described in the XZ plane which is close to the convention used for NACA section data where X is the chord line axis.

2.3 Pitch

As described previously, a propeller blade is formed from fully developed sections located at specified radii R_i placed so that their chord lines are at angles Φ_i to the horizontal X axis in the XZ plane (Figure 7). This angle Φ_i called the pitch angle, follows the right hand convention of positive angles counterclockwise about the Y or pitch axis. A view of the section on the XZ plane is also shown in Figure 8 which represents a surface of a cylinder of radius R_i rolled out to its fully developed length $2\pi R_i$. When this surface is wrapped around a cylinder of radius R_i, the chord line and its extension form a true helix.

In one revolution of the cylinder, a point on the helix travels a negative Z distance $P_i = 2\pi R_i \tan \Phi_i$

where P_i = pitch of section i

R_i = radius of the cylinder for section i

Φ_i = pitch angle of section i

The pitch P_i is the same axial distance a screw, of the same cylindrical diameter and pitch angle, would advance in one revolution.

The propeller designer must therefore specify the pitch P_i or the pitch angle Φ_i for each section at its radius R_i. He must also specify the full diameter D of the propeller. Frequently, these data are supplied as dimensionless values P_i/D called pitch to diameter ratios for each radius R_i. The pitch angles, if not given, can be extracted from this information.

$$\Phi_i = \tan^{-1} \frac{P_i/D}{2\pi R_i/D} \qquad (2.1)$$

A pitch diagram (Figure 9) can be drawn to show the pitch distribution with respect to the cylindrical radii. The diagram should form a smooth curve when plotted with the pitch P_i as the abscissa and the radii R_i as the ordinate, thus acting as a check of the data and the design.

Pitch is also the parameter for specifying the angle to which a controllable pitch propeller is set. Generally, for this purpose, the pitch angle ϕ_i of the blade section at the radius R_i of 0.7 of the full radius is used as a reference angle, and the setting is specified as a P/D ratio where

$$\frac{P}{D}_{0.7} = 0.7\pi \tan \phi_{0.7} \qquad (2.2)$$

2.4 Skew

Blades of simple geometry have the midchords or, in some cases, points of maximum thickness of the blade sections coincident with the blade pitch or spindle axis (Figure 6). Other blades of more sophisticated geometry have their sections displaced along their chord lines from the spindle axis. This displacement is called skew. It is applied to reduce the unsteady hydrodynamic loading on the blade when the propeller is operating in a highly non-uniform wake behind the ship's hull. Skew can be shown on the fully developed unwrapped view of the blade section (Figure 10) as a linear dimension along the section chord line from the intersection with the Z axis. The skew of the section is referenced to its midchord or point of maximum thickness (ie. the blade section reference point). Positive skew moves the section around the shaft axis in the flow direction, opposite to the direction of shaft rotation.

Skew can also be shown as a true linear dimension on the diagram of the expanded view of the blade (Figure 11). The fully developed sections are laid out flat on the XY plane with their section reference points displaced in the direction of the negative X axis a distance equal to the skew along the blade reference line. Positive skew is, therefore, measured along the negative X axis and negative skew along the positive X axis.

Skew is sometimes specified in angular form as an angle α_i, called the skew angle, projected on the XY plane, through which the wrapped section reference point is displaced (Figure 12). In this case, the linear skew dimension SK_i for the i^{th} section can be obtained by multiplying the projected skew angle α_i (radians) by the radius R_i and dividing by the cosine of the pitch angle ϕ_i, ie.

$$SK_i = \frac{R_i \alpha_i}{\cos \phi_i} \qquad (2.3)$$

2.5 Rake

Rake is an aft or forward displacement of the blade section reference point from the XY plane. It is used to increase clearance between the hull and propeller. As shown in Figure 13, a blade section can be caused to have rake in two ways. In the case of skewed blades, rake is induced when the sections are displaced along the pitch helix. The Z distance between the section reference point and the intersection of the chord and the YZ plane is called the skew induced rake. The conventional method to introduce applied rake (RK) is to offset each blade section an additional distance in the Z direction from the X axis. The rake distance RK is from the X axis datum (actually the XY plane) to the chord line YZ plane intersection. Offset aft produces positive rake and offset forward produces negative rake.

2.6 Warp

When a blade is given skew, it automatically moves the blade sections aft along the helix imposing the skew induced rake on the blade (Figure 13). If it is desirable to maintain the blade sections in the plane of rotation, then the blade sections are moved back by imposing rake on the sections equal to the negative of the skew induced rake (Figure 14a). Warp can thus be defined as the angular displacement equal to the projected skew angle of the blade sections measured from the Y axis with any axial displacement such as skew induced rake removed (Figure 14b). The linear dimension for the warp of section i (Figure 14a and 14c) on the unwrapped view of the blade and the expanded outline is

$$WK_i = R_i \alpha_i \qquad (2.4)$$

where α_i = warp or the projected skew angle of the i^{th} section.

2.7 Blade Sections

The logical process for determining the geometry of a propeller begins with the selection of the blade section. The designer may develop his own or choose from a variety of proven sections whose hydrodynamic performance have been characterized and recorded.

A blade section (Figure 15) is initially described in its fully developed or expanded form[2]. In this form, the shape is most often described by offsets TF and TB at fractions c of the section chord line length S. The section contour formed by the upper offsets TB measured from the chord line or datum is called the back or suction side of the section. The section contour formed by the lower offsets TF is called the face or pressure side. The chord line in addition to forming the datum for the section offsets acts as the line by which the section is angled to the horizontal datum, referred to as the pitch angle. Normally, the face of the section is convex with the offsets below the chord line which are considered to be positive values. When the offsets are all on or above the chord line such that the face is flat or concave, then the chordline is also referred to as the face pitch line (Figure 16).

2.7.1 Airfoil Section

If airfoil sections are chosen for marine propeller blades, then airfoil convention is used with the leading edge to the left, and the offsets are given normal to a camber or mean line rather than normal to the chord line (Figure 17). When blade section data are presented in the format for NACA airfoil sections, the data have to be converted to offsets from the chord line. The basic thickness forms and mean line data for a series of sections can be found in Reference 3.

In addition to the basic form and the mean line data, the lift coefficient and the leading edge and trailing edge radii are also supplied. For example, a section from the NACA five digit series may be specified as NACA 16-309 with a mean line NACA a = 1.0 where a is the fraction of the chord uniformly loaded at the ideal angle of attack. This is interpreted as a 16 series basic form with a thickness to chord ratio T/S of .09 or nine percent as indicated by the last two digits. The first digit after the dash is the lift coefficient C_ℓ expressed as a decimal. The lift coefficient in this example is 0.3. Referring to Figure 17 and page 382 of Reference 3, the mean line curve is plotted from the data table or from an equation such as the following, which is for a uniform chordwise pressure distribution at zero angle of attack.

$$y_f = C_\ell \frac{y_m}{S} = -\frac{C_\ell}{4\pi}\left[\left(1-\frac{x_m}{S}\right)\ln\left(1-\frac{x_m}{S}\right)+\frac{x_m}{S}\ln\frac{x_m}{S}\right] \quad (2.5)$$

$$\frac{dy_c}{dx_c} = \frac{C_\ell}{4\pi}\left[\ln\left(1-\frac{x_1}{S}\right)-\ln\left(\frac{x_1}{S}\right)\right] \quad (2.6)$$

where

x_1 = fixed point on the x axis

x_m = distance along chord from the leading edge

y_m = mean line ordinate at lift coefficient of 1.0

x_c = blade section chord station distance as percent of chord

y_c = mean line offset ordinate at x as percent of chord

y_f = mean line ordinate as fraction of the chord

$\frac{dy_c}{dx_c}$ = slope of the mean line at x_c

S = length of chord line

C_ℓ = lift coefficient

A mean line equation which may be more suitable for marine propellers but is too complex to describe here can be found on page 74 of Reference 3 as equation 4.27.

The basic thickness form data, page 309 Reference 3, is applied to the mean line ordinates and normal to the mean line local slope dy_c/dx_c. Other design lift coefficients can be accommodated by multiplying the tabulated values of y_c and dy_c/dx_c by the ratio of the desired lift coefficient to the given lift coefficient.

The normals to the mean line are then transformed to offsets from the chord line for the upper (back) and the lower (face) surface lines of the section by the following expressions. The expressions are based on a local axis system that agrees with airfoil convention (Figure 17). The expressions for TB and TF, the back and face offsets are introduced at this point as these symbols were used previously for non airfoil sections. In addition, their use allows the algebraic signs for the offsets to be made consistent with those chosen for the blade axis system where offsets above the chord line are negative and below the chord line are positive. This approach thus allows the data tables from Reference 3 to be used directly to obtain TB and TF values.

$$\Theta = \tan^{-1}(C_{\ell_r} \cdot dy_c/dx_c) \quad (2.7)$$

$$C_{\ell_r} = \frac{C_{\ell_{in}}}{C_\ell} \quad (2.8)$$

$$x_m = x_c.S/100$$

$$y_t = y.S/100$$

$$y_\ell = y_c \cdot C_\ell .S/100 \quad (2.9)$$

$$X_U = x_m - y_t \sin\Theta \quad (2.10)$$

$$Y_U = y_\ell + y_t \cos\Theta \quad (2.11)$$

$$TB = -Y_U \quad (2.12)$$

$$X_L = x_m + y_t \sin\Theta \quad (2.13)$$

$$Y_L = y_\ell - y_t \cos\Theta \quad (2.14)$$

$$TF = -Y_L \quad (2.15)$$

For the situation where the data tables of Reference 3 do not provide the desired thickness to chord ratio T_S, the basic form offsets can be scaled as follows

$$y_t = \frac{y}{2y_{\ell max}} .S\ T_S \tag{2.16}$$

The expressions for calculating the section offsets when using a varying T_s ratio become

$$X_U = x_m - \frac{y_t}{2y_{tmax}} .\ T_s \sin\Theta \tag{2.17}$$

$$Y_U = y_\ell + \frac{y_t}{2y_{tmax}} .T_s \cos\Theta \tag{2.18}$$

$$X_L = x_m + \frac{y_t}{2y_{tmax}} .\ T_s \sin\Theta \tag{2.19}$$

$$Y_L = y_\ell - \frac{y_t}{2y_{tmax}} .\ T_s \cos\Theta \tag{2.20}$$

2.7.2 Interpolation of Section Coordinates

Interpolation of additional section coordinates is frequently required. One reason is that the upper and lower offsets, when converted to the chord line datum from NACA section data, do not always originate from the same chord locations for each section. Though in this form section coordinates are useful for graphically presenting the geometry of the blade in spatial X, Y, Z cartesian coordinates, they are not suitable for stress analysis, particularly when using the finite element method. It is desirable therefore to generate upper and lower section offsets TB, and TF at identical chord locations.

A computer program BLADGM has been developed for generating the desired section coordinates and is described in Section 4 of this report. The program uses cubic spline interpolation which, together with a section plotting program, is available to display fully developed blade sections on a Tektronix graphics display terminal.

2.7.3 Leading Edge

The leading edge radius (LER), Figure (18), is generally given with the basic form data. The radius specified in the basic form data is the hypothenuse of the triangle at the leading edge with an included angle Θ, where Θ is the slope of the mean line at an X value of 0.5 percent of the chords. Thus, the coordinates for the center of the leading edge radius are

$$X_r = \text{LER} \cdot \cos\left[\tan^{-1}\left(C_\ell \cdot \left.\frac{dy_c}{dx_c}\right|_{0.5\%} \right)\right] \tag{2.21}$$

$$Y_r = \text{LER} \cdot \sin\left[\tan^{-1}\left(C_\ell \cdot \left.\frac{dy_c}{dx_c}\right|_{0.5\%} \right)\right] \tag{2.22}$$

The trailing edge radius if not otherwise specified is given by TER = .01T where T is the maximum thickness of the basic form.

Often insufficient data are supplied to accurately describe the leading edge region of a propeller blade section. There are a number of methods that can be used for generating the curve through the leading edge from the data supplied for the section. One such method depends on estimating the points of tangency (PT) of the leading edge arc (Figure 18) and the curves through the remaining points of the back and face contours. This method is awkward to program and depends on trial and error fitting of the curves using a spline fit.

2.7.3.1 Transformation Method

A second and better method uses the same basic thickness form data and the leading edge radius LER to fit a smooth curve through the data points and the leading edge. A cubic spline fit was chosen for this method as it is compatible with the thickness form equations given on page 117 of Reference 3.

The spline fitting technique is based on passing a set of cubic functions through the points using a new cubic in each interval. The equation for the i^{th} interval between points x_i and x_{i+1} is[4]

$$y = A_i + B_i (x-x_i) + C_i (x-x_i)^2 + D_i (x-x_i)^3 \tag{2.23}$$

The cubic must fit the two end points of the interval $x_i \le x \le x_{i+1}$ (Figure 19a). Therefore at $x = x_i$

$$y_i = A_i \tag{2.23a}$$

Simularly at $x = x_{i+1}$

$$y_{i+1} = A_i + B_i (x_{i+1}-x_i) + C_i (x_{i+1}-x_i)^2 + D_i (x_{i+1}-x_i)^3 \tag{2.23b}$$

An additional requirement is that there must be slope compatibility at the end points of adjoining intervals. For the interval $x_i \leq x \leq x_{i+1}$

$$\text{at } x = x_i \quad \frac{dy}{dx} = B_i \tag{2.23c}$$

$$\text{at } x = x_{i+1} \quad \frac{dy}{dx} = B_i + 2\,C_i\,(x_{i+1} - x_i) + 3D_i(x_{i+1} - x_i)^2 \tag{2.23d}$$

the A_i, B_i, C_i, D_i coefficients can be obtained from the data supplied provided the offsets y_i and the slopes can be found at the end points of the intervals. Unfortunately, at the leading edge, $x_i = x_1 = 0$, (Figure 19 (a)), the slope is infinite and cannot be used to solve for the coefficients. This problem can be overcome by a transformation of the x axis in the interval $x_1 \leq x \leq x_2$ of $x = \zeta^2$. The derivation of the transformation is given in Appendix A.

For the transformed coordinate system, the spline equation is

$$y = A_1 + B_1\,(\zeta-\zeta_1) + C_1\,(\zeta-\zeta_1)^2 + D_1\,(\zeta-\zeta_1)^3 \tag{2.24}$$

$$\text{at } \zeta = \zeta_1 = 0 \tag{2.24a}$$

$$y_1 = A_1 = 0 \tag{2.24b}$$

$$\frac{dy}{d\zeta} = B_1 = \sqrt{2r} \tag{2.24c}$$

where: r = specified leading edge radius LER.

$$\text{at } \zeta = \zeta_2$$

$$y_2 = B_1\,\zeta_2 + C_1\,\zeta_2^2 + D_1\,\zeta_2^3 = A_2 \tag{2.24d}$$

$$\frac{dy}{d\zeta} = B_1 + 2C_1\,\zeta_2 + 3D_1\,\zeta_2^2 = 2B_2\,\zeta_2 \tag{2.24e}$$

As can be seen from Figure 19(b), the transformed curve now has less than infinite slope at $\zeta = 0$ and A_1, B_1, C_1 and D_1 can be obtained in the interval $\zeta_1 \leq \zeta \leq \zeta_2$.

Therefore, since $\zeta = \sqrt{x}$

$$A_1 = 0 \tag{2.25}$$

$$B_1 = \sqrt{2r} \tag{2.26}$$

$$C_1 = \frac{1}{\sqrt{x_2}}\left[\frac{3A_2}{\sqrt{x_2}} - 2B_2\sqrt{x_2} - 2B_1\right] \tag{2.27}$$

$$D_1 = \frac{1}{x_2}\left[2B_2\sqrt{x_2} - \frac{2A_2}{\sqrt{x_2}} + B_1\right] \tag{2.28}$$

and in the first interval

$$y = A_1 + B_1\sqrt{x} + C_1 x + D_1(\sqrt{x})^3 \tag{2.28a}$$

where r = leading edge radius LER

x_2 = end point of interval $x_1 \leq x \leq x_2$

A_2 = the basic form offset at x_2 generally at 0.5 percent of chord

B_2 = the slope at the end point x_2

The slope B_2 is obtained from a cubic spline fit of the offsets y_2 to y_n of the basic form data in the intervals x_2 to x_n. Additional ordinate values are thereby obtained in this first interval from equation 2.28a. The cubic spline fitting equation defining the section from the leading edge over the interval x_1 to x_2 has been programmed and incorporated in the program BLADGM.

2.7.3.2 Elliptic/Hyperbolic Leading Edge

A third method for forming the leading edge is to use an ellipse or hyperbola (Figure 20). In this method the leading edge is formed from two elliptic/hyperbolic portions for which the semi-axes may be different; one for the face and one for the back. The ellipses or hyperbolae (face and back) must have the following properties:

(a) Their axes must be aligned with the chord line,

(b) They must pass through the leading edge point,

(c) They must pass through the offset y_1 at a point x_1 nearest to the leading edge, and

(d) The slope of the ellipse or hyperbola at the offset nearest to the leading edge is the same as the slope B_1 of the smooth curve at that point.

To form the ellipse or hyperbola, offsets at the leading edge, which are a smooth continuation of the section form (Figure 20), must be given. These blade edge offsets are used together with the other available offsets to obtain the slope of the section form, which is the coefficient B_1 of the cubic spline passed through the points to satisfy (d). If the blade edge offsets are not available, then the value of leading edge radius may be substituted.

The basic equation for generating the elliptic/hyperbolic leading edge coordinates is developed from the basic equation for ellipses or hyperbolae,

$$\frac{(a - x)^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (2.29)$$

The slope at the offset nearest the leading edge is

$$B_1 = + \left.\frac{dy}{dx}\right|_{x = x_1} = - \frac{b^2}{a^2} \left(\frac{a - x_1}{y_1}\right) \qquad (2.30)$$

or

B_1 = the slope of the spline *function* fitted through the upper or lower blade edge point.

From the ordinate and slope at x_1 the semi axes can be determined as

$$a = \frac{x_1 (B_1 x_1 + y_1)}{2B_1 x_1 + y_1} \qquad (2.30a)$$

and

$$b^2 = y_1 (y_1 - B_1 (a - x_1)) \qquad (2.30b)$$

to give the final equation for generating additional coordinates for the leading edge in the interval $0 \leq x_i \leq x_1$

$$y_i = \sqrt{b^2 \left(1 - \frac{(a - x_i)^2}{a^2}\right)} \qquad (2.31)$$

2.7.3.3 Leading Edge Extension and Final Coordinates

When the section has camber, the leading edge will extend beyond the basic form chord station $x_1 = 0$ (Figures 18 and 21). Fitting the leading edge to an already cambered section will introduce more complications than just curve fitting the leading edge to the basic form before camber is added. If not already given, the offset coordinates have to be shifted temporarily to the line running through the extreme leading edge point where there is a leading edge radius r. The chord length must be increased by the amount it extends beyond station x_1 to produce the modified chord length S_m which will become the datum for all the chord stations and section offset. Then the back and face contours are curve fitted separately for the contour of the nose. The interpolation stations for identical back and face positions can be input as selected values or generated within the program by the following equation where β is incremented from 0° to 180° according to the required number of stations.

$$x_i = \frac{S_m}{2} (1 - \cos \beta_i) \tag{2.32}$$

After curve fitting the nose, the vertical coordinates are shifted back to the original chord line position.

2.8 Expanded View

The expanded view for a right handed propeller (Figure 11) has been used previously to illustrate the skew of the blade sections of a propeller. The view also shows the fully developed blade sections revolved about their chord lines to form projections on the XY plane. The sections are in true proportion at their relative radial positions with the back surfaces of the sections up and the faces down. Their section reference points are aligned with the Y axis or a specified X distance from it equal to the skew, if it is present. If the section coordinates have been specified from the leading edge as fractions of the chord CF_j, then they are transformed to the new origin as follows (Figure 22a). The j^{th} chord location using mid chord as datum for the i^{th} blade section is given by

$$X_{ij} = S_i /2 - (CF_j S_i + SK_i) \tag{2.33}$$

or using the point of maximum thickness as datum expressed as a fraction of the chord length

$$X_{ij} = S_i . PMT_i - (CF_j S_i + SK_i) \tag{2.33a}$$

where

$X_{ij} = j^{th}$ offset station for i^{th} section

S_i = chord length for i^{th} section

CF_j = chord fraction for j^{th} offset

PMT_i = point of maximum thickness as new datum for i^{th} section as a fraction of the chord

SK_i = skew (see Section 2.4)

A further complication can arise if the chord stations CF_j are specified as fractions of the dimensions from the PMT datum to the leading and trailing edges (Figure 22b). For this case, the chord stations are calculated in two steps. From leading edge to PMT

$$X_{ij} = S_i \cdot PMT_i \cdot CF_j - SK_i \tag{2.33b}$$

From PMT to trailing edge

$$X_{ij} = -(S_i (1-PMT_i) CF_j + SK_i) \tag{2.33c}$$

where CF_j = fraction of chord length from PMT datum to leading and trailing edges

The sections are thus arranged in the proper relationship to the blade datum which is the Y or pitch axis. A continuous curved line drawn through the leading and trailing edges of the sections forms what is called the expanded outline of the blade projected on the XY plane (Figure 11). This is generally chosen as the first graphic representation of the propeller blade geometry and gives the chords, section offsets, section radii and skew in true proportion in accordance with the scale of the drawing.

