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SYNTHETIC OCEAN WAVEFORMS FOR TESTING SONOBUOY SUSPENSIONS

David M. F. Chapman

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Director / Sonar Division

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ABSTRACT

Four synthetic ocean waveforms are proposed for the testing of sonobuoy suspensions. They are based on the Bretschneider ocean wave spectrum, with significant waveheights chosen from the sea state code of the World Meteorological Organization (WMO) and corresponding peak periods based on statistical data from the Canadian North Atlantic Ocean in winter conditions. The waveforms are presented as sums of 20 sine-wave components of specified frequency, amplitude, and phase. There is one waveform each for WMO sea states 3, 4, 5, and 6, corresponding to significant waveheights of 1.25 m, 2.5 m, 4 m, and 6 m, respectively; the associated peak periods are 9 s, 10 s, 12 s, and 14 s, respectively. The maximum and minimum frequencies for each waveform are chosen to bracket 98% of the wave energy. All the information required to generate the waveforms is provided; waveform plots and numerical values at 0.5 s intervals are included for checking purposes.

RÉSUMÉ

Quatre formes d'ondes océaniques synthétiques sont proposées pour les tests de suspensions de bouées acoustiques. Elles sont fondées sur le spectre d'ondes océaniques de Bretschneider, les hauteurs d'onde significatives étant choisies à partir du code d'état de la mer de l'Organisation météorologique mondiale (OMM) et des périodes de crête correspondant aux données statistiques de l'Atlantique nord canadien pendant l'hiver. Les formes d'ondes sont présentées sous forme de sommes de 20 composantes sinusoïdales, de fréquence, d'amplitude, et de phase données, pour chacun des états de la mer 3, 4, 5 et 6 de l'OMM, qui correspondent respectivement à des hauteurs d'ondes significatives de 1.25 m, 2.5 m, 4 m et 6 m. Les périodes de crête associées sont respectivement 9 s, 10 s, 12 s et 14 s. Les fréquences minimale et maximale pour chaque forme d'onde sont choisies de manière à comprendre 98% de l'énergie de l'onde. Toutes les données nécessaires à la production des formes d'ondes sont données; on donne les graphiques de formes d'ondes et les valeurs numériques à intervalles de 0.5 s à des fins de vérification.

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NOTATION

A	parameter of the Bretschneider spectrum
A_n	amplitude of n^{th} sine wave component
B	parameter of the Bretschneider spectrum
CFMETR	Canadian Forces Maritime Experimental &Test Ranges
α	fraction of wave energy above f_{max} and below f_{min}
f	ocean wave frequency
$f_{max} = f_N$	maximum frequency of N sine waves
$f_{min} = f_1$	minimum frequency of N sine waves
f_n	frequency of n^{th} sine wave component
f_o	peak frequency of ocean wave spectrum
H_s or $H_{1/3}$	significant wave height
KEY	large integer seed for FORTRAN intrinsic function RAN
n	wave component index
N	number of sine wave components
RAN(...)	FORTRAN random number intrinsic function
$S(f)$	wave spectral density
s.d	standard deviation
t	time
T_o	modal period of ocean wave spectrum
$T_p = 1/f_o$	peak period of ocean wave spectrum
WMO	World Meteorological Organization
$z(t)$	sea surface elevation
z_{rms}	root-mean-square sea surface elevation
Δf	nominal frequency separation of sine waves
ϕ_n	phase of n^{th} sine wave component
$\pi = 3.1415926535...$	rounded value of pi

1. INTRODUCTION

1.1 Overview

Sonobuoys are expendable underwater acoustic sensors that are deployed and monitored from submarine-hunting aircraft. The acoustic sensor is suspended in some manner from the surface buoy; data are transferred by electric cable from the sensor to the surface buoy and then to the aircraft by radio telemetry. Two of the dominant noise components at low frequencies are self-noise due to vibrations of the suspension system itself and hydrodynamic flow noise due to sensor motion through the water, so it is imperative to design the suspension system with care in order to achieve good acoustic performance.

Two methods have been devised as alternatives to at-sea testing of sonobuoy suspensions: (a) computer modelling of sonobuoy suspension dynamics using a numerical hydro-mechanical simulation and (b) controlled dynamical testing of suspension systems in tanks, harbours, and inlets using a mechanical winch-and-cable sea state simulator. For both methods, one needs to specify several time series of typical vertical motions associated with a range of anticipated sea states. For method (a), the time series would be one of the inputs to the numerical algorithm; for method (b) the time series would provide signals to a controlling mechanism that would drive the winch back and forth.

In this document we propose some ocean waveforms for the testing of sonobuoy suspensions over a range of typical sea states. Each waveform is characterized by a set of 20 sinusoidal components of specified frequency, amplitude, and phase. The waveforms can be generated easily from these parameters. As a check, we have provided graphs of surface elevation vs. time for the first 50 s of each waveform and tables of the numerical values of surface elevation for the first 20 s of each waveform at 0.5 s intervals.