2.9 Generating the Blade Tip

2.9.1 Tip Outline

The geometric details of the propeller blade frequently do not give sufficient details of the tip outline curve. The last radius and section specified are generally at about 0.95 of the full radius of the propeller (Figure 23). It is necessary therefore to complete the blade outline at the tip. This is not a difficult job for the pattern marker or the draftsman as they can fit a curve by eye using drawing instruments. This process is not practical for a computer geometric definition, and another method must be used. The method chosen is similar in most respects to that used for curve fitting the blade section leading edge as described in 2.7.3 of this report. In this case, the expanded outline is used except that skew is not included.

The outline is generated from the end points of the chords. The chord mid points or points of maximum thickness are located on the spindle or Y axis. Because the tip radius is not specified and is a requirement for the solution, it can be shown (see Appendix B) that based on a parabolic fit and by using the leading and trailing edge coordinates of the last given section, the radius of curvature at the tip can be given by

$$r_t = \frac{x_1^2}{2y_1} \tag{2.34}$$

where

x_1 = the distance from the leading or trailing edge to the Y axis at the last given section radius.

y_1 = distance from the last section radius R_i to the full blade tip radius RR.

The tip outline is then generated using the cubic spline equation described in Section 2.7.3.1. For this operation, it is convenient to shift the origin of the axis system to the tip. In the final calculations, the tip coordinates are transformed back to the original blade axis system and the skew, if any, is added.

2.9.2 Tip Thickness

Once the blade outline at the tip has been formed, then the blade thickness from the last specified radius to the tip must be determined. This is accomplished by using the offsets TB_{max} and TF_{max} at mid chord or at the point of maximum thickness for each of the specified blade sections. The offsets when plotted produce a section which is a good spanwise representation of the blade section shape just as the horizontal blade sections are of the chordwise shape (Figure 24). From these data the section at the blade tip is generated in radial or elliptic/hyperbolic form in the same manner as the leading edge of the chordwise sections in 2.7.3.1 and 2.7.3.2. If the radial form of tip profile generation is chosen then a radius for the tip profile must be specified. If an elliptic/hyperbolic profile is chosen then a known or estimated thickness for the semi-minor axis must be given for generating the axes and thickness offsets using the equations presented in Section 2.7.3.2. In the case where the center of the blade tip (without rake and skew) does not coincide with the vertical reference axis a value CT for the offset may be given. If it is not available then it must be estimated. (Figure 24).

On occasion, the computer generated drawing of the blade thickness diagram, without rake, may show the blade tip profile hooked to the right or left. This can be removed by experimenting with values for CT.

2.9.3 Tip Section Data

Blade chordwise section data for the additional blade radius fractions at the tip are obtained by scaling the last given section data by the ratios of the new and given chord lengths and the ratios of the new and given maximum section thicknesses. Figure 25 shows these data plotted in the form of expanded sections from the last given section to the blade tip. Pitch, skew, and rake for the additional chord lengths, are obtained by interpolating the original data provided values for these parameters are given for the tip in the original data.

2.10 Blade Fillets

The transition between blade and hub is made by fitting a fillet tangent to the blade surface and to the hub or palm. A variety of shapes are used for propeller hubs and palms. For variable pitch propellers, the palm is sometimes of spherical form (Figure 3). Using the same spherical shape for the palm guarantees a smooth uninterrupted surface at the blade hub regardless of blade pitch angle. This is not aways achievable for various reasons, in which case, the spherical radius for a palm can be larger than the radius of the hub.

In the case of fixed pitch propellers, the forms most generally used for hubs are cylindrical and conical. Distorted versions of these forms are called bullet and barrel shaped hubs (Figure 3).

The requirement to fit a fillet for smooth transition between the hub and blade for such a variety of hub forms is most easily accomplished by the pattern maker in the case of cast propellers. It is much more difficult to do in the case of blades machined on an N.C. mill. For N.C. milling, all surfaces must be mathematically described so that machine tool paths can be generated.

The proportions of the fillets are based on hydrodynamic and strength requirements, the idea being to generate a fillet of such proportions as to give a constant stress through the transition while meeting hydrodynamic and manufacturing requirements.

One standard for a constant stress fillet proportions in common use is a compound radius $3T_f$ and $T_f/3$ where T_f is the blade thickness at its intersection with the hub (Figure 26). Unfortunately, this fillet form is not suitable for N.C. machining because it is discontinuous in the second derivative at the tangency points of the two radii and the tangency points with the blade and hub. This problem has been overcome by the use of fillets generated by a continuous function.

2.11 Data Verification

Data plots of the blade data can be generated by the computer program BLADIN. These plots allow inspection of the supplied tabular data in graphic form so that any discontinuities in the basic data can be detected. Computer generated plots of the following data can be produced:

(a) Pitch angle, ϕ, versus blade section radius, R_i (Figure 27);

(b) Skew, SK, versus blade section radius, R_i (Figure 28);

(c) Rake, RK, versus blade section radius, R_i (Figure 29);

(d) Section chord, S, versus blade section radius, R_i (Figure 30);

(e) Leading edge radius, LER versus blade section radius, R_i (Figure 31);

(f) Trailing edge radius, TER, versus blade section radius, R_i (Figure 32);

(g) Maximum camber, y_{max}, versus blade section radius, R_i (Figure 33);

(h) Maximum thickness, T, versus blade section radius, R_i (Figure 34);

(i) Individual section plot (Figure 35).

2.12 Generation of Spatial Coordinates

The spatial coordinates for the blade are generated by transforming the two dimensional coordinates of the expanded view and the blade sections into the three dimensional coordinates of the blade in its final form (Figure 36). The fully developed sections in the expanded view (Figure 11) show skew as an offset SK_i of the mid chord from the Y axis. As stated previously, in some cases, the point of maximum thickness PMT_i is used rather than the mid chord point. If the blade has rake, the unwrapped sections on the XZ plane are offset a distance along the Z axis, an amount equal to the rake RK_i (Figure 13). Each section is rotated about the point of intersection of the chord line with the YZ plane to its specified pitch angle Φ_i, measured between the pitch chord line and the XY plane. Angles measured counterclockwise from the XY plane are positive. If there is no rake, the projection of the line on the XZ plane runs through the origin of the XZ axis system which becomes the center of pitch rotation (Figure 10).

The X and Z coordinates of the mid chord $S_i/2$ of the unwrapped section (Figure 36a)

$$ZK_i = SK_i \sin \Phi_i + RK_i \tag{2.35}$$

$$XK_i = - SK_i \cos \Phi_i \tag{2.36}$$

when warp is specified instead of skew (Figure 14) then

$$ZK_i = 0$$

$$XK_i = - R_i \alpha_i = - WK_i \tag{2.36a}$$

The shape of the section about the pitch line is described in the local coordinate system $X' Z'$ (Figure 7) by the thickness offsets TB_{ij} and TF_{ij} at fractions c_j of the chord line S_i (Figure 36a). The coordinates of the face and back can be transformed from the local system to the global XYZ system using the following equations and remembering that TB_{ij} and TF_{ij} are negative above the chord line and positive below.

$$ZB_{ij} = ZK_i + S_i\,(c_j - ½)\sin\Phi_i + TB_{ij}\cos\Phi_i \tag{2.37}$$

$$R_i = RF_i \cdot RR \tag{2.38}$$

$$\omega_{B_{ij}} = -XD_{ij}/R_i \tag{2.39}$$

$$= -\frac{1}{R_i}\,(XK_i - S_i(c_j - ½)\cos\Phi_i + TB_{ij}\sin\Phi_i) \tag{2.39a}$$

Wrapping the section about the specified cylindrical radius R_i gives

$$XB_{ij} = -\,R_i\sin\omega_{B_{ij}} \tag{2.40}$$

$$YB_{ij} = R_i\cos\omega_{B_{ij}} \tag{2.41}$$

where

RF_i = fraction of full propeller radius, RR;

c_j = fraction of chord length, S_i for the jth offset location;

R_i = cylindrical radius for section i;

TB_{ij} = back of blade section offset at c_j of radius, R_i;

XB_{ij} = X coordinate of back of blade section offset, TB_{ij};

YB_{ij} = Y coordinate of back of blade section offset, TB_{ij};

ZB_{ij} = Z coordinate of back of blade section offset, TB_{ij};

ZK_i = Z coordinate of mid chord;

XK_i = X coordinate of mid chord of unwrapped section;

S_i = chord length of section at R_i;

Φ_i = pitch angle of chord, S_i, at R_i, positive when measured counterclockwise from the X axis;

XD_{ij} = projected arc length to TB_{ij} from YZ plane (Figure 36b); (Also X coordinate of point TB_{ij} of unwraped section (Figure 36a))

$\omega_{B_{ij}}$ = projected angle in radians to offset TB_{ij} from the Y axis at R_i.

The same equations are used for the face (pressure side) coordinates of the blade except that TF_{ij} are substitutes for TB_{ij}, etc.

In this manner, the two dimensional data supplied by the designer is converted to three dimensional cartesian coordinates, and projections of the blade on the XY, XZ and YZ planes can be produced.

2.13 Conventional Views of the Blade

2.13.1 Transverse View

Once the propeller blade is described in three dimensional cartesian coordinates, various views can be produced through the use of computer graphics. One conventional view is a projection on the XY plane which is referred to here as the transverse view. It is a view of the blade an observer would see if he were standing astern of the ship looking forward at the propeller set at its design pitch (Figure 37).

2.13.2 Longitudinal View

This is a projection of the blade on the YZ plane. It is a view an observer would see when looking at the propeller from the starboard side (Figure 38).

2.13.3 Plan View

The plan view is not always given. When supplied, it is a projection of the blade on the XZ plane (Figure 39).

2.13.4 Expanded View

The expanded view (Figure 11) is generally included in the suite of graphic representations of the propeller geometry as it provides a compact method of displaying the blade sections and the skew.

2.13.5 Pitch Diagram

The pitch diagram (Figure 9) is drawn to show the pitch variation with blade radius. It should form a smooth curve from which the P/D ratio can be determined.

2.13.6 Blade Thickness Diagram

The blade thickness diagram (Figure 40) illustrates the blade spanwise maximum thickness neglecting rake or skew. The thickness for each specified blade section is shown rotated into the YZ plane to give the true scaled dimension for the thickness and its location relative to the Y axes.

2.13.7 Combined Blade Thickness and Rake Diagram

The applied rake of the blade is shown in Figure 41 in combination with the maximum thickness, in true scaled dimension, on the YZ plane. Positive rake is aft as indicated by the arrowhead representing the propeller shaft axis. The angle formed by a line from the tip to the point of intersection of the shaft axis and the Y axis is the global rake angle.

2.13.8 Net Rake and Blade Thickness Diagram

The net rake is the resultant of the imposed rake plus the skew induced rake. It is shown in Figure 42 in combination with the maximum thickness, in true scaled dimension, on the YZ plane.

2.13.9 Combined Drawing of Propeller Blade

The conventional views of the blade are frequently combined to provide all the blade information on a single drawing as shown in third angle projection in Figure 43. Thus, the transverse view superimposed on the expanded outline and the rake and thickness superimposed on the longitudinal view are presented with the pitch diagram. Also, the outline of the swept area of interest is shown superimposed on the longitudinal view. The swept area is especially useful information to have for highly skewed blades as it shows the area that the blade will cut from the YZ plane. Required clearances around the blade can be shown in this manner.

2.14 Rotation of Blade Axes

It is useful to have the ability to rotate the propeller blade and geometric axes system to new positions to produce a variety of views which will give a better representation of the blade than straight orthographic projections. Rotations can be about either the Global XYZ axes or the Local body xyz axes systems using the same coordinate axis transformation matrices. However, the particular axes system selected will dictate the size of the rotation angles and the order in which the transformation matrices are applied. For example, in accordance with the right hand rule, isometric drawings can be produced on the XY plane either by first rotating -45° about the vertical global Y and then 30° about global X or by rotating -45° about local y, -24.94° about local z and 24.94° about local x. In the latter case, however, the local axes system rotates with the body and the multiplications of the rotational transformation matrices have to be done in the reverse order; first the x matrix, then the z matrix and finally the y matrix. A discussion of the three rotational transformation matrices is given briefly.

2.14.1 Rotation Φ About the Global Y or Local y Axis.

The global coordinates of the end points of the new position of the original xyz propeller body axis system are obtained by using the rotational transformation matrix.

$$[\Phi] = \begin{bmatrix} \cos\Phi & 0 & -\sin\Phi \\ 0 & 1 & 0 \\ \sin\Phi & 0 & \cos\Phi \end{bmatrix} \qquad (2.42)$$

Assuming the local axes are unit vectors, the new global coordinates are simply

$$[\{X\}\{Y\}\{Z\}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [\Phi] \qquad (2.43)$$

The same transformation matrix can be applied to any single point. With existing xyz coordinates, the new coordinates of the point would be

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}^T = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}^T [\Phi] \qquad (2.44)$$

2.14.2 Rotation α About the Global Z or Local z axis

The coordinate transformation matrix for rotation about Z (or z) is

$$[\alpha] = -\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.45)$$

2.14.3 Rotation Θ About the Global X or Local x Axis

The coordinate transformation matrix for rotation about the X (or x) axis is

$$[\Theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & \sin\Theta \\ 0 & -\sin\Theta & \cos\Theta \end{bmatrix} \qquad (2.46)$$

As mentioned, the rotations can be combined and the resulting coordinates of the geometry referenced to the global axis position. The sequence of rotations, however, is important when rotating about the local axes because the coordinates are obtained by working back from the final axes position. Thus, for rotations about the y, z and x axes, the transformation matrices are multiplied in the order of $[\Theta]$, $[\alpha]$ and $[\Phi]$. The net mutiplication $[\Theta]\,[\alpha]\,[\Phi]$ of the three transformation matrices is sometimes referred to as the literal matrix $[L] = [\Theta]\,[\alpha]\,[\Phi]$.

3. COORDINATES FOR MANUFACTURE AND MEASUREMENT

The coordinates that are usually generated are for the propeller blade at the design pitch angle. The pitch angle at the 70% radius is used as the datum for the pitch settings when a variable pitch blade is rotated about its spindle axis. At the design pitch setting, the coordinates can be used for stress analysis and graphic representation of the blade. The design pitch is not, however, the best angle for machining with a numerically controlled milling machine or for measuring the blade surface contours. For these two processes, it is often desirable to present the maximum projected area possible to reduce machine tool and measurement gauge travel. Zero pitch at the 70% radius gives close to the maximum projected area for most blades. Other angles may be used if they are found to be more suitable.

3.1 Cartesian Coordinates

To present the face of the blade for measurement or machining, it is rotated about its spindle axis through the angle $-\phi$ to approximately zero pitch. Right handed blades, i.e., blades which advance when the propeller shaft is rotated clockwise, are rotated clockwise about their spindle axis. Left hand blades are rotated counterclockwise. The rotation $-\phi$ requires a coordinate transformation of the same form as was described in Section 2.14.1. The values of YB and YF are not changed by the rotation. The values of XF, XB and ZF and ZB, however, are modified as follows

$$XMF_{ij} = XF_{ij} \cos\phi + ZF_{ij} \sin\phi \tag{3.1}$$

$$ZMF_{ij} = -XF_{ij} \sin\phi + ZF_{ij} \cos\phi \tag{3.2}$$

To gain access to the back surface, the blade must be rotated an additional 180°. Thus,

$$XMB_{ij} = XB_{ij} \cos(\phi+\pi)\ ZB_{ij} \sin(\phi+\pi) \tag{3.3}$$

$$ZMB_{ij} = -XB_{ij} \sin(\phi+\pi) + ZB_{ij} \cos(\phi+\pi) \tag{3.4}$$

The coordinates of the blade are now in a form that permits easy measurement of the blade front and back surfaces provided the measurement device can measure accurately along three axes, X, Y and Z. An N.C. milling machine or a jig borer are devices that can be used for this purpose. A typical example of blade geometry in cartesian coordinates form is shown in Table 1.

3.2 Cylindrical Coordinates

The blade to this point has been described in spatial cartesian coordinates. Cylindrical coordinates are better suited for measurement by a gauge of the type shown diagramatically in Figure 44. For this purpose, the coordinates are converted to an angle measured from the Y axis, a radius R from the center of rotation of the shaft of the propeller and a Z axis dimension for every section offset given in the designer's original data. The measurement angles are

Face Angle

$$\Theta_F = -\tan^{-1} (XMF_{ij} / YF_{ij}) \tag{3.5}$$

Back Angle

$$\Theta_B = -\tan^{-1} (XMB_{ij} / YB_{ij}) \tag{3.6}$$

Face Radii

$$RMF_{ij} = \sqrt{XMF_{ij}^2 + YF_{ij}^2} \tag{3.7}$$

Back Radii

$$RMB_{ij} = \sqrt{XMB_{ij}^2 + YB_{ij}^2} \tag{3.8}$$

The measurement of the Z coordinates can be more complex if a dial gauge is used and negative values cannot be read directly. If negative values occur in the column of Z offsets, an origin shift is required to convert all the offsets to positive values. This is accomplished by subtracting the largest negative Z from all the others (Figures 44 and 45). The subroutine RXYZ of program BLADGM carries out the origin shift when required and prints out the modified Z offsets as a data column titled "GAGE".

4. COMPUTER PROGRAMS

A computer program, BLADE, consisting of three subprograms BLADAT, BLADGM and BLADPL has been developed for generating and displaying propeller blade geometry.

BLADAT is an interactive program which prompts the user in entering propeller geometry data. It can accept the data in the three following basic forms which are used to describe the blade sections.

a. blade section data in non-dimensional form as ratios of section half thickness over maximum thickness and camber over maximum camber at specified fractions of the section chord.

b. NACA section data in the form of basic and mean line data.

c. section offsets from drawings or tables.

Upon completion of the basic data entry, the data can be examined by plotting or by directly examining the input. At this stage any errors which are identified can be corrected using BLADAT.

BLADGM is also an interactive program. It accepts the data from BLADAT and completes the blade section description by generating the additional leading and trailing edge coordinates when required. It also completes the data required to describe the blade tip and generates the three-dimensional coordinates ,in tabular form for the propeller, required for blade manufacture and checking. The three-dimensional coordinates in this form can also be used to generate finite element models of the blade for strength and vibration analysis.

The subprogram BLADPL uses the data generated by BLADGM to plot the propeller geometry. It provides a menu of the plotting options for selecting the graphics required to produce conventional drawing of an individual blade or a complete propeller. In addition, it offers an option to view the blade in a solid geometry form by generating a session file for the solid modelling program PATRAN. An example of views of blade geometry generated by PATRAN from a BLADPL produced session file is shown in Figure 46.

The versatility of the program is shown by its ability to produce drawings of a variety of blade forms. Figure 47 is the plan view of the blade sections of a moderately skewed blade. Figure 48 is the same view of a long narrow blade with high skew and Figure 49 shows a transverse projection of a six bladed propeller with moderate skew.

5. CONCLUSIONS

The geometry of marine propellers has been described and the relationships have been derived for generating propeller blade geometry in the form of three dimensional coordinates from two dimensional blade section data including pitch, rake and skew.

The problem of curve fitting the leading and trailing edges and the blade tip has been overcome by use of a special coordinate transformation and cubic spline interpolation for non elliptic and elliptic form.

The equations derived for defining the geometry of a blade have been used to develop a computer program for blade coordinate generation. The program includes the ability to produce computer generated graphic displays of blade geometry for checking hydrodynamic form and the detection of data errors. It also can be used for producing conventional drawings and tabular data for manufacture and blade form measurement.

TABLE 1: COORDINATES OF THE BLADE BACK SURFACE ROTATED FOR EASY MEASUREMENT

3 DIMENSIONAL X,Y,Z CARTESIAN COORDINATES FOR CHECKING.

BLADE ROTATED 180 DEGREES FROM FACE SETTING OF 0.000 DEGREES

FRACTION RADIUS 0.3000 BLADE RADIUS 502.95 MM.