The waveforms are based on the Bretschneider analytic ocean wave spectrum with significant waveheights taken from the sea state code proposed by the World Meteorological Organization and peak periods taken from statistical data collected in the Canadian North Atlantic Ocean during winter. The calculations were performed using the FORTRAN code CSTATE developed at DREA; this code and the analysis associated with it will be the subject of a separate DREA publication.

1.2 Brief Literature Review

The classic work on ocean surface waves generated by wind is that of **KINSMAN 1965**. Although this is quite readable, we have found the more up-to-date work by **SARPKAYA and ISAACSON 1981** to be more practical for our purposes, especially as a reference for the Bretschneider spectrum and other analytic spectra. **BERTAUX 1976** also has some introductory material on ocean surface waves. A set of waveforms for sonobuoy testing was produced by **BRETT 1977**; these are currently being used in the test facility at the Canadian Forces Maritime Experimental & Test Ranges (CFMETR) in British Columbia. [There is no relation between the Bretschneider spectrum and the waveforms discussed by Brett, despite the similar-sounding names.] **GRAHAM 1984** and **CAMPBELL and GRAHAM 1988** describe how the Bretschneider spectrum is used to generate irregular seas for ship motion studies at DREA. The sea state code of the World Meteorological Organization is discussed by **LLOYD 1989**.

1.3 Motivation

As the simulation of random ocean waves by the summation of discrete sine wave components is not a new technique, and as a set of waveforms already exist for sonobuoy suspension tests, some justification is required for the production of this document.

The waveforms described in **BRETT 1977** - which we will call Brett waveforms for convenience - were based on a discrete representation of an analytic wave spectrum, but the phases were not chosen randomly. In fact, the phases were chosen so that all the components would be in phase at a particular instant (980 s), forming a "freak" or "rogue" wave of unusual height. At other parts of the waveform, the sea state seems unusually low. However desirable such a rogue wave might be in the simulation, the non-random choice of phases results in a waveform which has non-stationary statistical properties. Also, the sea surface elevation statistics are non-Gaussian if the rogue wave is included. For these reasons (and possibly other reasons), there seems to be a general impression that dynamic tests based on the Brett waveforms do not prepare sonobuoy engineers for what their sonobuoys encounter in the real ocean.

The significant waveheights of the published Brett waveforms do not seem to coincide with any sea state code. The waveforms were based upon the Pierson-Moskowitz spectrum and the sea state code published in **BERTAUX 1976**, but the significant waveheights of the resulting waveforms seem to be 10% - 16% lower than expected, even with the rogue wave included. Without the rogue wave, they are reduced a further 4% in height.

When compared with the mean waveheights from the sea state code proposed by the World Meteorological Organization (WMO), the Brett waveforms give higher waveheights for some sea states and lower waveheights for other sea states. Following the lead of the naval architects, we propose to adopt the WMO sea state code as a basis for synthesizing waveforms for sonobuoy suspension testing. Furthermore, rather than using a sea state code to designate ocean wave height, we wish to promote the use of significant wave height as an unambiguous quantitative measure of sea state, to avoid confusion between different sea state codes.

As part of a Sonobuoy Suspension Design Study, DREA has built its own suspension test facility to be operated near DREA in Bedford Basin; an identical facility is being constructed for use at CFMETR. For these new facilities, we decided to prepare some new ocean waveforms whose characteristics we could specify.

2. SEA STATE SPECIFICATION

For the purpose of simulating the vertical motion of the floating component of a deployed sonobuoy, only the time history of the motion at a given location is required, so we ignore the spatial and directional characteristics of ocean waves in this work. It is convenient to regard the time series of sea surface elevation as the integral of a continuous spectrum of sinusoidal components having different temporal frequencies and uncorrelated phases. Various analytic forms of ocean wave spectra have been proposed, based partially on the hydrodynamic theory of waves and partially upon ocean wave data. We have chosen to use the Bretschneider spectrum because DREA also uses this spectrum for the study of ship dynamics. Once a wave spectrum is chosen, it is easy to generate representative time series of sea surface elevation.

2.1 The Bretschneider Ocean Wave Spectrum

As shown in **SARPKAYA and ISAACSON 1981**, the Bretschneider spectrum has the form :

$$S(f) = (A/f^5) \exp(-B/f^4), \tag{1}$$

in which f is frequency, S(f) is the spectral density of the sea surface elevation, and A and B are

$$A = 5\, z_{rms}^2 f_0^4 \approx 5\, H_s^2 f_0^4 /16 \tag{2a}$$

and

$$B = 5\, f_0^4 /4, \tag{2b}$$

in which z_{rms} is the root-mean-square sea surface elevation, H_s is the significant wave height and f_0 is the peak frequency in the spectrum. The approximate relation $H_s \approx 4\, z_{rms}$ will be discussed below; H_s is the more commonly-used parameter.