FRACTION CHORD	XMB	BACK YB	ZMB
0.0000	-188.13	466.44	363.87
0.0010	-185.16	467.63	364.52
0.0038	-181.34	469.12	363.94
0.0075	-177.01	470.77	362.67
0.0100	-174.35	471.76	361.70
0.0250	-161.34	476.37	354.24
0.0500	-143.07	482.17	339.89
0.1000	-110.44	490.67	308.87
0.1500	-81.31	496.33	275.82
0.2000	-55.09	499.92	241.16
0.2500	-30.27	502.04	205.72
0.3000	-6.93	502.90	169.49
0.3500	15.01	502.73	132.56
0.4000	35.70	501.68	95.01
0.4500	55.20	499.91	56.89
0.5000	73.54	497.54	18.23
0.5500	90.66	494.71	-21.00
0.6000	106.46	491.55	-60.87
0.6500	120.93	488.20	-101.35
0.7000	134.15	484.73	-142.44
0.7500	146.12	481.26	-184.13
0.8000	156.84	477.87	-226.43
0.8500	166.44	474.61	-269.29
0.9000	174.92	471.55	-312.71
0.9500	182.60	468.63	-356.53
0.9750	186.08	467.26	-378.62
0.9875	187.39	466.74	-389.90
0.9967	187.96	466.51	-398.37
1.0000	187.01	466.89	-402.10

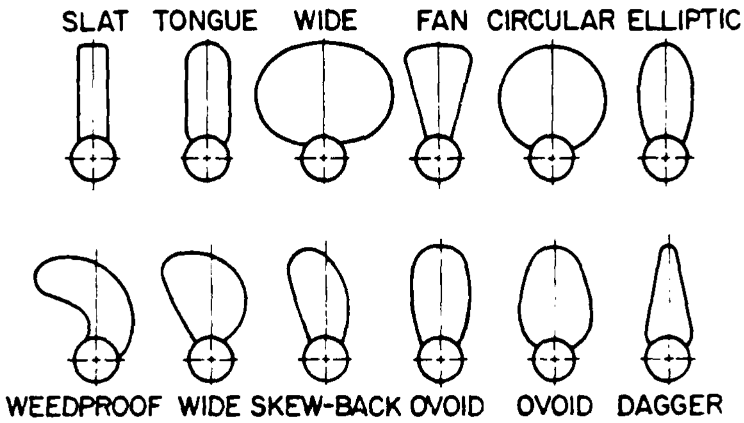


FIGURE 1: PROFILE SHAPES FOR SCREW PROPELLERS (REF.1)

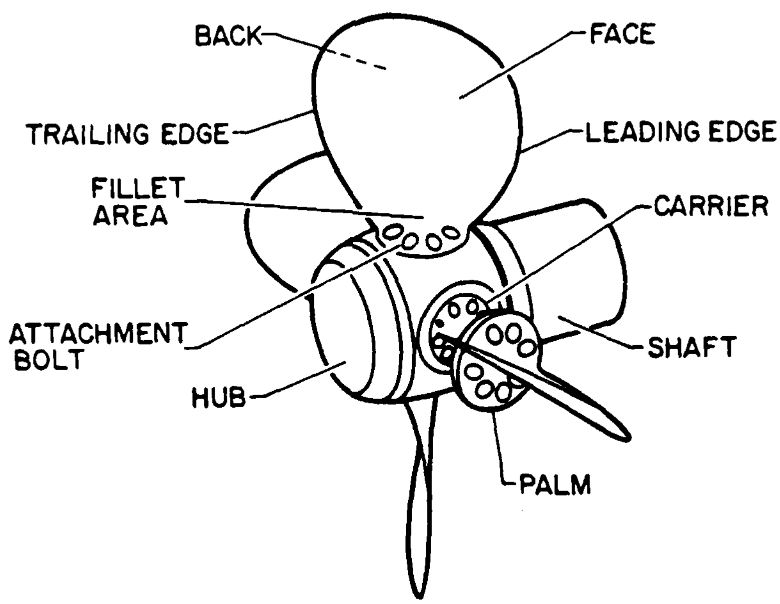


FIGURE 2: PARTS OF A MARINE PROPELLER

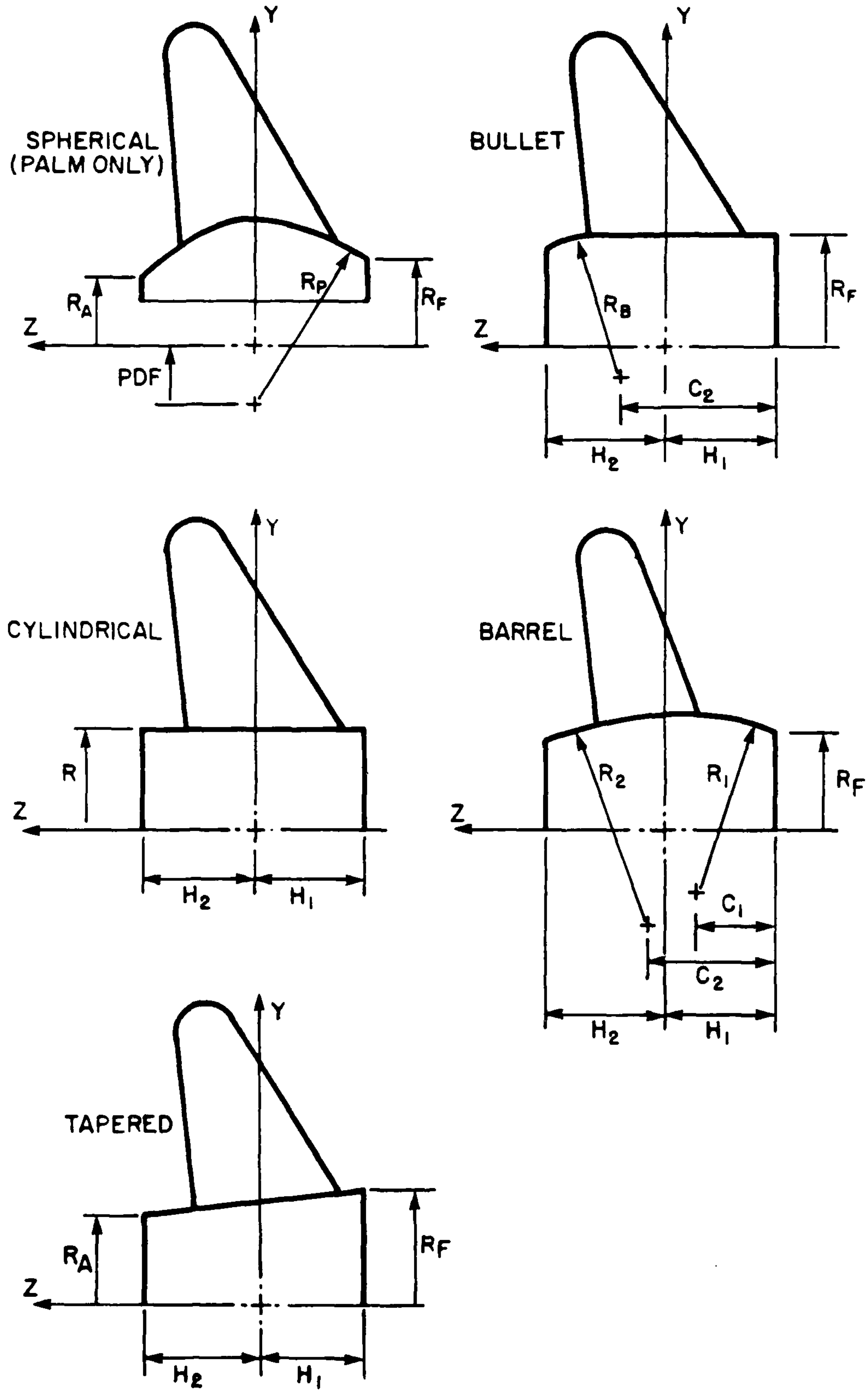


FIGURE 3: TYPES OF BLADES PALM/HUB MODELS

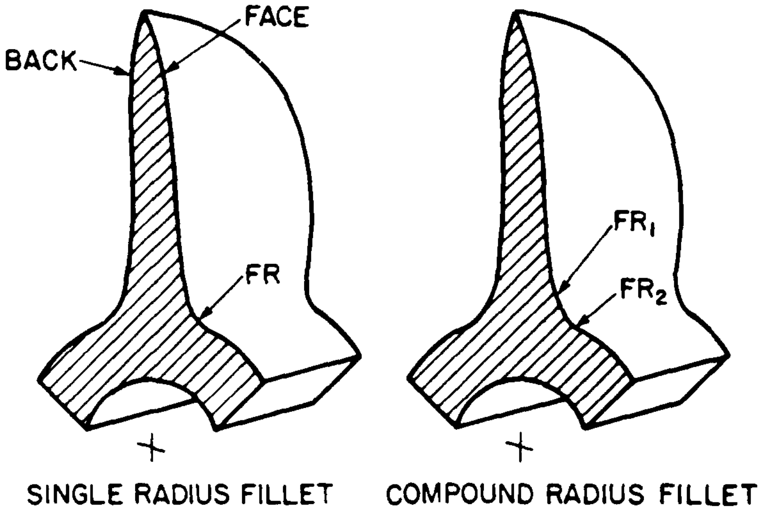


FIGURE 4: BLADE TO HUB INTERSECTION FILLETS

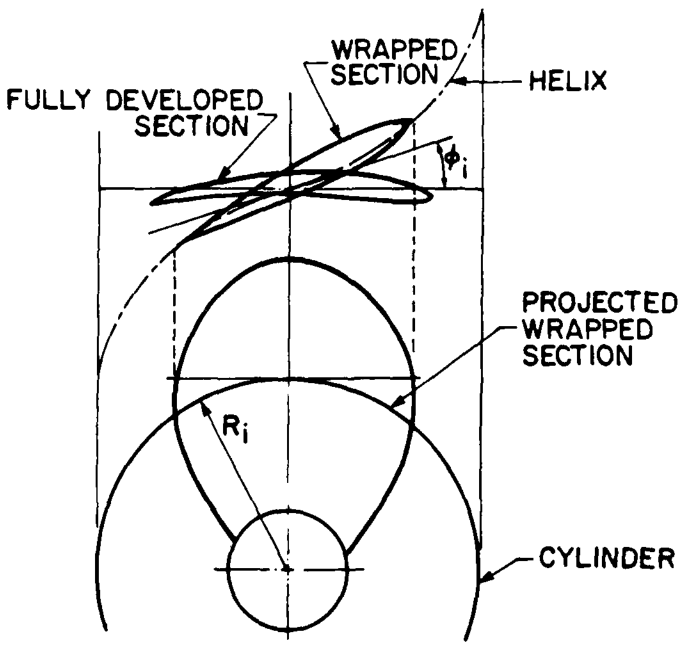


FIGURE 5: ROTATING AND WRAPPING OF FULLY DEVELOPED SECTIONS

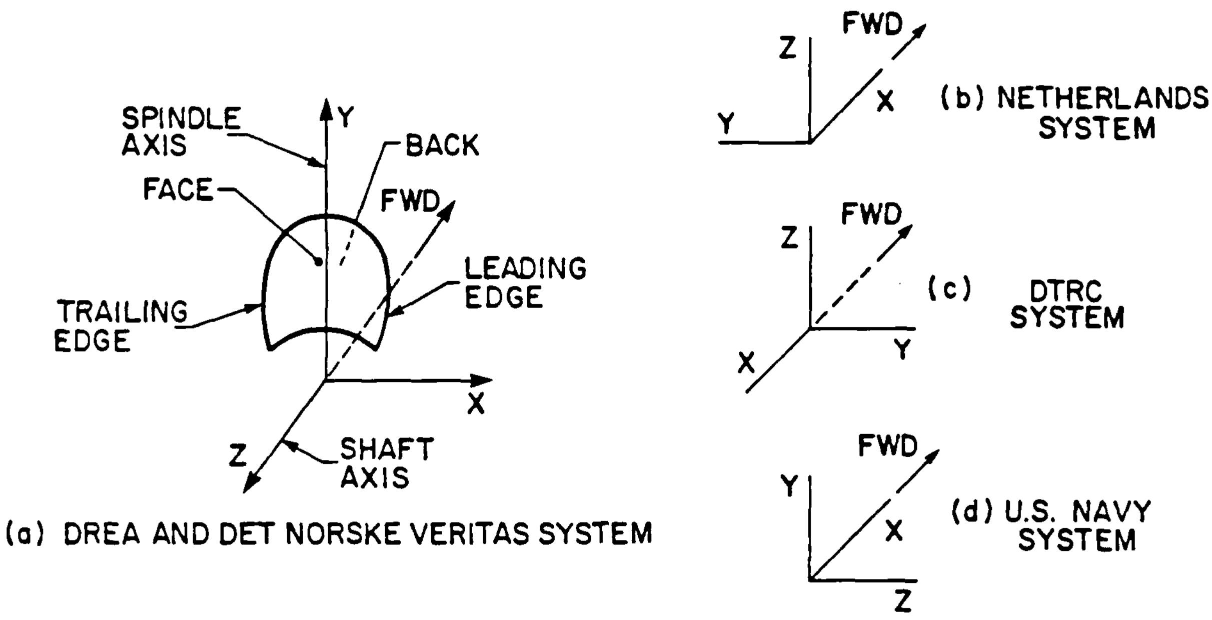


FIGURE 6: PROPELLER AXES SYSTEMS

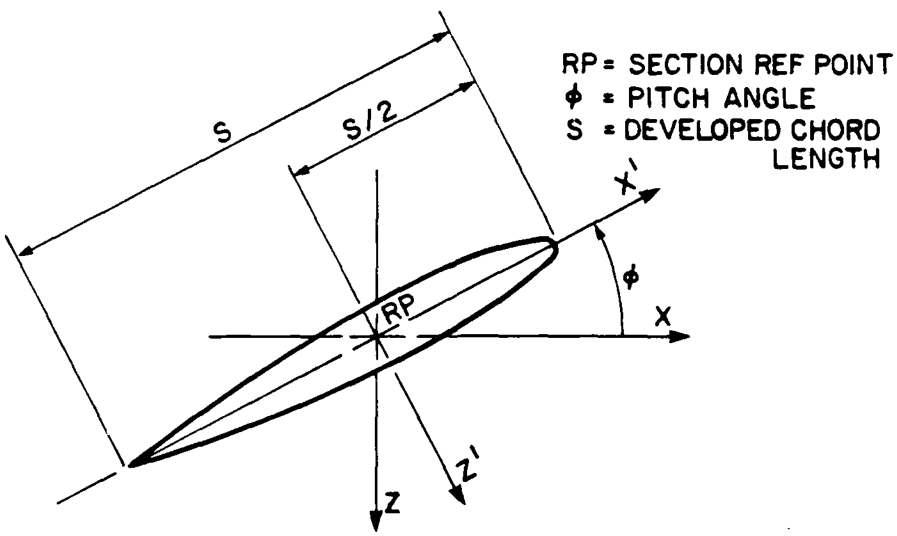


FIGURE 7: FULLY DEVELOPED SECTION PROJECTED ON XZ PLANE WITH ORIGIN SHIFTED TO MIDCHORD

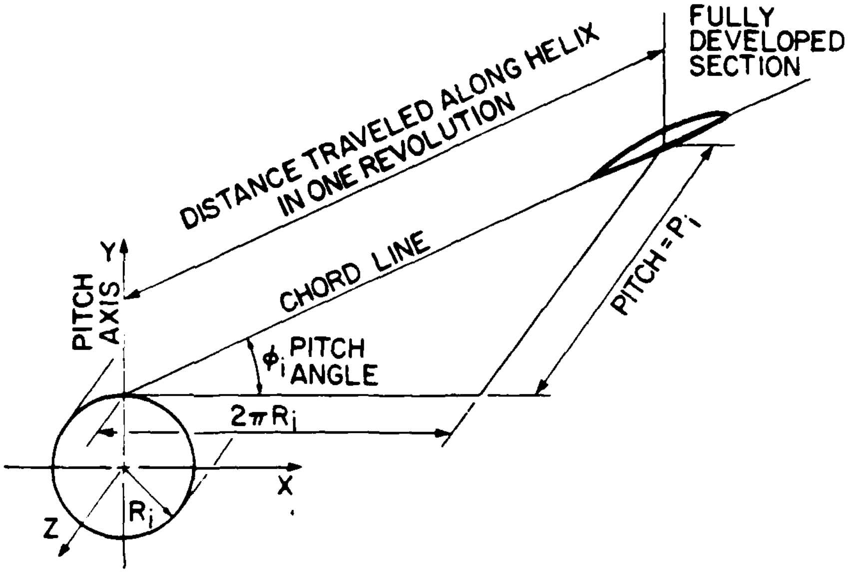


FIGURE 8: PITCH OF AN UNWRAPPED BLADE SECTION BY DEVELOPMENT OF AN XZ PLANE FROM CYLINDRICAL SURFACE

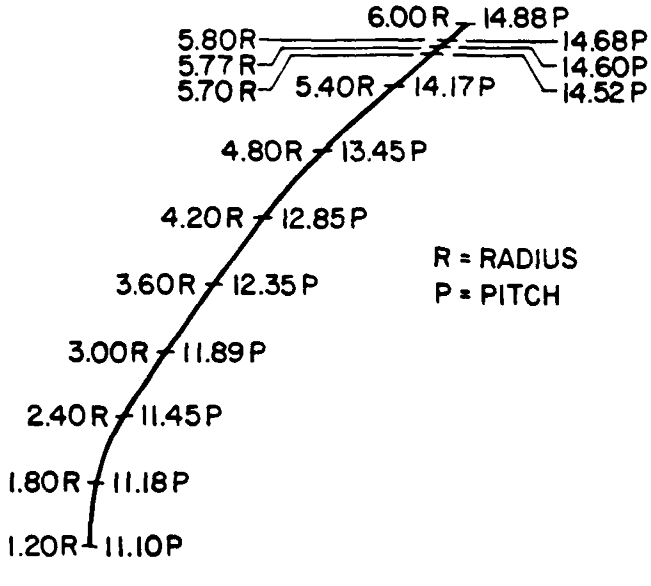


FIGURE 9: PITCH DIAGRAM

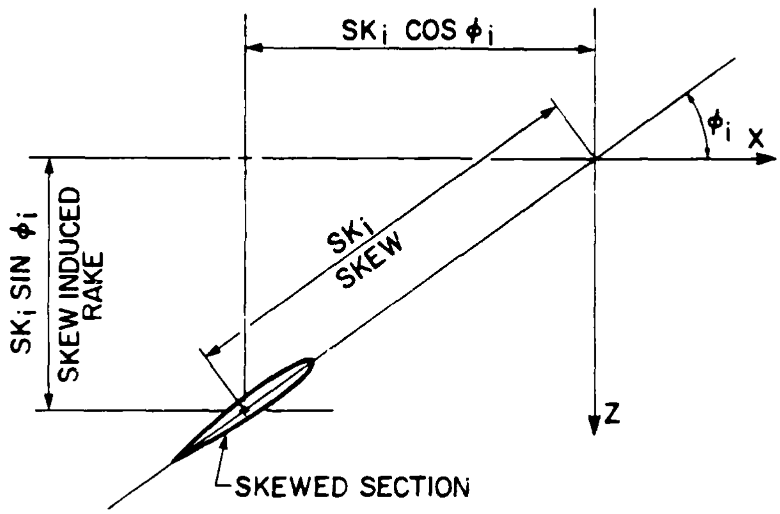


FIGURE 10: PROPELLER SECTION UNWRAPPED ON THE XZ PLANE

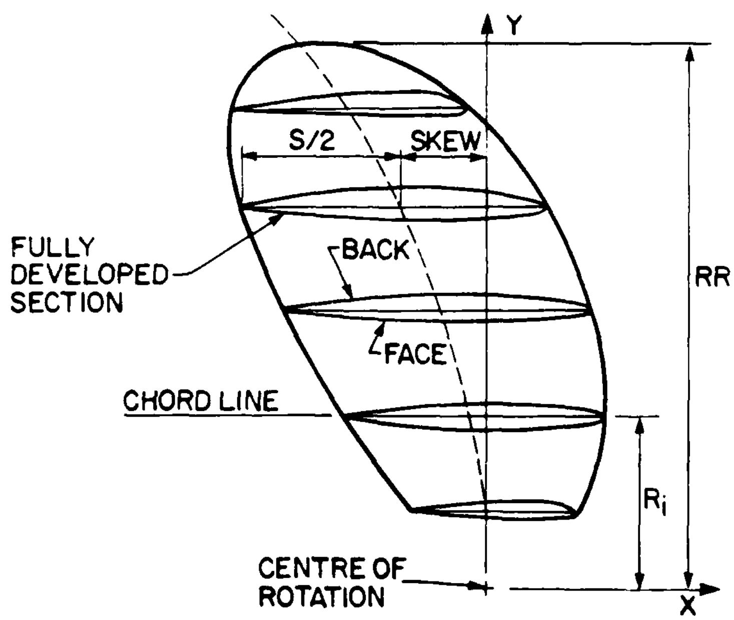


FIGURE 11: EXPANDED VIEW OF BLADE

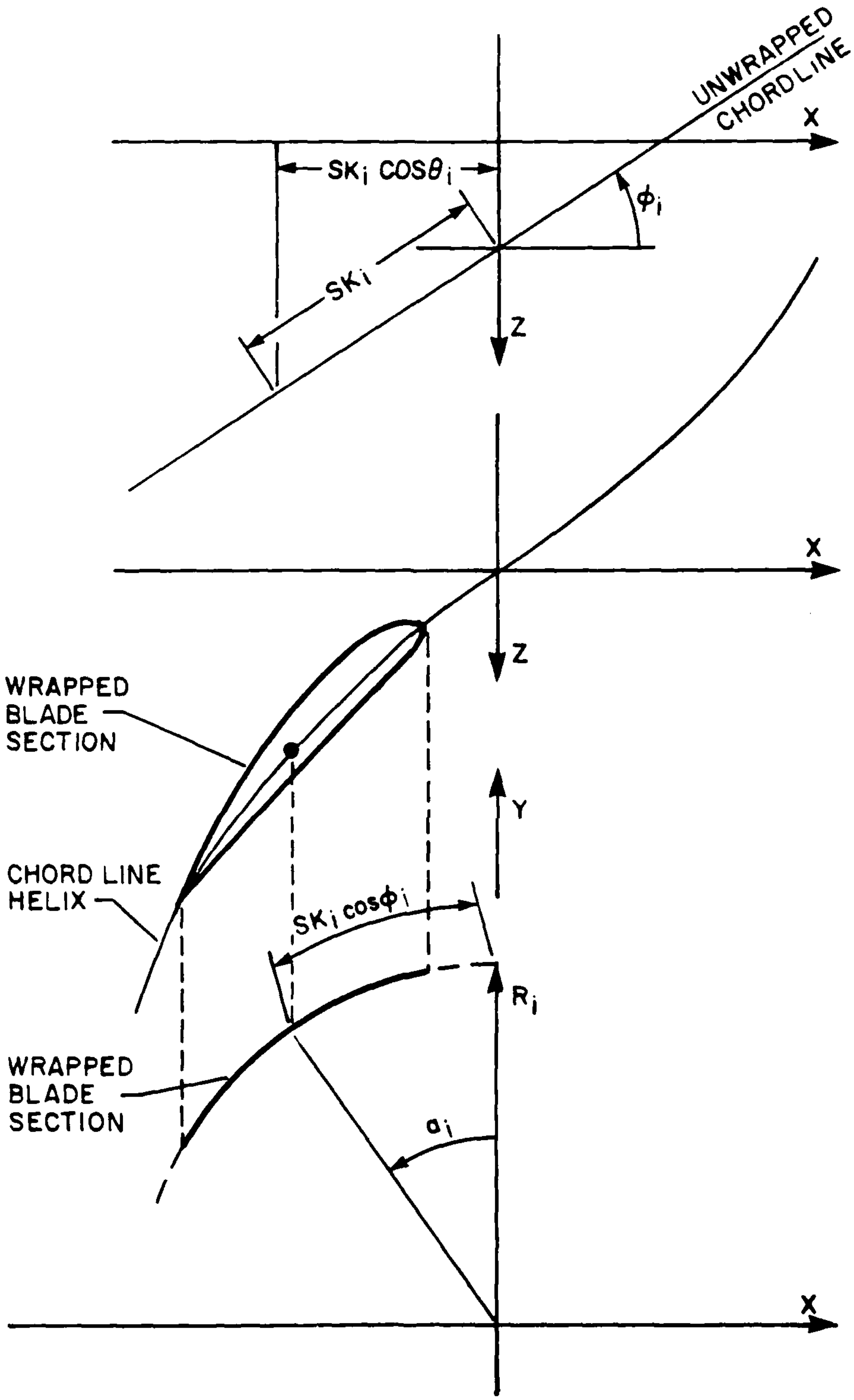


FIGURE 12: WRAPPED BLADE SECTION SHOWING SKEW ANGLE α_i

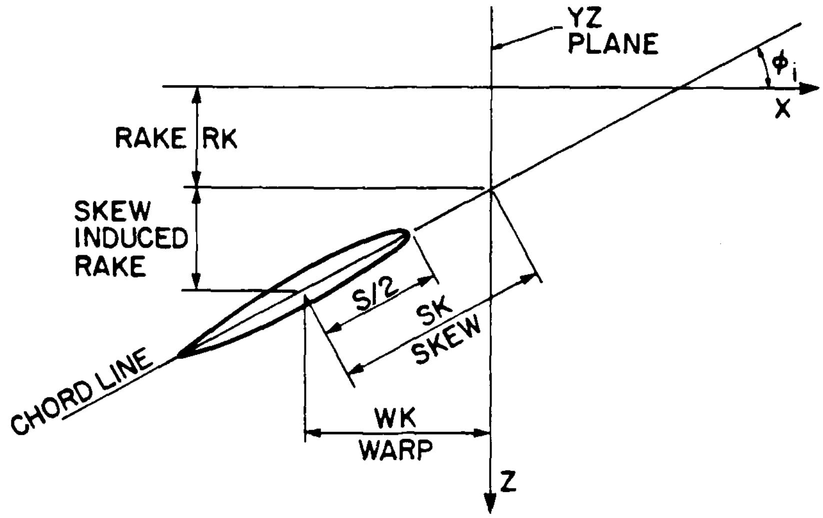
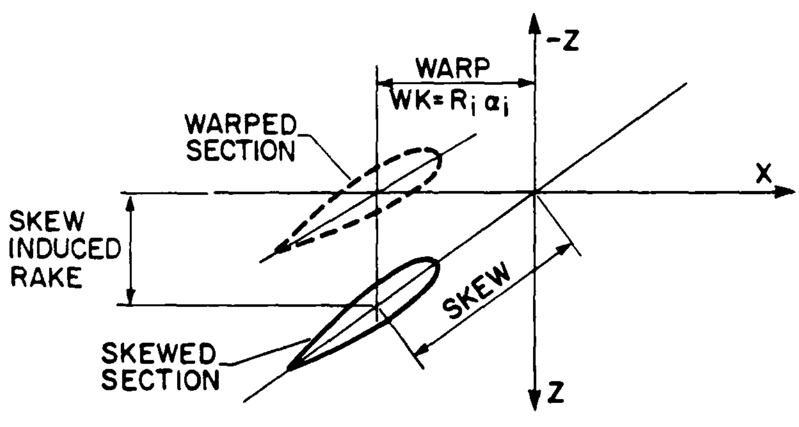
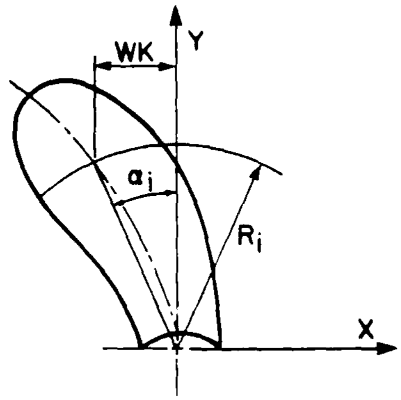


FIGURE 13: FULLY DEVELOPED SECTION ON XZ PLANE SHOWING SKEW AND RAKE

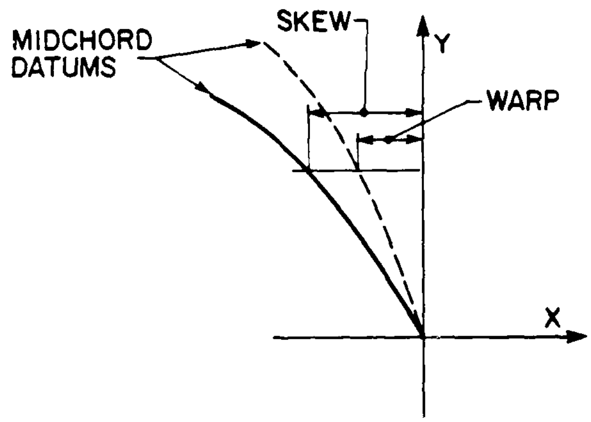


a) UNWRAPPED VIEW OF SECTION SHOWING RELATIONSHIP OF WARP AND SKEW

FIGURE 14: BLADE WARP

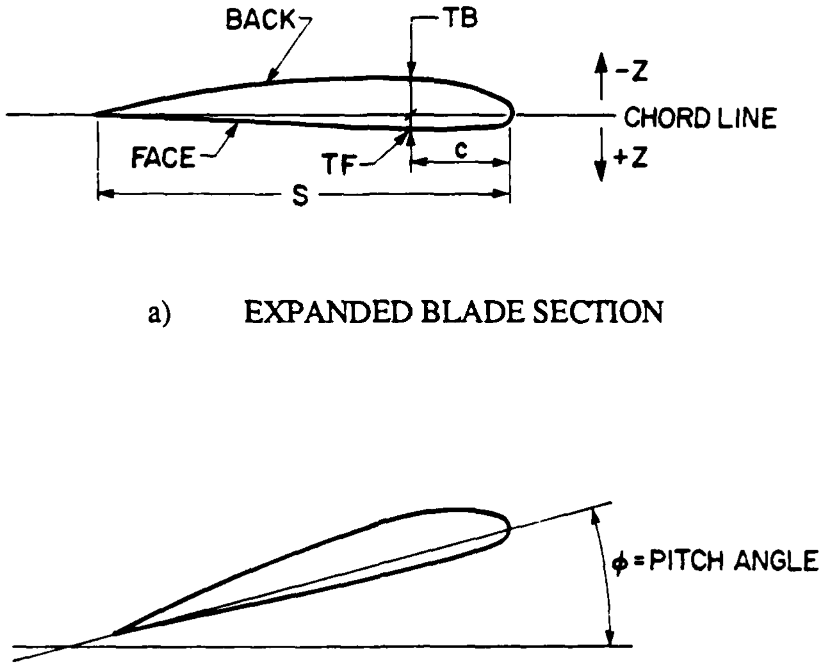


b) WARP ANGLE SHOWN ON PROJECTED VIEW OF BLADE



c) LINEAR DIMENSION FOR WARP AND SKEW
ON EXPANDED MIDCHORD DATUM

FIGURE 14: BLADE WARP



a) EXPANDED BLADE SECTION

b) SECTION PITCH ANGLE

FIGURE 15: INITIAL DESCRIPTION OF BLADE SECTION

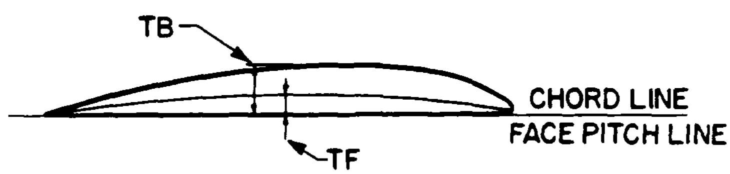


FIGURE 16: FACE PITCH LINE FOR FLAT OR CONCAVE SECTIONS

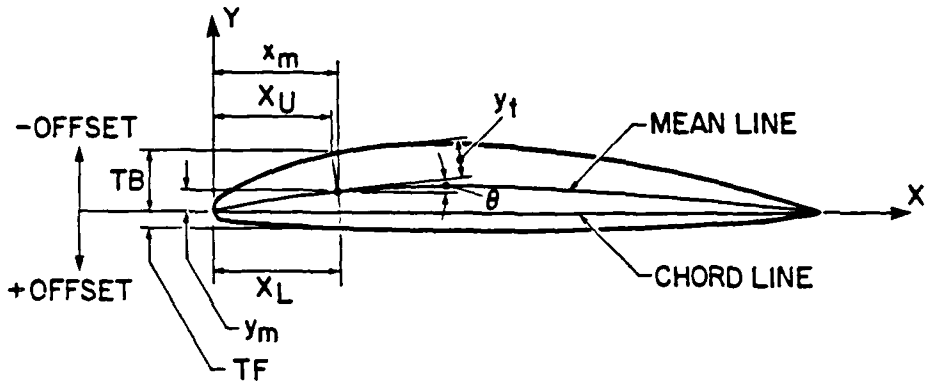


FIGURE 17: CONVERSION OF MEAN LINE AND BASIC FORM DATA TO SECTION OFFSETS

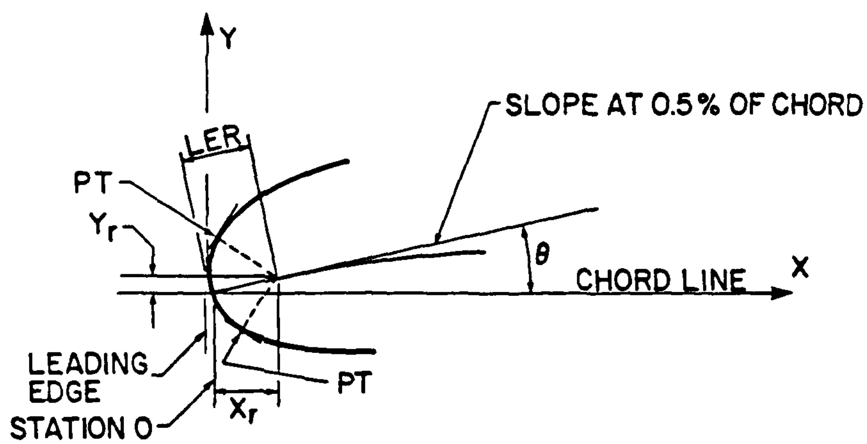
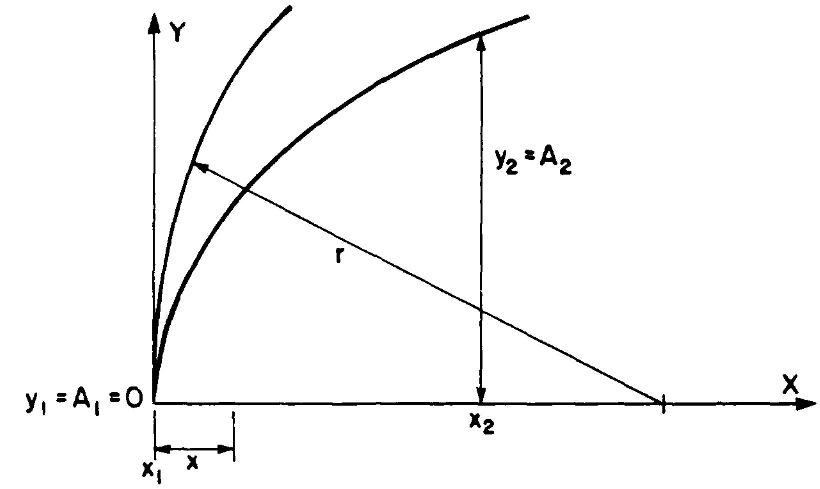
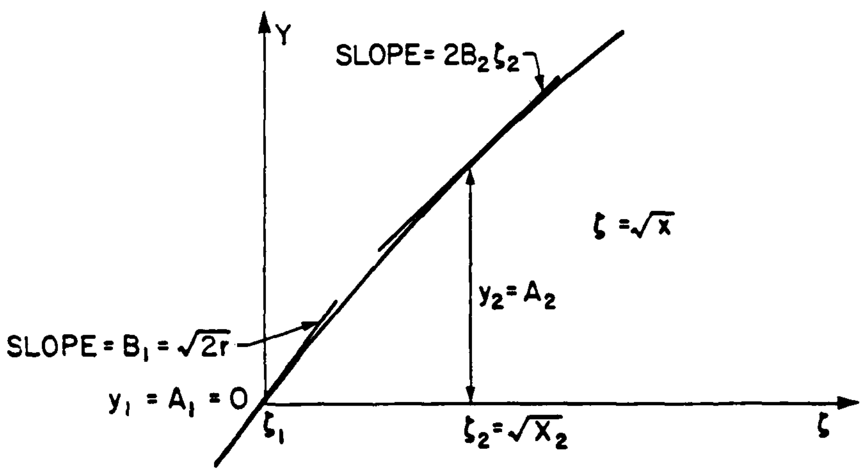


FIGURE 18: LEADING EDGE GEOMETRY



a) CURVE FITTING INTERVALS FOR LEADING EDGE FIT



b) TRANSFORMATION OF X-AXIS TO ζ-AXIS

FIGURE 19: SPLINE FITTING TECHNIQUE FOR DEFINING LEADING EDGE

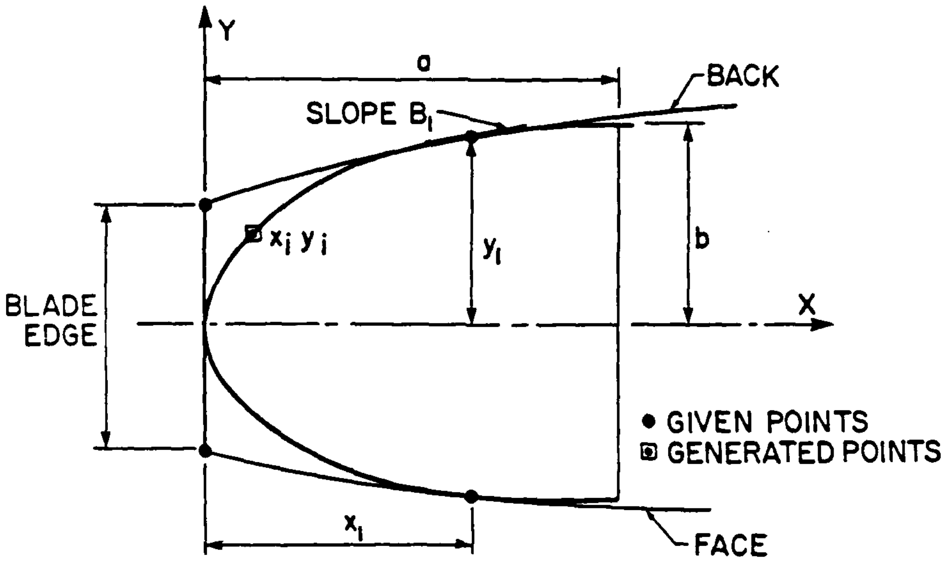


FIGURE 20: ELLIPTICAL LEADING EDGE

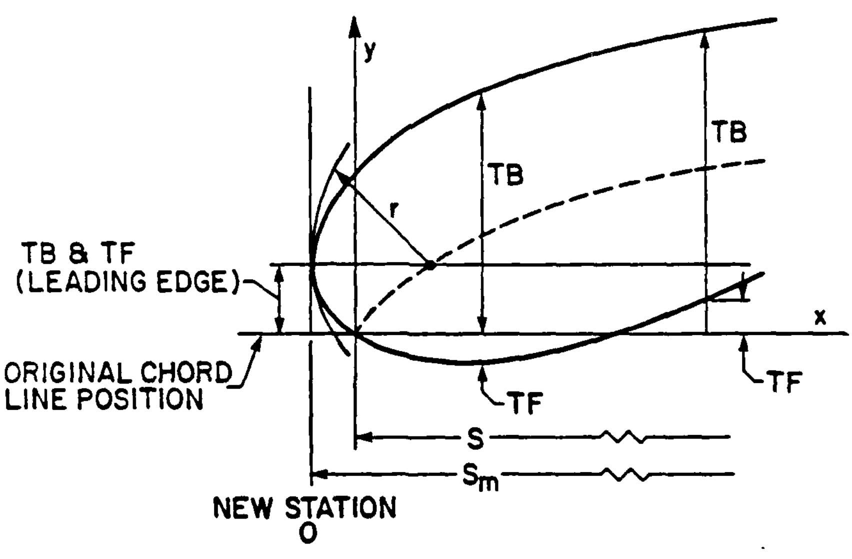
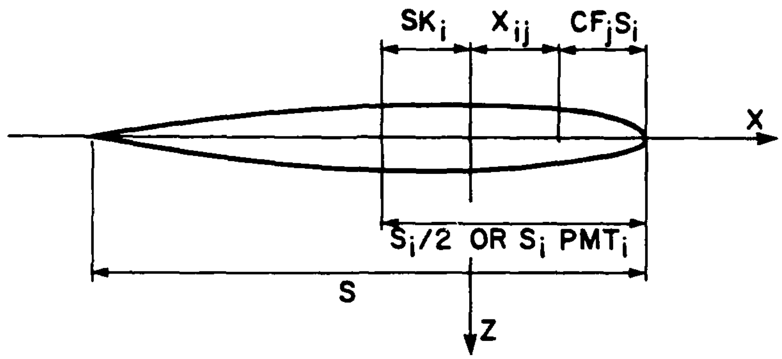
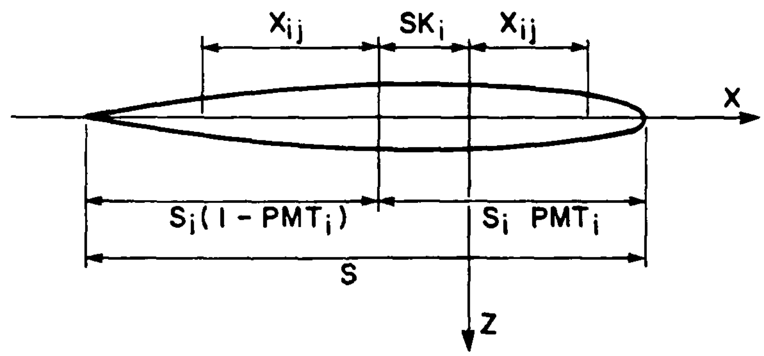