The function S(f) is defined so that the integral of S over all frequencies is exactly the mean-square elevation of the ocean surface, that is,

$$\int_0^\infty S(f)\,df = A/4B = z_{rms}^2 \approx H_s^2/16. \tag{3}$$

The well-known Pierson-Moskowitz spectrum has the same generic form, but the coefficients A and B are different and depend on one adjustable parameter only (usually wind speed). In contrast, the Bretschneider spectrum is a two-parameter spectrum, as both f_o and H_s are separately adjustable.

2.2 Wave Height

In a waveform of sea surface elevation, one observes an apparently random succession of peaks and troughs of varying size. The *waveheight* of an individual wavelet is defined to be the vertical distance between the wave crest and the following wave trough. The sequence of waveheights in a given sea is therefore a random process and is unpredictable, except in a statistical sense. The *average waveheight* is simply the average of all the waveheights. A more useful measure of the sea state is the *significant waveheight*, which is the average of the highest one-third of the waves. The significant waveheight, denoted H_s or $H_{1/3}$, is clearly larger than the average waveheight, but has been judged to be the quantitative measure that is closest to subjective waveheight estimates made by experienced observers.

Mathematically, a waveform whose sea surface elevation statistics derive from a narrowband Gaussian process has a significant waveheight that is related to the root-mean-square sea surface elevation by

$$H_s = 4.0051\ z_{rms} \approx 4\ z_{rms}. \tag{4}$$

One could quibble with the narrowband Gaussian assumption for ocean waves, but it is acceptable for our purpose. The effect of spectral bandwidth on waveheight statistics is dicussed by **SARPKAYA and ISAACSON 1981.** The effect of finite spectral bandwidth is to reduce slightly the actual significant waveheight from the narrowband value. In the context of this work, it is convenient to regard H_s simply as four times z_{rms}.

2.3 Wave Period

The peak period T_p is the inverse of the peak frequency f_o, the frequency at which the wave spectrum is a maximum. [When regarded as a function of wave period rather than wave frequency, the wave spectrum peaks at the modal period T_o, which is about 12% less than the peak period T_p for the Bretschneider spectrum.] A *fully-developed* sea is the sea that results from a constant wind blowing for an infinite time over an infinite fetch. Compared to the fully-developed sea, *developing* seas have spectra which peak at short periods (high frequencies) and *decaying* seas have wave spectra which peak at long periods (low frequencies). Since the Bretschneider ocean wave spectrum has independent parameters controlling waveheight and peak period, it can model a wide variety of sea conditions. For a given sea state (i.e. for fixed H_s) one can adjust the frequency distribution of the waves by varying f_0 (or T_p). Table 2 in **CAMPBELL and GRAHAM 1988** shows a joint probability distribution of significant waveheights and modal periods of waves in the Canadian North Atlantic Ocean in winter.

2.4 The World Meteorological Organization Sea State Code

Table I , from **LLOYD 1989**, defines the sea state code according to the WMO. Each "sea state" spans a range of significant waveheights. The minimum and maximum values shown specify the bounds of each sea state category; the mean value shown is simply the average of these two. Naval architects within NATO use this code as part of an international standard for seakeeping studies. We propose to adopt the same sea state code for our Sonobuoy Suspension Design Study.

TABLE I: The WMO Sea State Code

Sea State	Significant Waveheight [m]		
	min	mean	max
3	0.5	0.875	1.25
4	1.25	1.875	2.5
5	2.5	3.25	4.0
6	4.0	5.0	6.0
7	6.0	7.5	9.0
8	9.0	11.5	14.0

Furthermore, due to the possible confusion that arises from using a sea state code, we wish to promote the use of significant wave height as an unambiguous quantitative measure of sea state. "sea state 5" could mean a sea with H_s lying anywhere from 2.5 m to 4 m in the WMO sea state code, and it may mean something entirely different to those unfamiliar with the WMO sea state code. We prefer the more precise terminology "a 3 m sea state" or "a 4 m sea state", with the understanding that significant waveheight is the commonly-used measure of waveheight. Even so, this is an incomplete specification of the sea, as it ignores the distribution of wave energy over frequency. It would be even better to specify a peak frequency in addition to the significant waveheight.

If one groups the waveheight data reported in **CAMPBELL AND GRAHAM 1988** according to the WMO sea state code, one finds the following percentage occurences of sea states during winter in the Canadian North Atlantic: sea state 3 and below (2%), sea state 4 (12%), sea state 5 (27%), sea state 6 (32%), sea state 7 and above (27%).

2.5 Discrete Sine-Wave Representation

For practical purposes, we may generate a realization of an ocean waveform corresponding to a particular analytical spectrum by forming a sum of N discrete sine waves having amplitudes A_n and frequencies f_n spanning a spectrum from a low frequency of $f_{min}=f_1$ to a high frequency of $f_{max}=f_N$; that is,

$$z(t) = \sum_{n=1}^{N} A_n \sin(\phi_n - 2\pi f_n t), \qquad (5)$$

in which z(t) is the height of the ocean surface above its mean level,

$$f_n = f_{min} + (n-1)\Delta f, \qquad (6a)$$

$$\Delta f = (f_{max}-f_{min})/(N-1), \qquad (6b)$$

$$A_n = \sqrt{2\, S(f_n)\, \Delta f}\,, \qquad (6c)$$

and the ϕ_n are phases randomly chosen within the range 0 to 2π. The sum in Eq.(5) should be regarded as an approximation to the integral of a continuous spectrum of sinusoidal waves. However, if a sufficient number of discrete sine-wave components are chosen from the significant band of the spectrum, the approximation can be made as accurate as one desires.

2.6 Some Numerical Considerations

In implementing such a discrete-component representation of a continuous ocean wave spectrum there are some numerical aspects to consider. To avoid unwanted periodicities that arise from precise equal spacing of component frequencies, it is common practice to "jiggle" the frequencies by shifting them slightly from the nominal frequencies prescribed by Eq.(6b). Also, if frequencies are specified only to n places after the decimal, then the waveform necessarily repeats after 10^n seconds, as the phase of each component will have advanced through an integer multiple of 2π radians.

Another numerical issue is the choice of minimum and maximum frequencies for the discrete representation and what to do with the wave energy that is essentially discarded outside these limits. Fortunately, the Bretschneider spectrum is analytically integrable, so this is easy to do. From Eq.(1), the integral of S(f) over frequency from 0 to ∞ is A/4B. Let us define the minimum and maximum frequencies of our discrete representation so that a fraction α of the wave energy lies above f_{max} and the same fraction α lies below f_{min}. Then f_{max} and f_{min} are given by

$$\int_0^{f_{min}} S(f)\,df = \int_{f_{max}}^{\infty} S(f)\,df = \alpha(A/4B), \tag{7}$$

or

$$f_{min} = [-(4/5)\ln(\alpha)]^{-1/4} f_o, \tag{8a}$$

and

$$f_{max} = [-(4/5)\ln(1-\alpha)]^{-1/4} f_o. \tag{8b}$$

The wave energy that would otherwise be discarded is added back into the waveform by adjusting the amplitudes of the lowest- and highest-frequency components:

$$A_i \Rightarrow \sqrt{A_i^2/2 + \alpha H_s^2/8}\ . \qquad (i\ = 1 \text{ and } N) \tag{9}$$

Finally, there is the issue of the number of components required to adequately simulate the ocean waveform. **BRETT 1977** discretized the Pierson-Moskowitz spectrum using 19 sine waves, choosing maximum and minimum frequencies to match the spectral content for each sea state. **GRAHAM 1984** discretized the Bretschneider spectrum using 40 sine waves, but he used the same maximum and minimum frequencies for all sea states: $f_{min} = .0318$ Hz and $f_{max} = .3183$ Hz. We feel that 20 components should suffice, if the above procedure is followed for choosing maximum and minimum frequencies.

3. WAVEFORM CALCULATIONS

We used the computer code CSTATE to generate component frequencies, amplitudes, and phases for simulated ocean waveforms corresponding to WMO sea states 3 through 6. As mentioned before, the CSTATE code will be described in a separate document, along with the relevant documentation and more mathematical details.

Given that the WMO sea state code admits a wide range of significant waveheights within each sea state category and that the Bretschneider spectrum allows independent choice of peak frequency, it is difficult to come up with a representative single pair of (H_S,T_P) parameters for each "sea state". On the other hand, it is desirable to have a small number of waveforms from which to choose for testing purposes. For each WMO sea state category, we have chosen the maximum value of H_S from Table I, rather than the mean value. (If one wants to claim that a particular sonobuoy suspension performs to specification in "Sea State 6", then the test waveform should be higher than *all* sea state 6 waveforms.) We used the statistical data presented in **CAMPBELL and GRAHAM 1988** to choose values of T_P that are slightly longer than the most probable value for a given sea state. The parameters we settled upon are shown in Table II.

TABLE II: Spectrum Parameters of Computed Waveforms

Sea State (WMO)	Significant Waveheight H_S [m]	Peak Period T_P [s]
3	1.25	9
4	2.5	10
5	4.0	12
6	6.0	14

This is the procedure we follow to compute the frequencies, amplitudes, and phases of the sine-wave components and to compute the corresponding waveforms:

- For each waveform, choose parameter values H_S and $f_o = 1/T_P$. [The values we use are shown in Table II.]

- Calculate f_{min} and f_{max} from Eqs.(8a) and (8b), to bracket 98% of the wave energy ($\alpha = .01$).

- Calculate 20 equally-spaced frequencies between f_{min} and f_{max}, using Eqs.(6a) and (6b); then "jiggle" them by adding values chosen randomly within the range $-\Delta f/40$ to $+\Delta f/40$.

- Calculate the wave component amplitudes A_n using Eq.(6c) and Eq.(9).