FIGURE 21: LEADING EDGE RADIUS FITTED TO CAMBERED SECTION DATA



a) CHORD STATIONS SPECIFIED AS FRACTIONS FROM LEADING EDGE



b) CHORD STATIONS SPECIFIED AS FRACTIONS FROM POINT OF MAXIMUM THICKNESS

FIGURE 22: TRANSFORMATION OF SECTION COORDINATES USING CHORD FRACTIONS

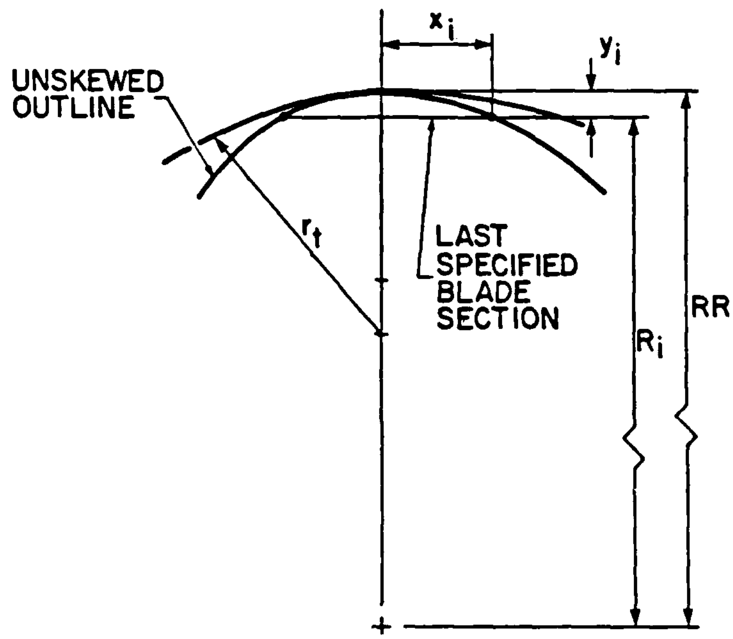


FIGURE 23: CURVE FITTING BLADE TIP

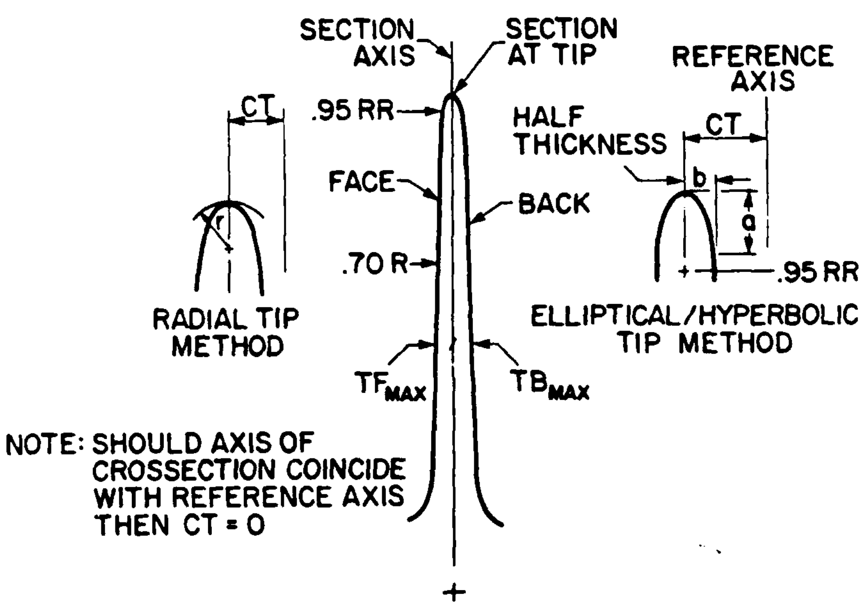


FIGURE 24: BLADE SPANWISE MAXIMUM THICKNESS SECTION

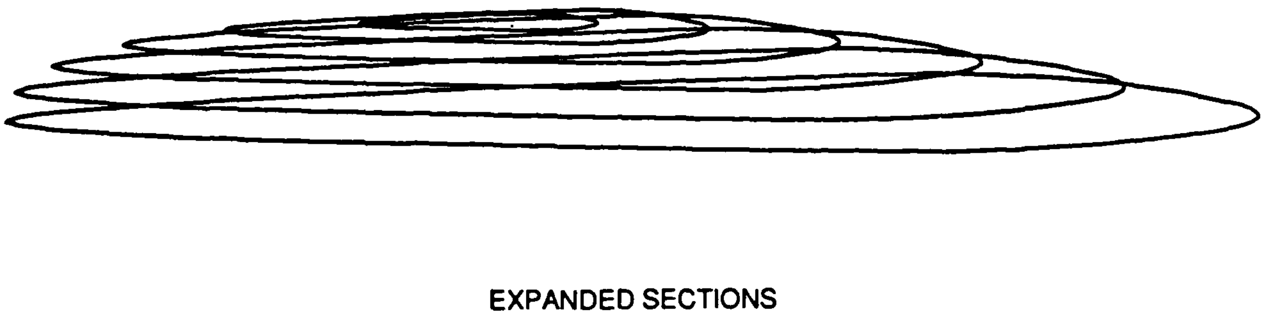


FIGURE 25: ADDITIONAL BLADE SECTIONS GENERATED FROM LAST GIVEN SECTION TO THE TIP OF THE BLADE

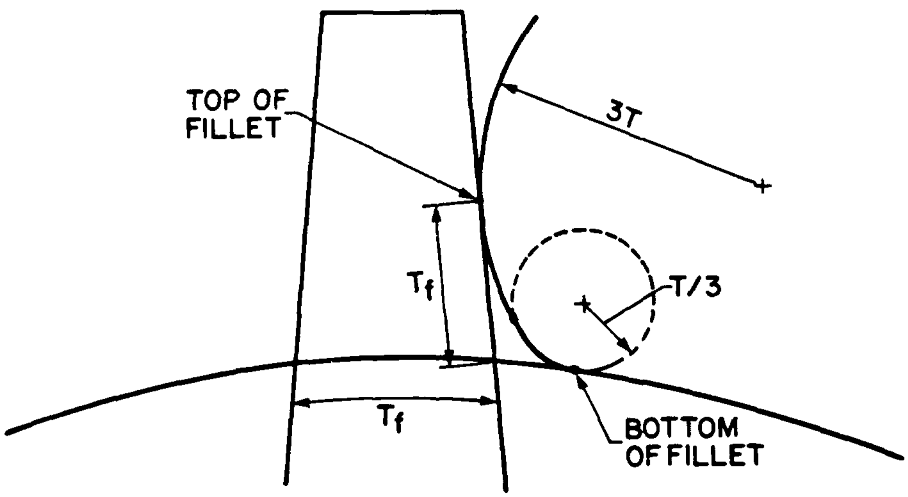


FIGURE 26: COMPOUND RADIUS TYPE FILLET

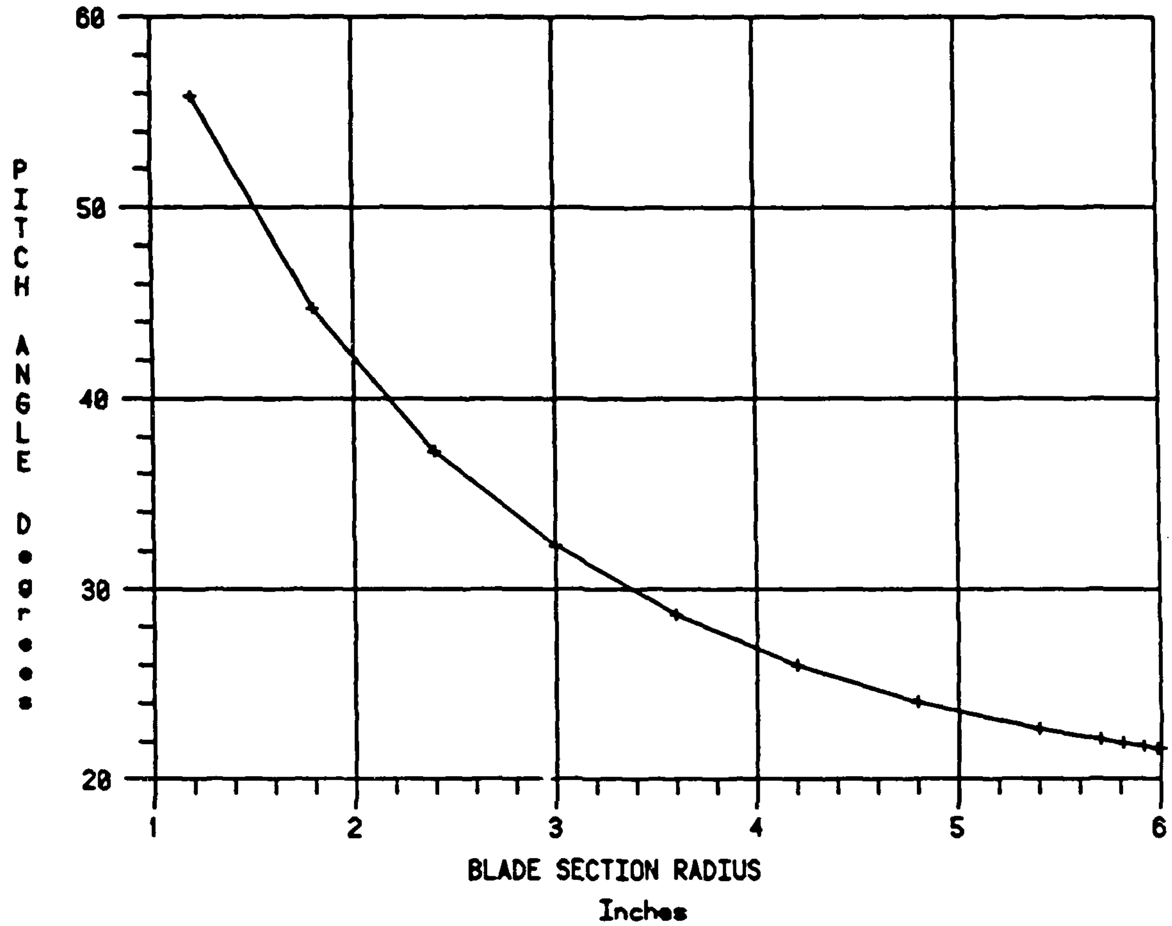


FIGURE 27: DATA PLOT OF PITCH ANGLE VERSUS BLADE SECTION RADIUS

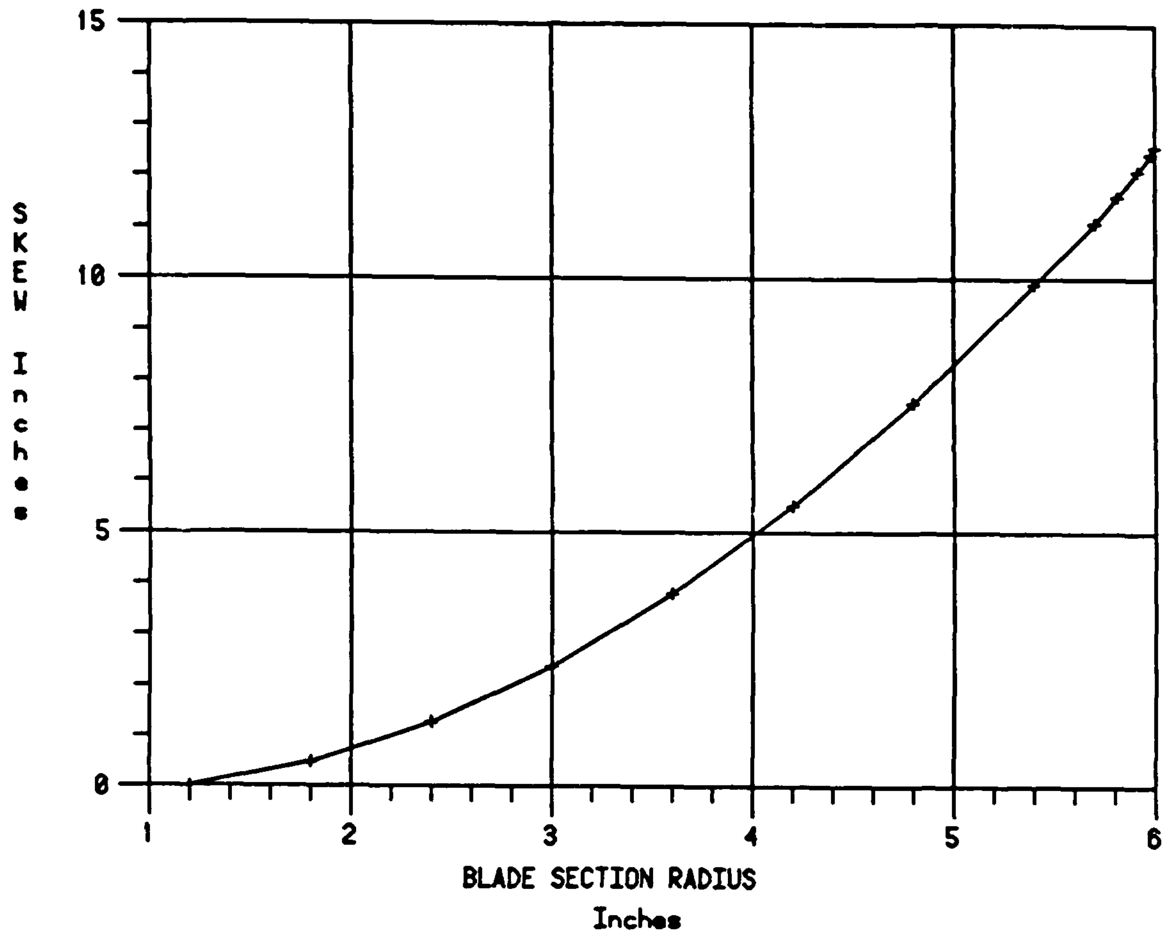


FIGURE 28: DATA PLOT OF SKEW VERSUS BLADE SECTION RADIUS

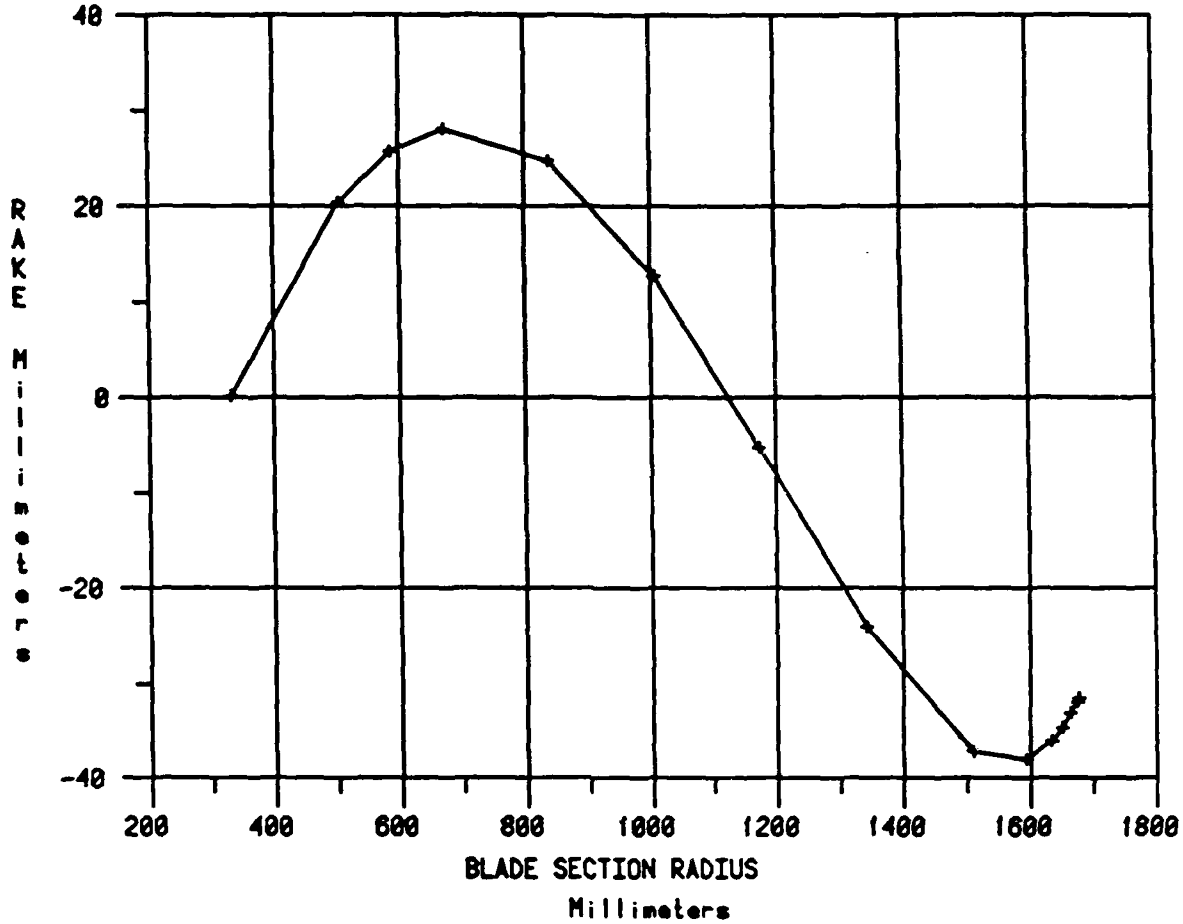


FIGURE 29: DATA PLOT OF RAKE VERSUS BLADE SECTION RADIUS

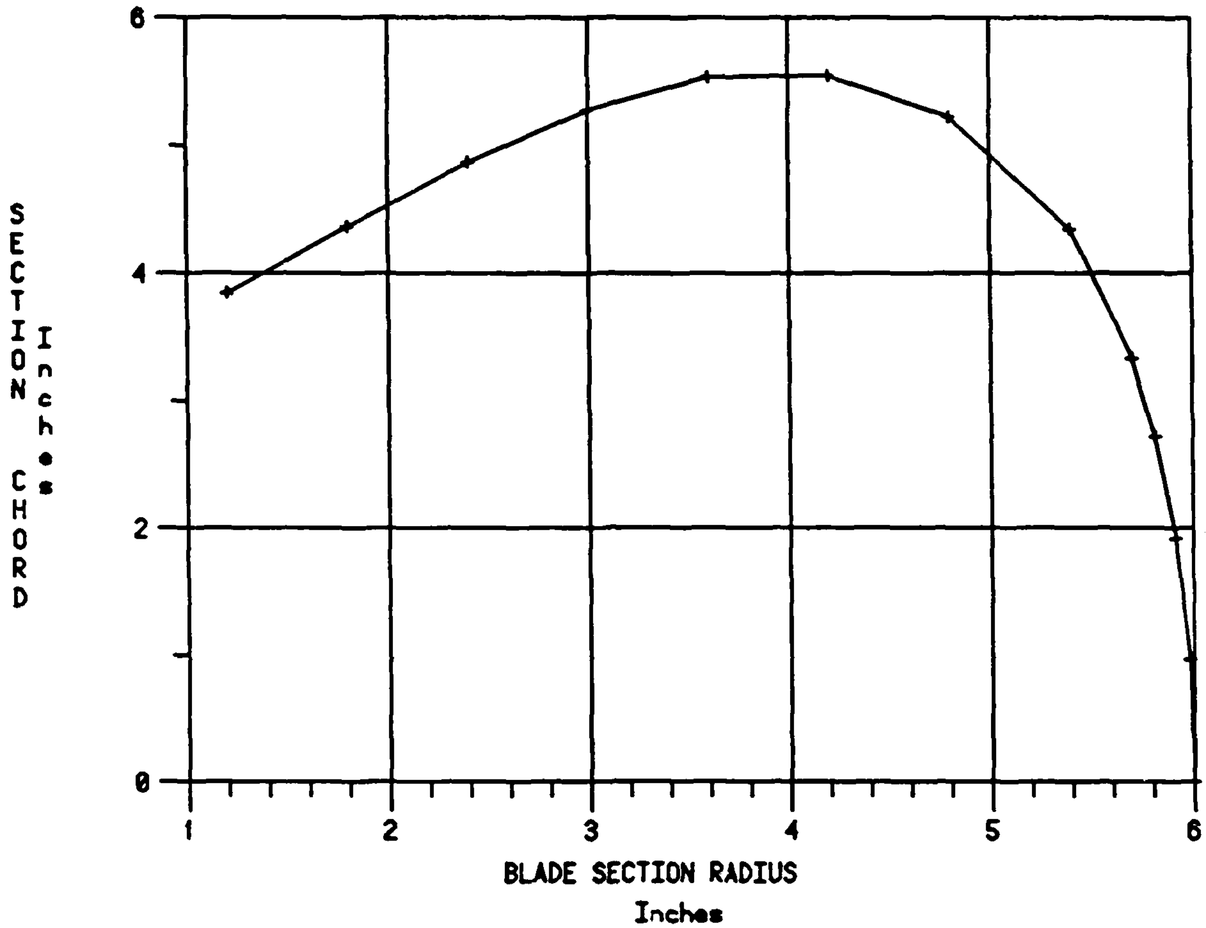


FIGURE 30: DATA PLOT OF SECTION CHORD VERSUS BLADE SECTION RADIUS

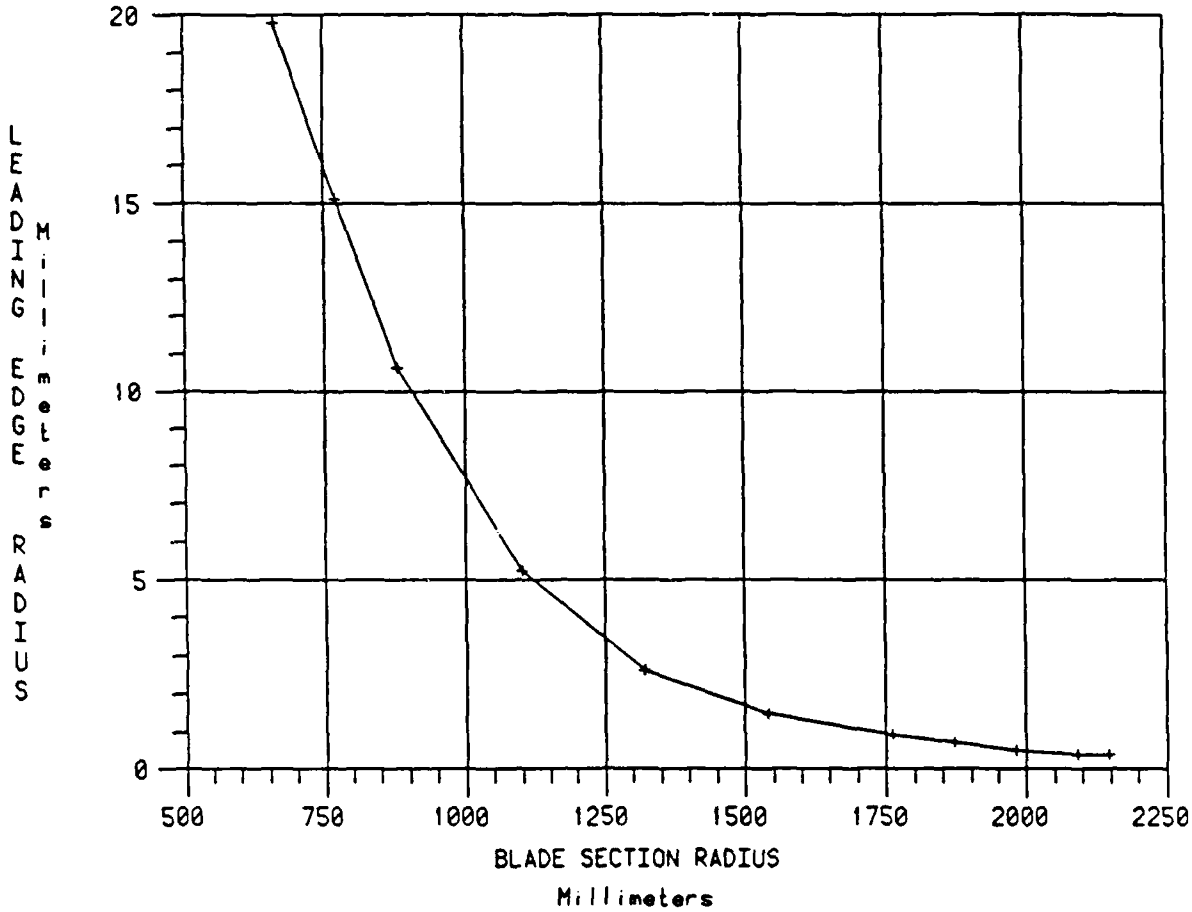