- Calculate the phases ϕ_n by randomly choosing numbers between 0 and 2π.

- Calculate synthetic ocean waveforms using Eq.(5), sampling every 0.5 s.
- Verify the basic statistical characteristics of the waveform: the ideal waveform would have zero mean, a standard deviation (s.d.) of $H_s/4$, zero skew, and a kurtosis that is exactly $3(\text{s.d.})^4$.

The sine-wave components for WMO sea states 3-6 are shown in Tables III-VI, respectively. All tabular values are as printed by the computer using the FORTRAN default output format. The first 50 s of the corresponding waveforms are plotted in Figures 1-4. The Appendix contains tables of numerical values of the first 20 s of these waveforms; these are provided for checking purposes, should the reader wish to implement a similar numerical computation of ocean waveforms.

We computed several moments of the synthesized sea surface elevation as a crude test for the normality of the distribution . For each waveform, we generated 3000 samples at time intervals of $T_p/4$. We calculated the mean, the standard deviation (s.d.), the skew and the kurtosis of the samples, and normalized the mean, skew, and kurtosis by (s.d.), $(\text{s.d.})^3$, and $(\text{s.d.})^4$, respectively. For all the waveforms, the error in the standard deviation (compared to one-quarter the desired value of H_s) was less than 1%. In all cases, the normalized mean was less than 3×10^{-4} and the normalized skew was less than 3×10^{-3}. The normalized kurtosis varied around the ideal value of 3 (for a Gaussian distribution), ranging from 2.8 to 3.1. Compare these values with the normalized kurtosis of the 1000 s samples of the Brett waveforms with the rogue wave included: they ranged from 3.7 (sea state 3) up to 6.0 (sea state 8). Although more formal statistical tests are available, we are satisfied that the statistics of the new waveforms are an improvement over the statistics of the Brett waveforms and that the new waveforms more closely represent a Gaussian process.

We calculated the random phases using the VAX/VMS FORTRAN intrinsic function RAN. The 20 phase values were $2\pi\times\text{RAN(KEY)}$, where KEY is a large integer used to seed the random number generator. We used the rounded value 3.1415926535 for π. The first value of KEY for every sequence is supplied by the user: we used the seed numbers 123456789, 456123789, 456789123, and 123789456 for WMO sea states 3-6, respectively. In this way, each waveform has a different set of starting phases, but we can re-create the same random sequences at will.

TABLE III: Components of a 1.25 m Sea State (WMO Sea State 3)

$H_s = 1.25$ m $T_p = 9$ s

n	f_n [Hz]	A_n [m]	ϕ_n [rad]
1	8.0089264E-02	7.2960034E-02	1.802866
2	9.5370702E-02	0.1699056	0.8529510
3	0.1105974	0.1963142	5.522275
4	0.1261697	0.1833094	3.270354
5	0.1417645	0.1575738	3.663849
6	0.1567392	0.1325387	0.8847657
7	0.1723868	0.1098316	5.602105
8	0.1870575	9.2279606E-02	0.8799245
9	0.2028357	7.7007420E-02	5.919548
10	0.2180873	6.5158397E-02	5.152680
11	0.2334537	5.5514280E-02	2.016389
12	0.2485950	4.7782790E-02	4.443599
13	0.2635380	4.1509736E-02	1.062167
14	0.2788676	3.6182538E-02	3.582838
15	0.2948716	3.1570427E-02	5.195513
16	0.3098810	2.7948666E-02	6.281874
17	0.3251981	2.4817949E-02	6.6411332E-03
18	0.3400603	2.2225510E-02	5.711411
19	0.3558834	1.9860348E-02	1.182253
20	0.3707672	4.5979332E-02	0.4022419

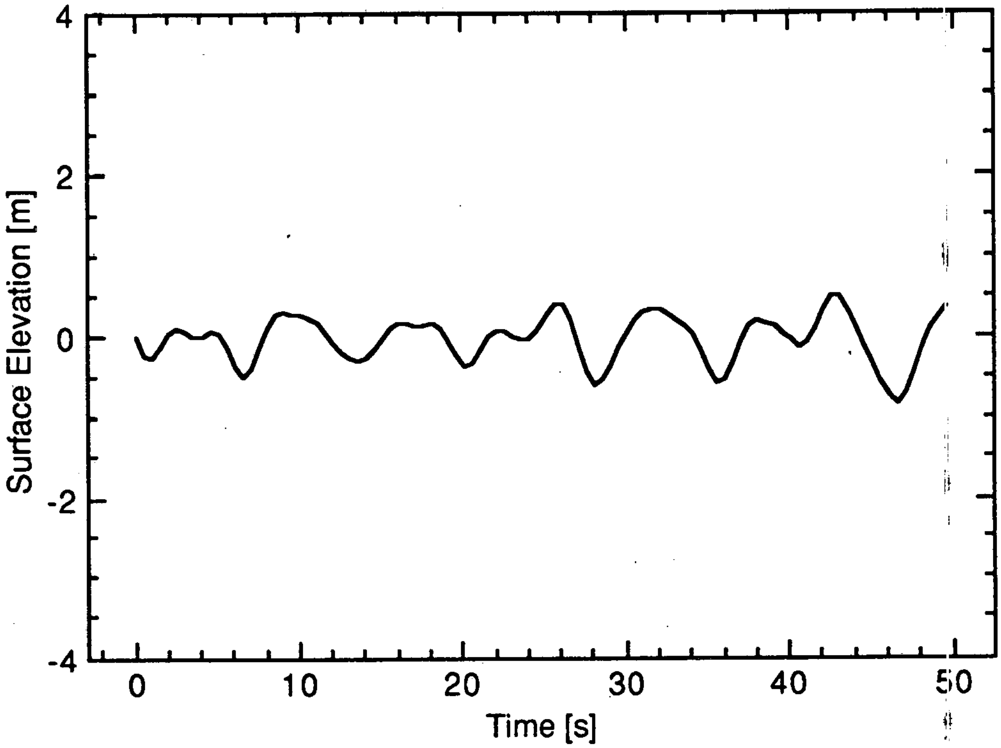

Figure 1. Simulated 1.25 m sea state (WMO sea state 3) waveform based on the Bretschneider spectrum: $H_s = 1.25$ m, $T_p = 9$ s.

TABLE IV: Components of a 2.5 m Sea State (WMO Sea State 4)

$H_s = 2.5$ m $T_p = 10$ s

n	f_n [Hz]	A_n [m]	ϕ_n [rad]
1	7.1904220E-02	0.1443992	4.784852
2	8.5823163E-02	0.3397239	4.532616
3	9.9779993E-02	0.3926614	0.2971983
4	0.1131726	0.3678329	5.414904
5	0.1271300	0.3169089	1.473751
6	0.1410419	0.2651594	4.943305
7	0.1545967	0.2212844	1.415740
8	0.1688080	0.1834685	3.346826
9	0.1824402	0.1542298	0.8706926
10	0.1959984	0.1307515	3.298077
11	0.2100781	0.1110665	1.875230
12	0.2239326	9.5363751E-02	1.577244
13	0.2372521	8.2961969E-02	1.538996
14	0.2514071	7.2066389E-02	2.230188
15	0.2652191	6.3237332E-02	3.150291
16	0.2785792	5.6052230E-02	1.642429
17	0.2927443	4.9608313E-02	3.664547
18	0.3061650	4.4411305E-02	1.194104
19	0.3201054	3.9778981E-02	3.030540
20	0.3341179	9.1936499E-02	1.059237

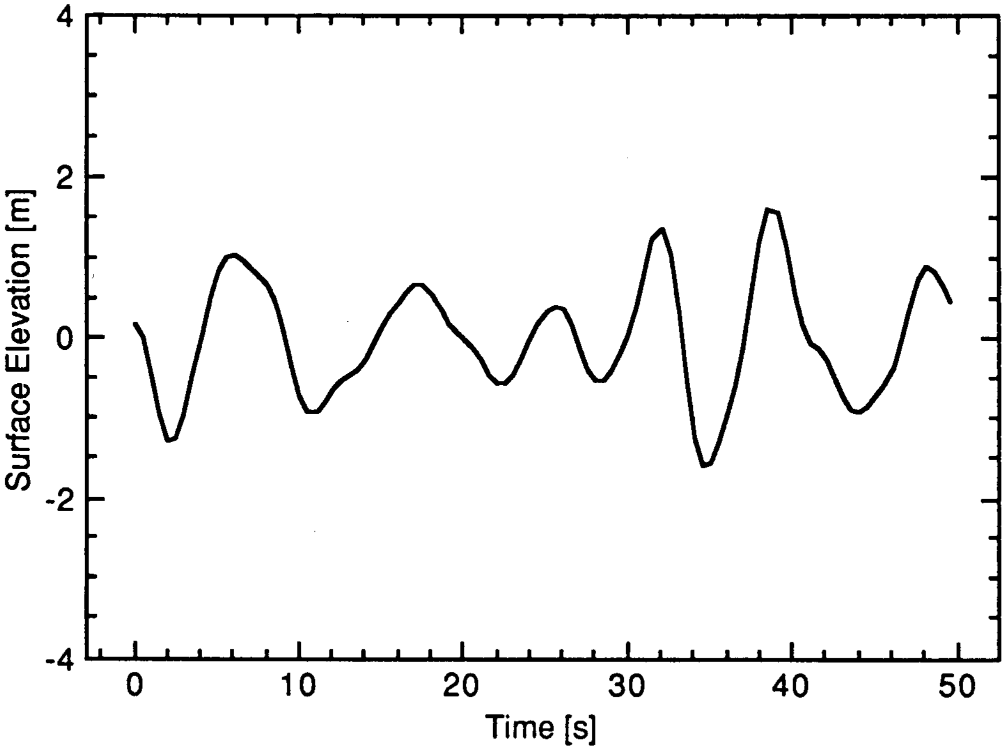

Figure 2. Simulated 2.5 m sea state (WMO sea state 4) waveform based on the Bretschneider spectrum: $H_s = 2.5$ m, $T_p = 10$ s.