FIGURE 31: DATA PLOT OF LEADING EDGE RADIUS VERSUS BLADE SECTION RADIUS

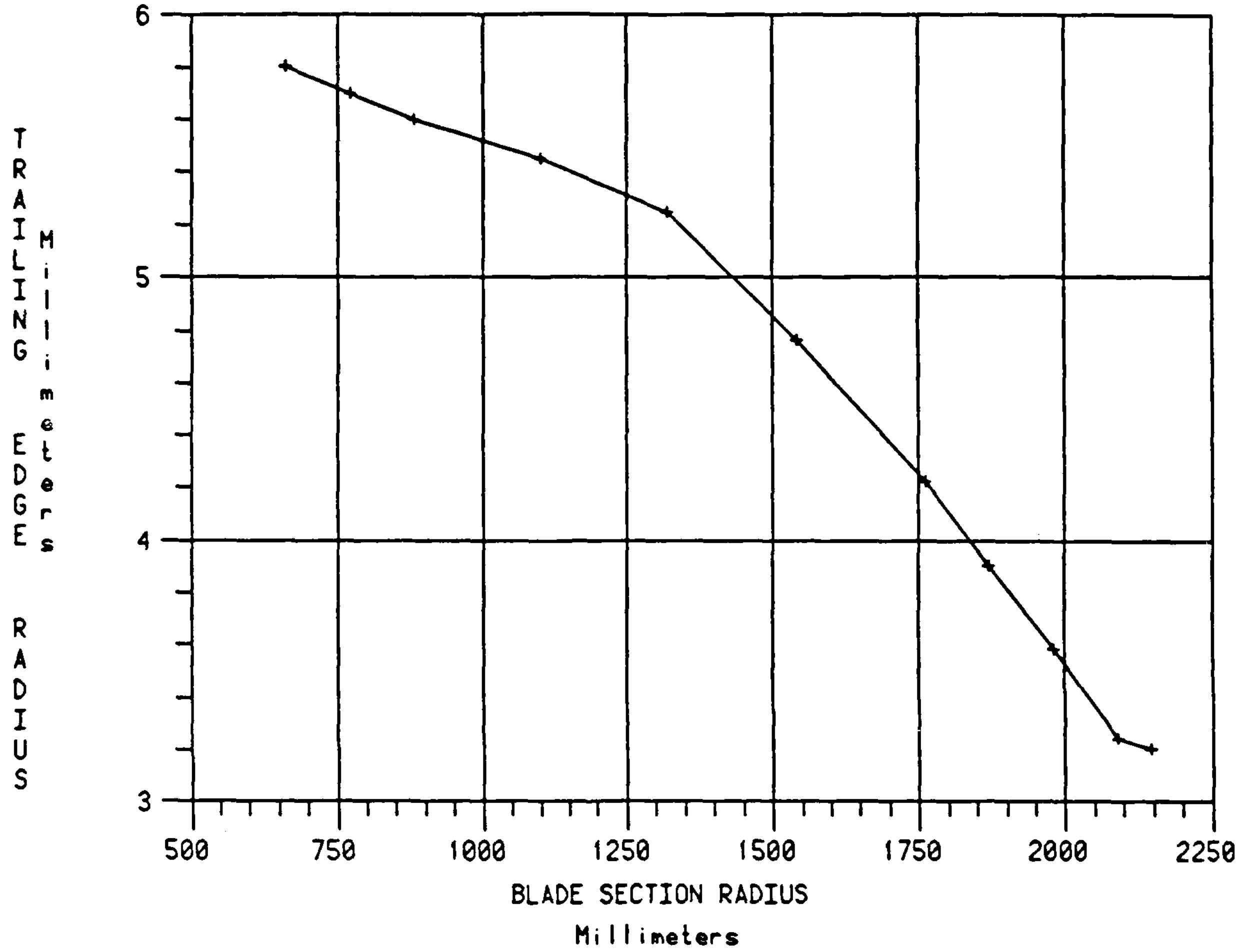


FIGURE 32: DATA PLOTS OF TRAILING EDGE RADIUS VERSUS BLADE SECTION RADIUS

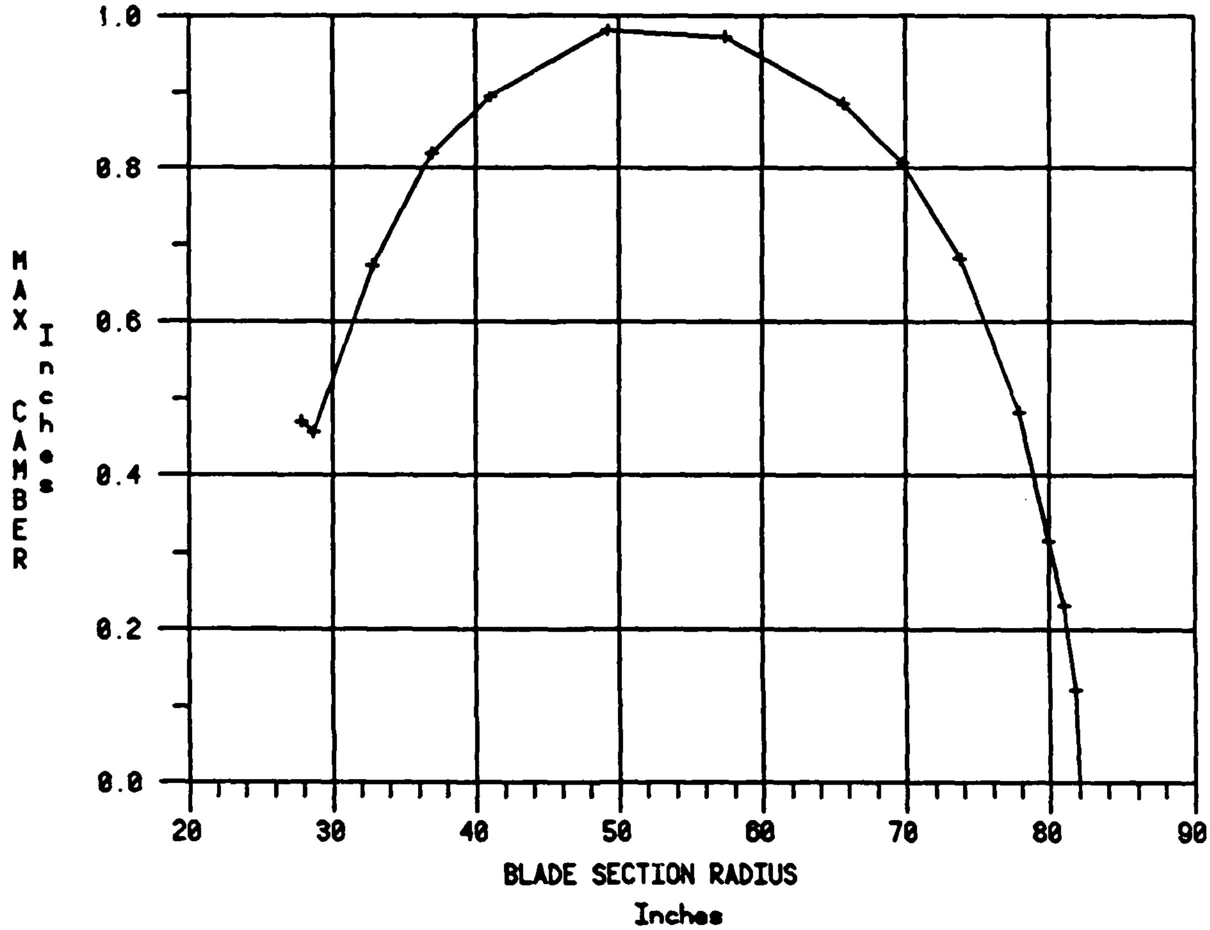


FIGURE 33: DATA PLOT OF MAXIMUM CAMBER VERSUS BLADE SECTION RADIUS

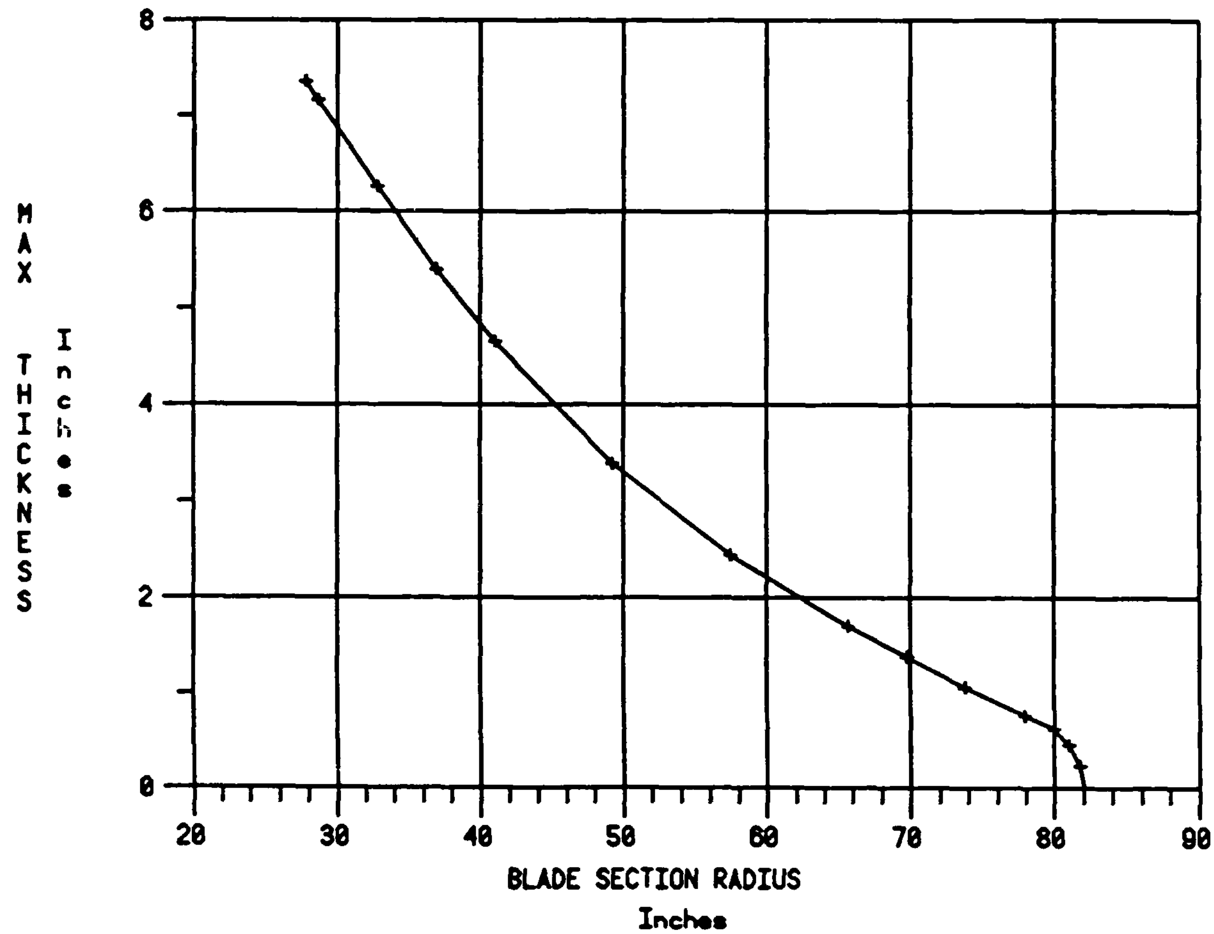
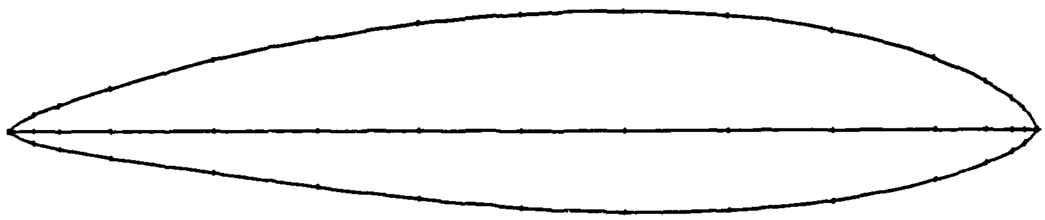


FIGURE 34: DATA PLOT OF MAXIMUM THICKNESS VERSUS BLADE SECTION RADIUS

BLADE SECTION AT RADIUS 360.0MM.   R/RR =0.2105

VERTICAL SCALE MULTIPLIER = 1.00

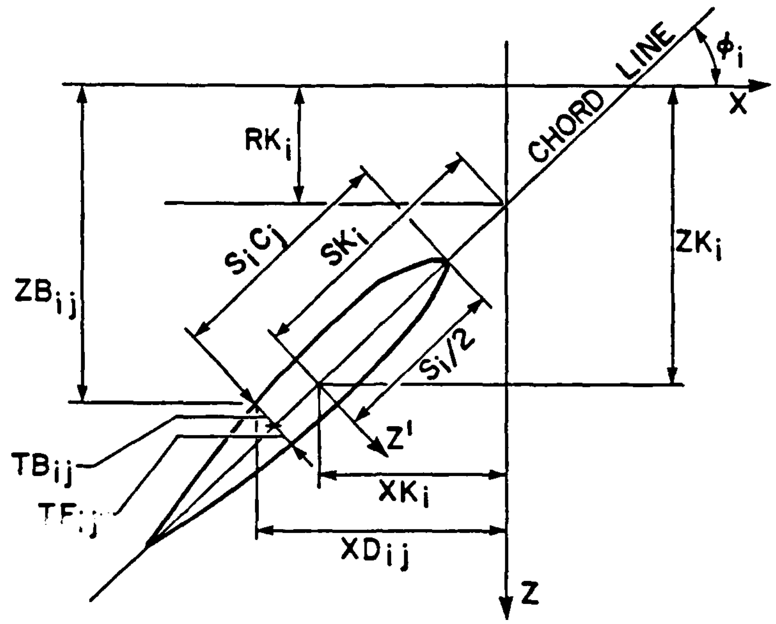
UNITS OF MEASURE ARE MILLIMETERS



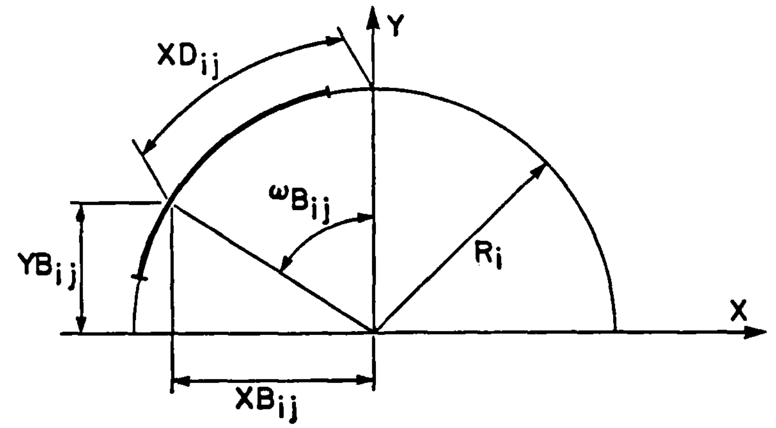
CHORD LENGTH= 656.0   RTE = 3.0   RLE = 0.0

X/C	1.0000	0.9500	0.9000	0.8000	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000	0.1000	0.0500	0.0250	0.0125	0.0000
CHORD	656.0	623.2	590.4	524.8	459.2	393.6	328.0	262.4	196.8	131.2	65.6	32.8	16.4	8.2	0.0
BACK	0.00	-16.60	-27.60	-46.40	-60.35	-70.50	-76.75	-78.06	-75.43	-65.73	-46.74	-31.40	-20.77	-13.00	0.00
FACE	0.00	11.77	17.70	27.77	37.38	45.50	51.37	54.24	53.26	47.10	33.73	22.43	14.50	8.33	0.00