TABLE V: Components of a 4 m Sea State (WMO Sea State 5)

$H_s = 4$ m $T_p = 12$ s

n	f_n [Hz]	A_n [m]	ϕ_n [rad]
1	6.0321733E-02	0.2377353	4.690284
2	7.1781054E-02	0.5476937	0.5438693
3	8.3158433E-02	0.6282595	3.792766
4	9.4670385E-02	0.5863233	6.148043
5	0.1060293	0.5064071	7.4503683E-02
6	0.1178403	0.4221999	5.127352
7	0.1289227	0.3535335	5.252677
8	0.1407820	0.2930529	6.154060
9	0.1520391	0.2467472	2.418825
10	0.1633958	0.2090120	0.4166687
11	0.1748492	0.1782272	1.761850
12	0.1864122	0.1529719	5.718413
13	0.1980098	0.1322531	5.473223
14	0.2093207	0.1155552	3.019164
15	0.2209903	0.1012084	1.902130
16	0.2323635	8.9480504E-02	0.9419994
17	0.2436150	7.9645619E-02	0.6433671
18	0.2552306	7.0994020E-02	3.522415
19	0.2669574	6.3526660E-02	5.572644
20	0.2781510	0.1471263	0.4824976

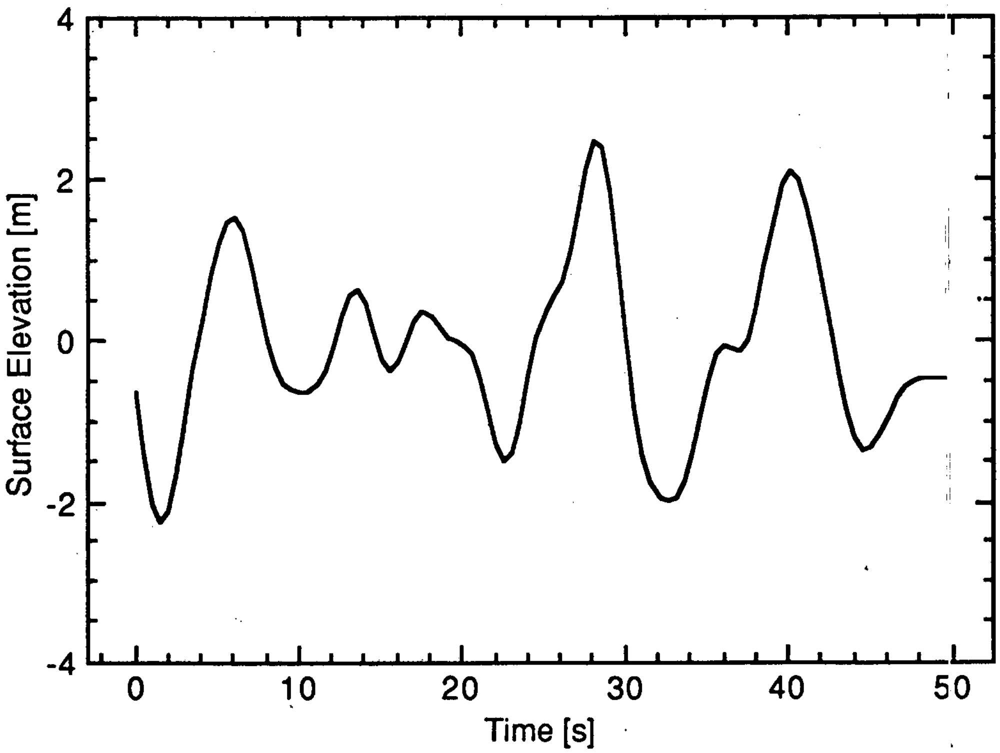

Figure 3. Simulated 4 m sea state (WMO sea state 5) waveform based on the Bretschneider spectrum: $H_s = 4$ m, $T_p = 12$ s.