FIGURE 35: INDIVIDUAL SECTION PLOT WITH BACK AND FACE OFFSETS



a) UNWRAPPED BLADE SECTION ON THE XZ PLANE



b) THE SECTION WRAPPED AROUND THE Z AXIS AT RADIUS Ri PROJECTED ON XY PLANE

FIGURE 36: GENERATION OF SPATIAL XYZ COORDINATES

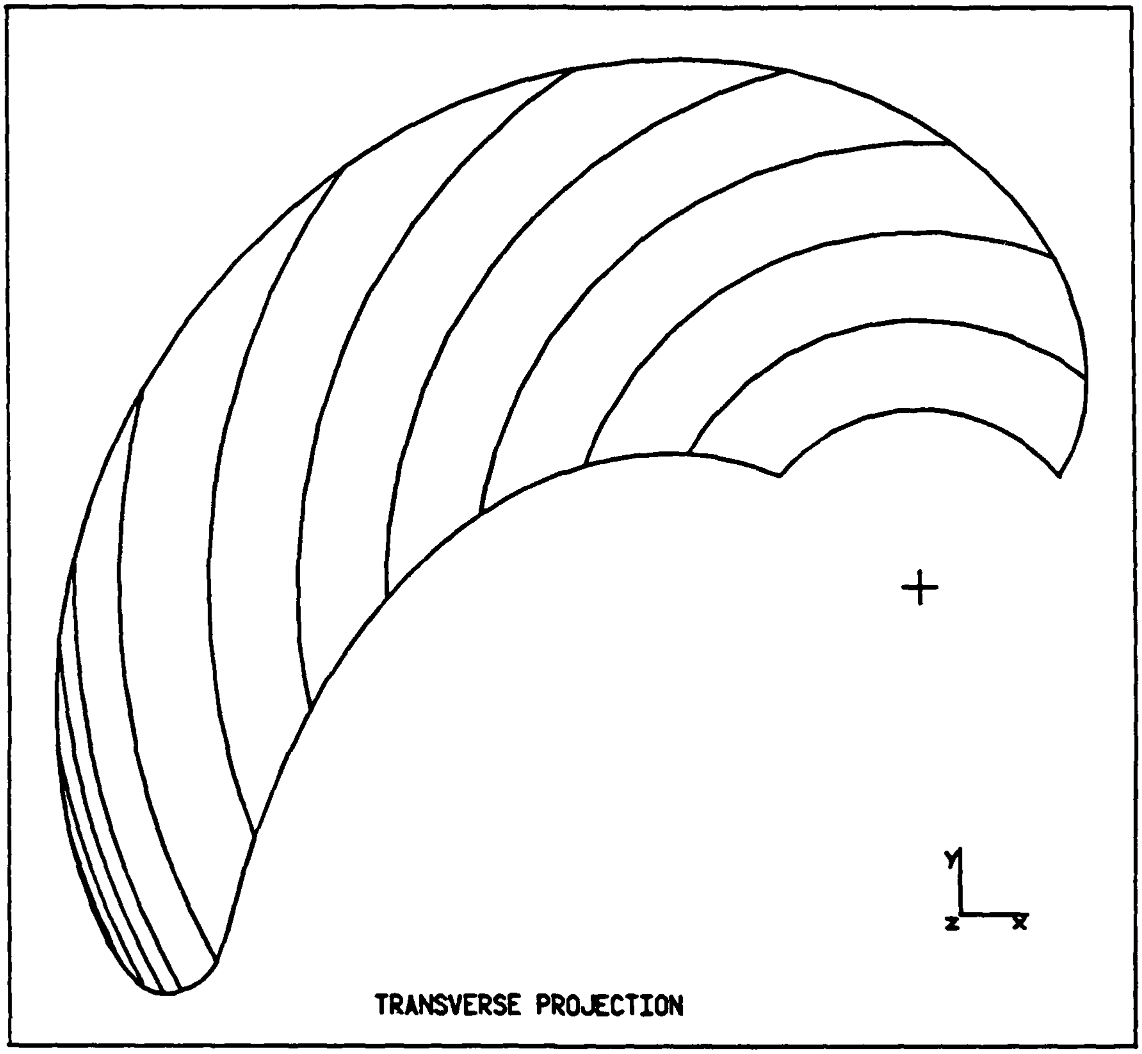


FIGURE 37: COMPUTER PLOT OF TRANSVERSE VIEW

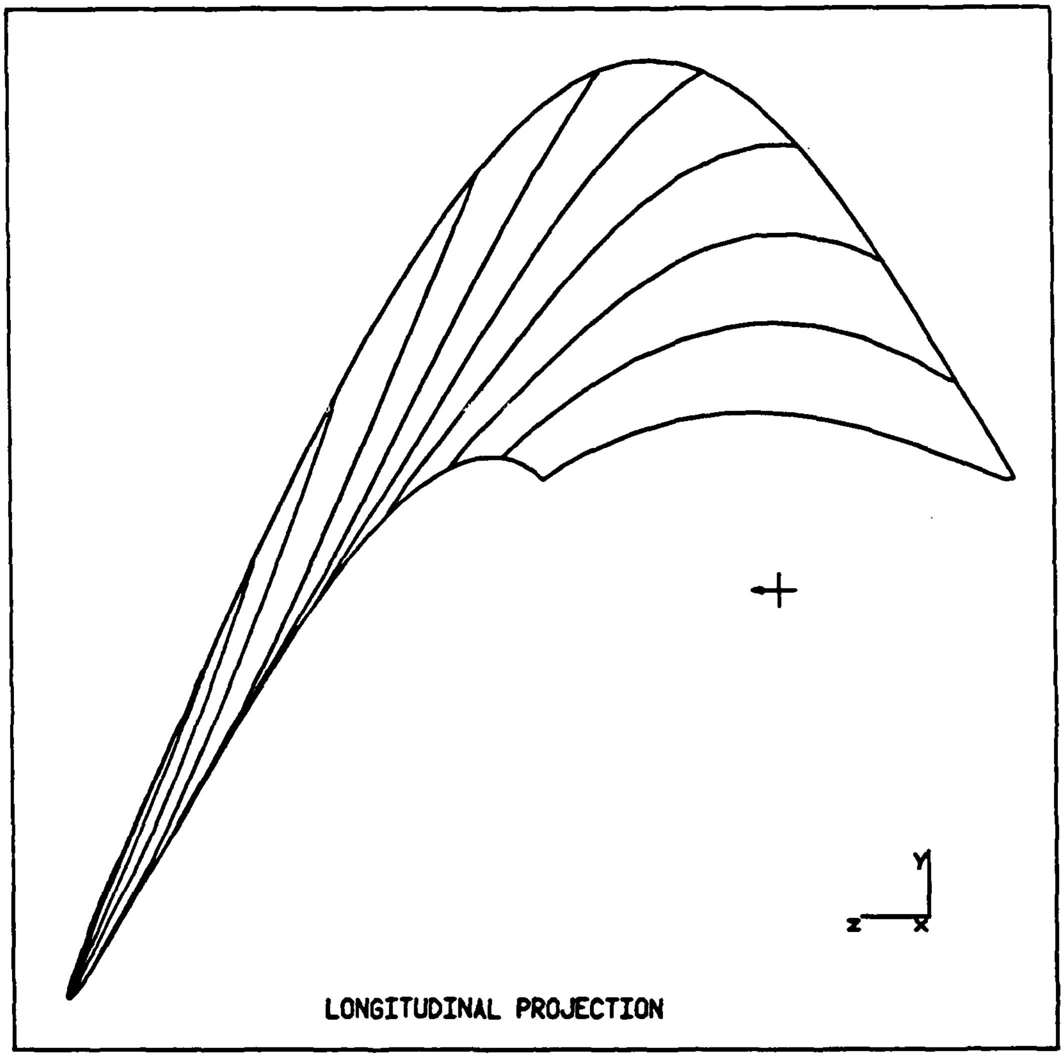


FIGURE 38: COMPUTER PLOT OF LONGITUDINAL VIEW

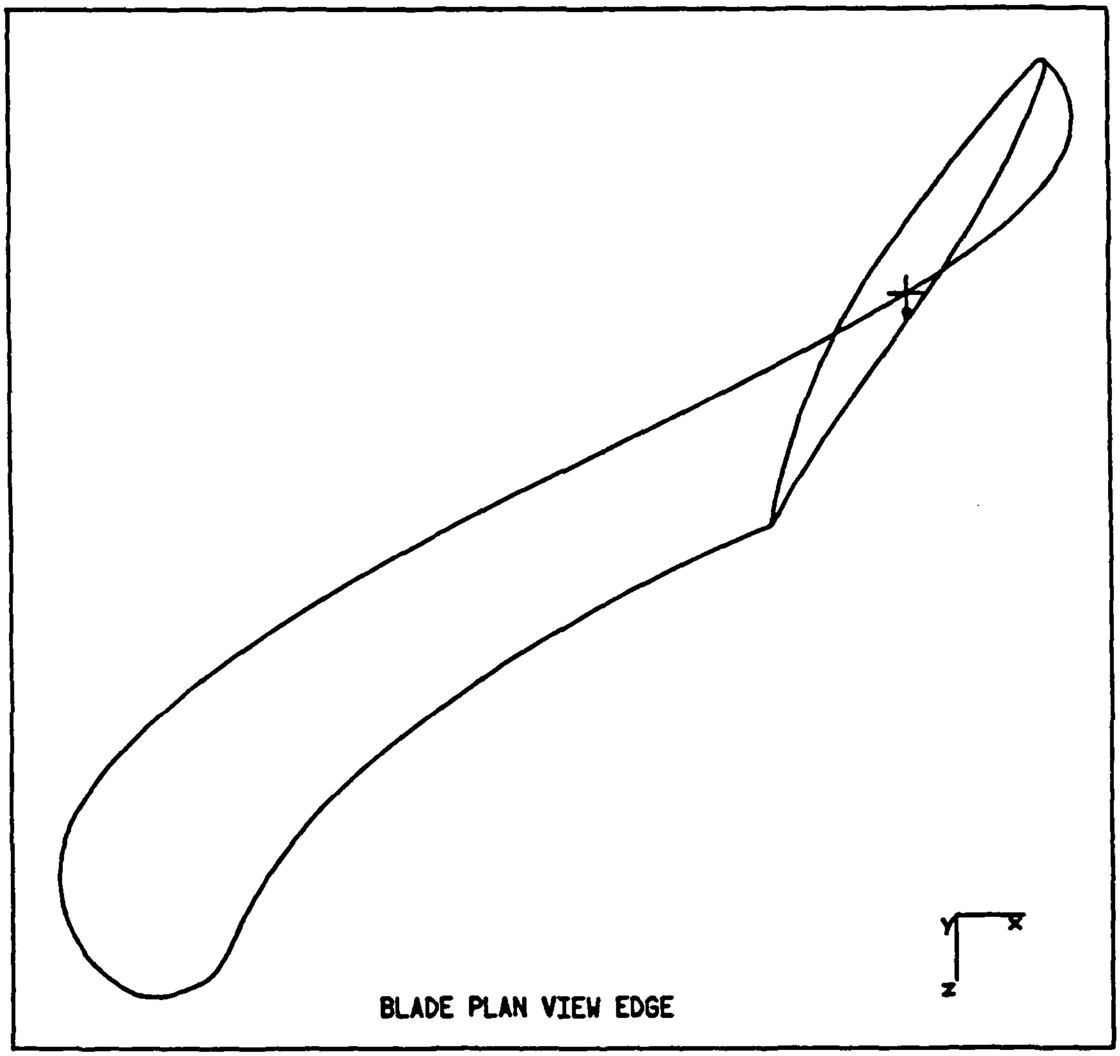


FIGURE 39: COMPUTER PLOT OF PLAN VIEW

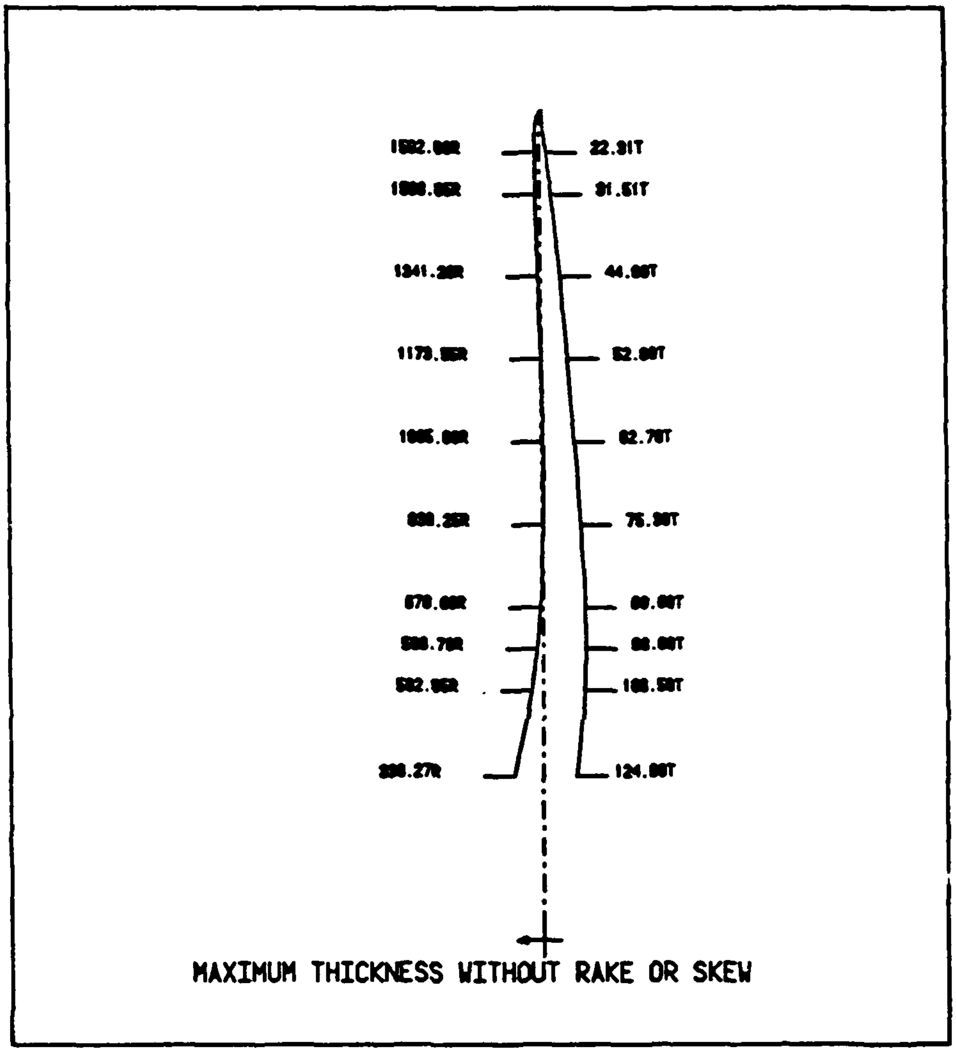


FIGURE 40: BLADE THICKNESS WITHOUT RAKE OR SKEW

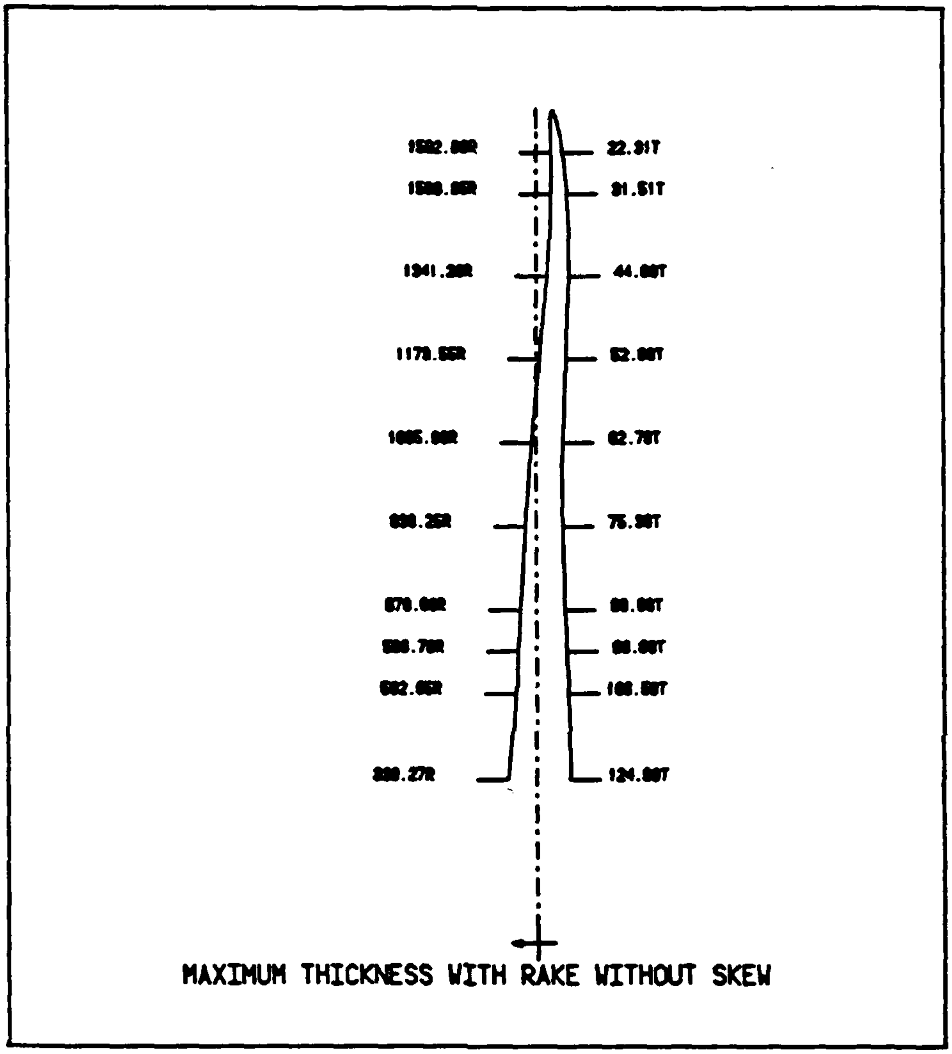


FIGURE 41: COMBINED BLADE THICKNESS AND RAKE

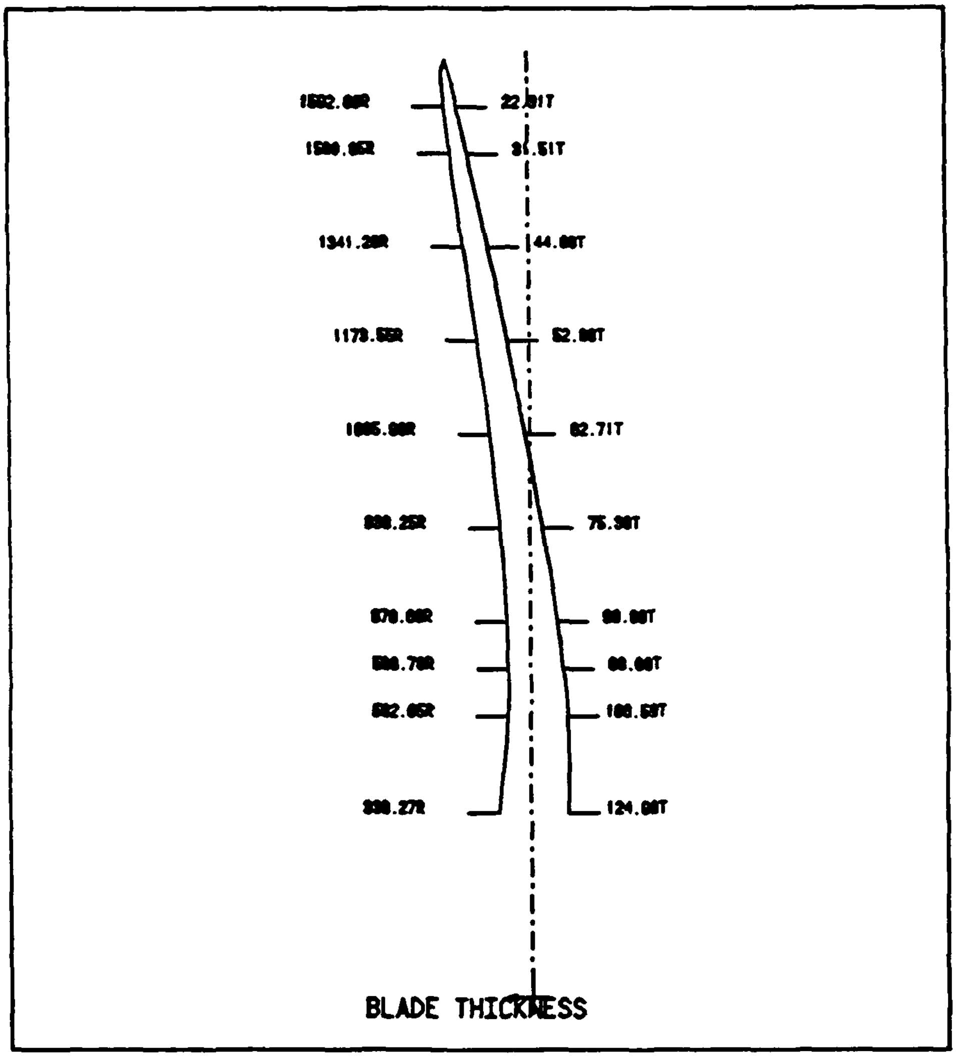


FIGURE 42: NET RAKE AND BLADE THICKNESS DIAGRAM

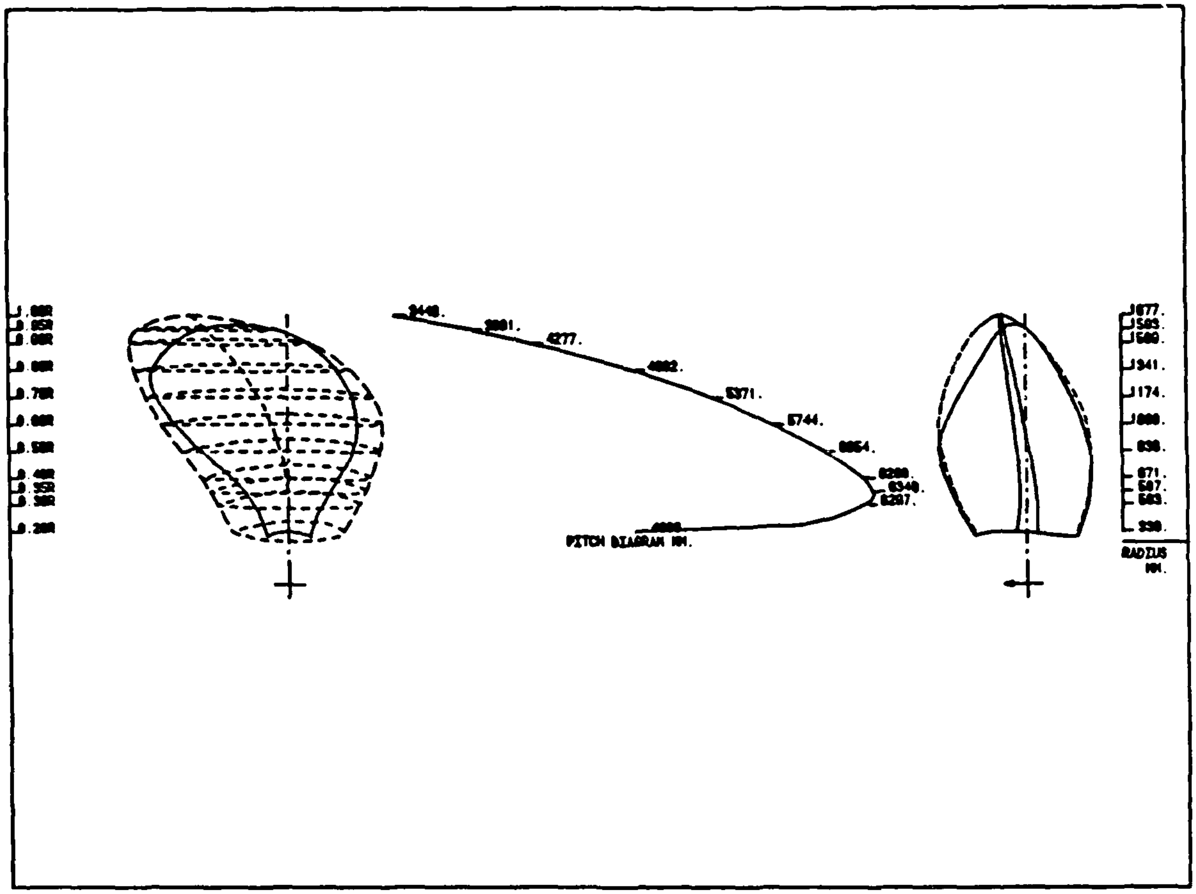


FIGURE 43: COMBINED DRAWING OF PROPELLER BLADE

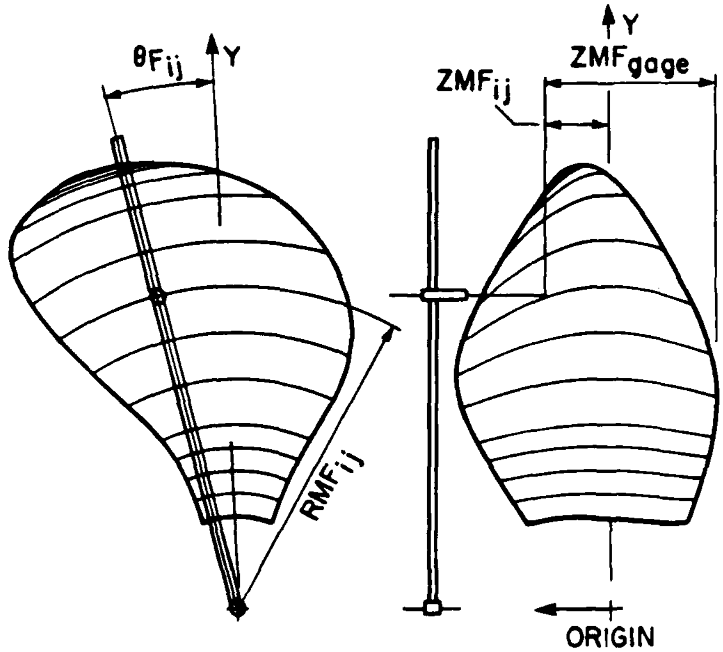


FIGURE 44: BLADE SURFACE MEASUREMENT

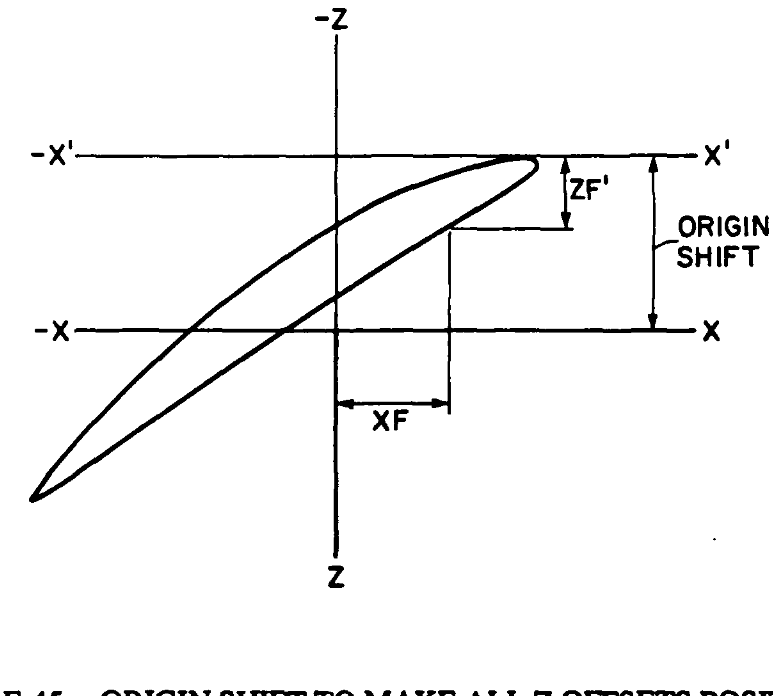


FIGURE 45: ORIGIN SHIFT TO MAKE ALL Z OFFSETS POSITIVE

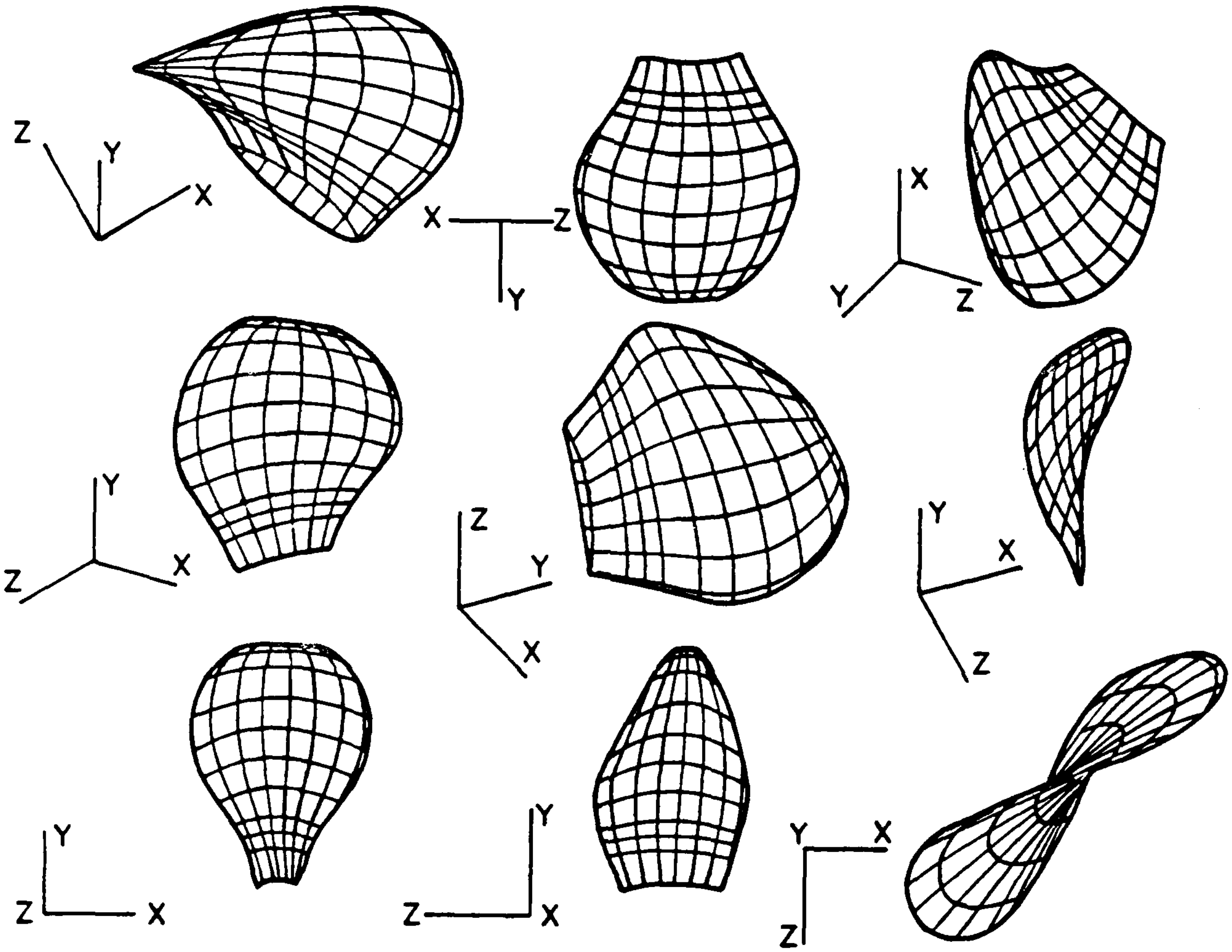


FIGURE 46: SOLID MODELLING OF A BLADE BY PATRAN FROM A PATRAN SESSION FILE PRODUCED BY BLADPL

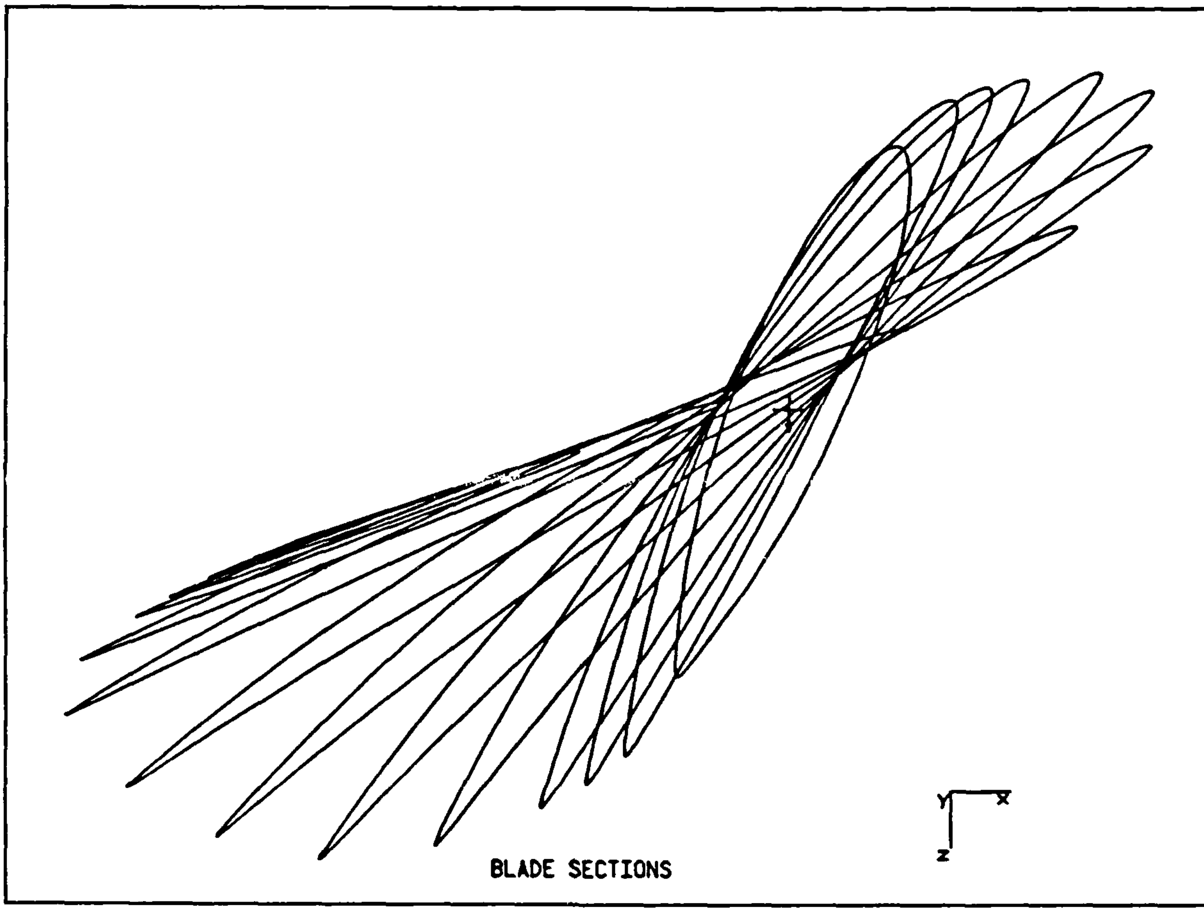


FIGURE 47: PLAN VIEW OF ASSEMBLED BLADE SECTIONS FOR A MODERATELY SKEWED BLADE

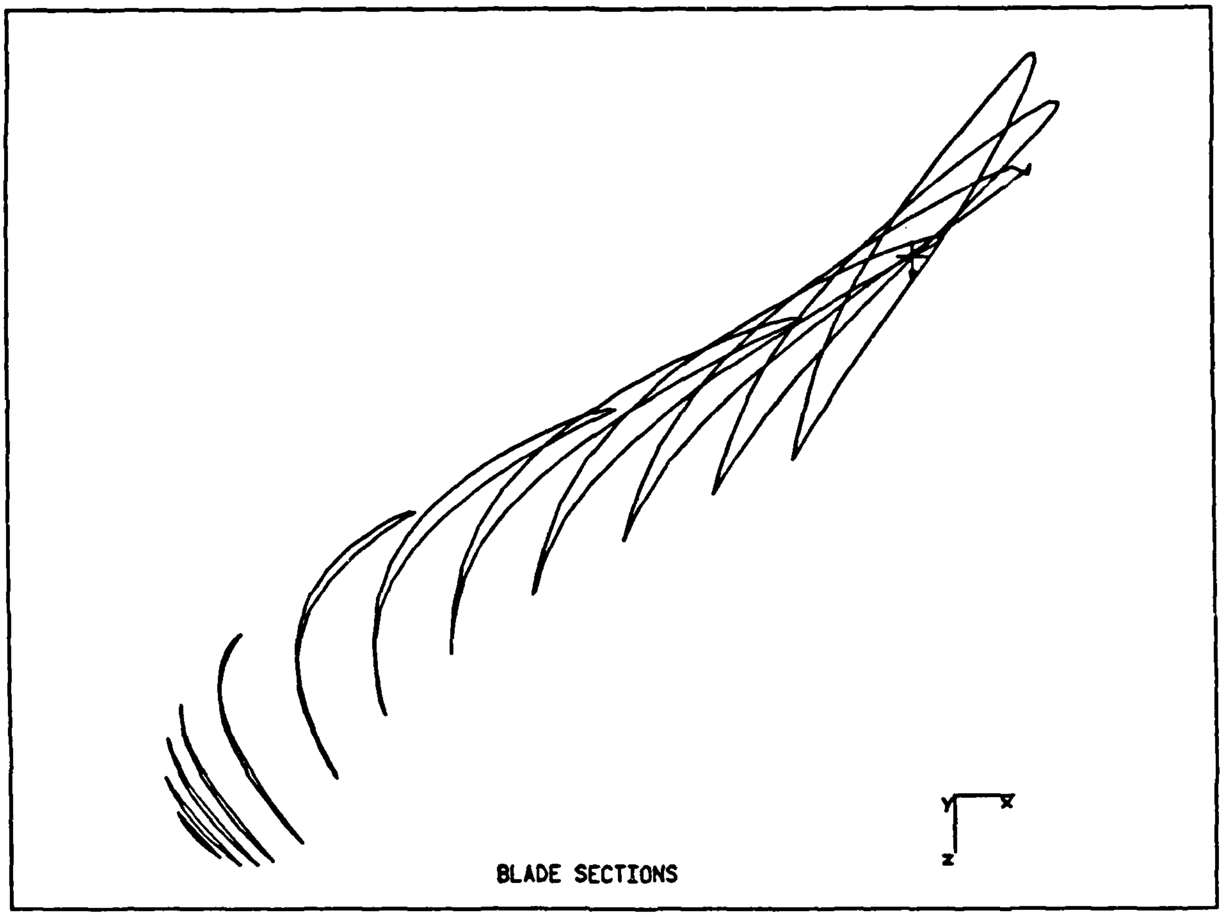


FIGURE 48: PLAN VIEW OF ASSEMBLED BLADE SECTIONS FOR A HIGHLY SKEWED BLADE

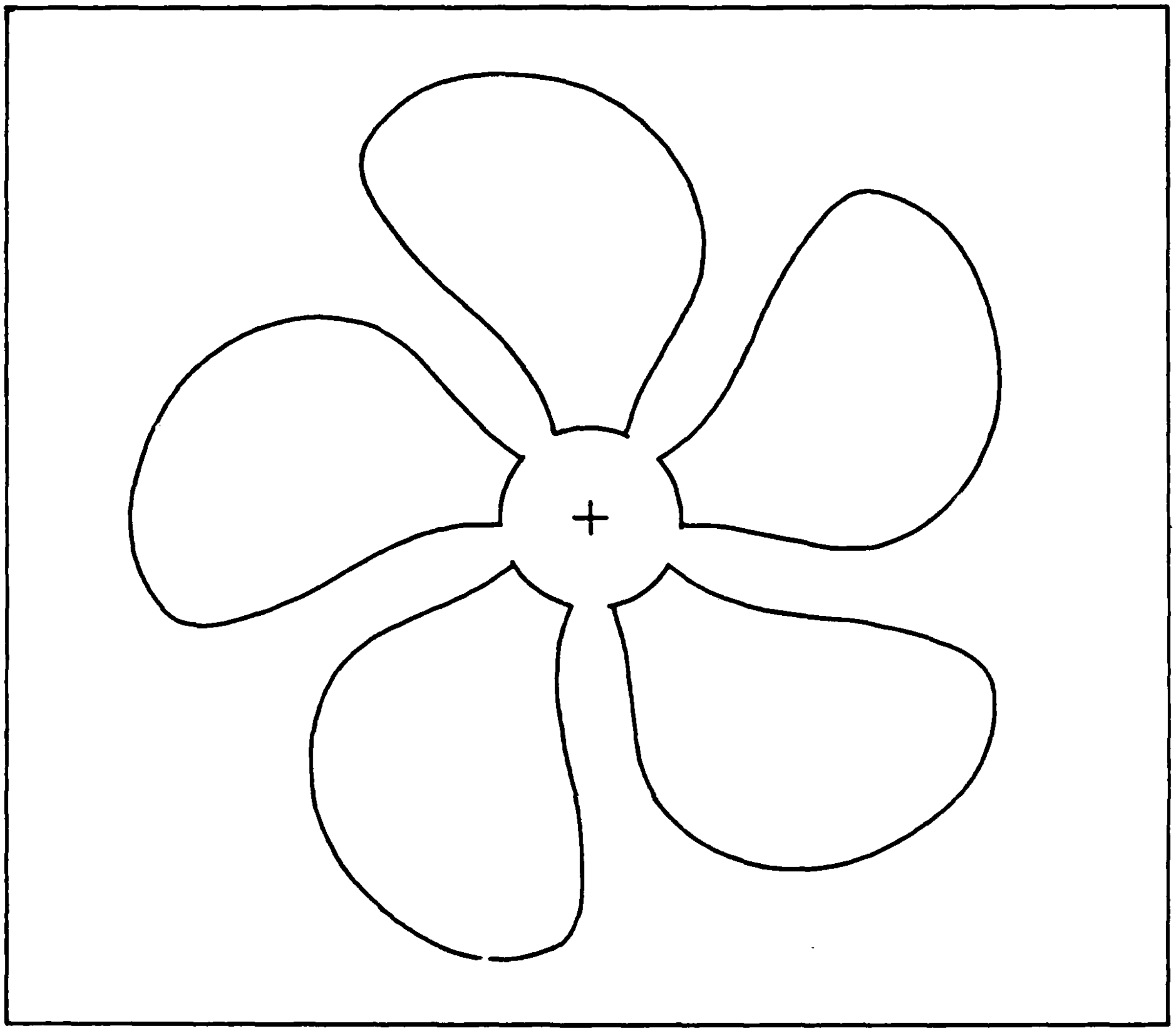


FIGURE 49: TRANSVERSE VIEW OF THE COMPLETE MODERATELY SKEWED PROPELLER

APPENDIX A

TRANSFORMATION FOR LEADING EDGE

Transformation of the x axis in the interval $x_1 \leq x \leq x_2$ using $x = \zeta^2$ will overcome the problem of infinite slope at x_1 and, therefore, infinite value of the slope coefficient B_1 for the x cubic spline function.

The transformation $x = \zeta^2$ was arrived at because of the approximation of the nose of the blade section to the arc of a circle with leading edge radius r (see also Figure 19a and 19b).

The equation for a circle with its centre on the x axis at radius r is

$$(x-r)^2 + y^2 = r^2 \tag{A.1}$$

Expanded and rearranging

$$y^2 = 2rx - x^2 \tag{A.1a}$$

for small x values (ie. equivalent to nose of blade section)

$$y^2 \sim 2rx \tag{A.1b}$$

or taking just the positive root

$$y \sim \sqrt{2r}\ \sqrt{x} \tag{A.1c}$$

Letting $\sqrt{x} = \zeta$ (ie. transformation $x = \zeta^2$) a linear relationship is produced.

$$y = \sqrt{2r}\ \zeta \tag{A.2}$$

Resulting in finite slope at $\zeta = 0$ of

$$\frac{dy}{d\zeta} = \sqrt{2r} \tag{A.2a}$$

In the interval $\zeta_1 \leq \zeta \leq \zeta_2$ (ie. equivalent to $x_1 \leq x \leq x_2$) a cubic spline function is used for interpolation and to generate additional ordinate values.

$$y = A_1 + B_1(\zeta-\zeta_1) + C_1(\zeta-\zeta_1)^2 + D_1(\zeta-\zeta_1)^3 \tag{A.3}$$

where, at $\zeta = \zeta_1 = 0$

$$y = A_1 = 0$$

$$\frac{dy}{d\zeta} = B_1 = \sqrt{2r}$$

and at $\zeta = \zeta_2$

$$y = B_1 \zeta_2 + C_1 \zeta_2^2 + D_1 \zeta_2^3 \qquad \text{(A.4a)}$$

$$\frac{dy}{d\zeta} = B_1 + 2C_1 \zeta_2 + 3D_1 \zeta_2^2 \qquad \text{(A.4b)}$$

by equating the offset ordinate and slope given by the last two equations to the values for the next interval (see Figure 19(a) and 19(b)) such as

$$y = A_2$$

$$\frac{dy}{d\zeta} = \frac{dy}{dx}\frac{dx}{d\zeta} = 2B_2 \zeta_2 \qquad \text{(A.5)}$$

and using the transformation $\zeta = \sqrt{X}$, the coefficients A_1, B_1, C_1 and D_1 can be obtained

$$A_1 = 0 \qquad \text{(A.6)}$$

$$B_1 = \sqrt{2r} \qquad \text{(A.7)}$$

$$C_1 = \frac{1}{\sqrt{X_2}}\left[\frac{3A_2}{\sqrt{X_2}} - 2B_2\sqrt{X_2} - 2B_1\right] \qquad \text{(A.8)}$$

$$D_1 = \frac{1}{X_2}\left[2B_2\sqrt{X_2} - \frac{2A_2}{\sqrt{X_2}} + B_1\right] \qquad \text{(A.9)}$$

APPENDIX B

CURVE FITTING THE BLADE TIP

To determine the radius of curvature at the tip of the blade, a parabola is fitted of the form (Figure 23)

$$y = ax^2 \tag{B.1}$$

The radius of curvature is equal to the reciprocal of the curvature K of a general curve y = f(x) given by

$$r_c = \frac{1}{K} = \left[\frac{1+(dy/dx)^2}{d^2y/dx^2} \right]^{3/2} \tag{B.2}$$

Evaluating the first and second derivatives of y at x = 0, the radius of curvature is given by

$$r_c = \frac{1}{2a} \tag{B.2a}$$

Using the coordinates x_1 and y_1 of the leading and trailing edge points of the last specified blade section,

$$a = \frac{y_1}{x_1^2} \tag{B.2b}$$

Thus, the radius of the blade's tip is

$$r_t = \frac{x_1^2}{2y_1} \tag{B.3}$$

This radius is used to determine the slope coefficient of the cubic spline function in the transformed coordinate system for the blade tip region similar to that outlined in Section 2.7.3.1 and Appendix A for the leading edge fit.

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The geometry of marine propellers is described in this report. The types of data used for blade section description are covered and methods for generating blade section leading and trailing edge forms are derived. A method for generating the blade tip from minimal data is given. The computer program BLADE developed for generating 3 dimensional propeller blade coordinates is described together with examples of its graphic output.

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