TABLE VI: Components of a 6 m Sea State (WMO Sea State 6)

H_s =6 m T_p =14 s

n	f_n [Hz]	A_n [m]	ϕ_n [rad]
1	5.1658042E-02	0.3552414	4.897175
2	6.1175890E-02	0.8117661	2.000170
3	7.1101956E-02	0.9423104	4.128466
4	8.1263289E-02	0.8782117	0.4953305
5	9.0679832E-02	0.7622269	6.105818
6	0.1006458	0.6375458	4.118382
7	0.1103675	0.5316657	1.237388
8	0.1202979	0.4425668	2.283542
9	0.1301508	0.3712110	3.552021
10	0.1399803	0.3139003	0.5703832
11	0.1497388	0.2678993	5.101292
12	0.1599721	0.2288036	0.2309982
13	0.1695458	0.1988814	6.170874
14	0.1794387	0.1732834	3.977326
15	0.1893034	0.1520424	1.429840
16	0.1993010	0.1340017	2.790066
17	0.2089108	0.1193303	1.637644
18	0.2186501	0.1066343	3.733974
19	0.2286231	9.5494002E-02	5.594897
20	0.2387221	0.2206361	0.1138726

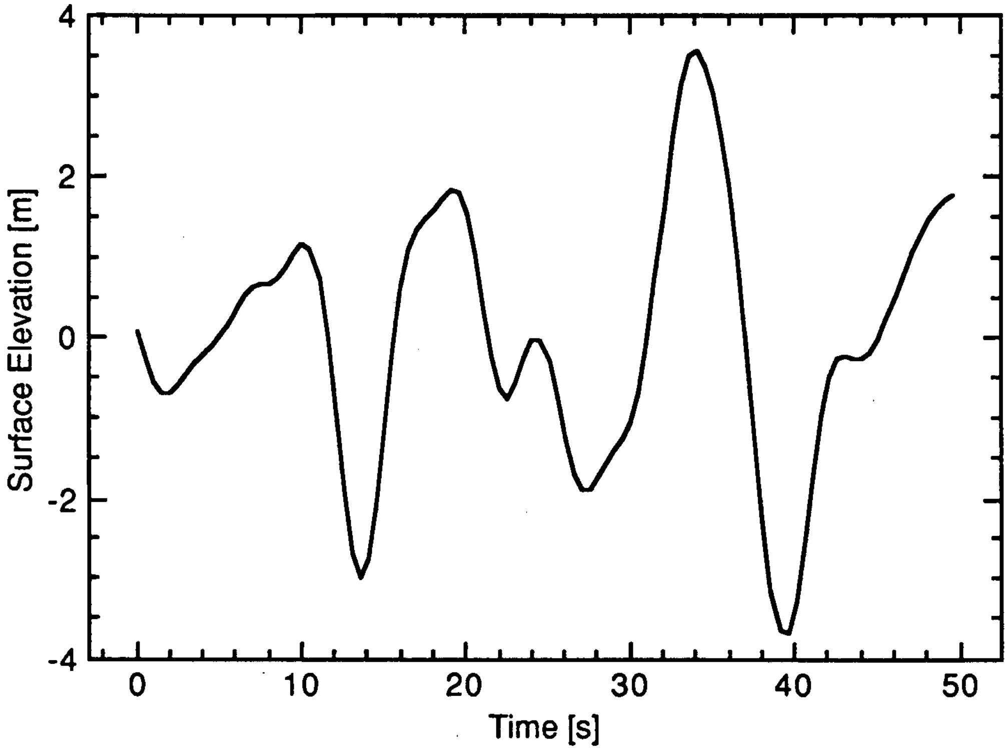

Figure 4. Simulated 6 m sea state (WMO sea state 6) waveform based on the Bretschneider spectrum: $H_s = 6$ m, $T_p = 14$ s.

4. SUMMARY

In this memorandum, we have presented some synthetic ocean waveforms for use in sonobuoy suspension design work. The waveforms are based on a Bretschneider two-parameter spectrum: the input parameters are significant wave height H_s and peak period T_p (i.e. the inverse of f_o, the peak frequency). The waveform for each sea state has the maximum value of significant wave height allowed by the sea state code of the World Meteorological Organization (also used by NATO) and a corresponding peak period based on statistical data from the Canadian North Atlantic. The waveforms are presented as sums of 20 sine-wave components of specified frequency, amplitude, and phase. We have presented four waveforms: one each for WMO sea states 3, 4, 5, and 6, corresponding to significant waveheights of 1.25 m, 2.5 m, 4 m, and 6 m, respectively; the associated peak periods are 9 s, 10 s, 12 s, and 14 s, respectively.

In addition, we have presented the frequencies, amplitudes, and phases of the 20 sine-wave components used to synthesize the waveforms. We have prescribed the method used to compute the waveforms and we have included all information needed to verify the computations. We have plotted graphs of the first 50 s of each waveform and we have generated tables of numerical values of the first 20 s of each waveform, sampled at 0.5 s intervals.

Finally, we have proposed that DREA adopt the WMO sea state code for the Sonobuoy Suspension Design Study and we have promoted the idea of using the significant waveheight as an unambiguous quantitative measure of sea state.

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Many thanks to Dr. Ross Graham of DREA for providing the FORTRAN code for calculating Bretschneider ocean wave spectrum components and for several discussions regarding sea state simulation. Thanks also to Mr. Jim MacEachern of the Naval Air Development Center, Warminster, Pennsylvania, for providing DREA with several relevant NADC Publications on sonobuoy suspension research.

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APPENDIX: TABLES OF SEA STATE ELEVATION FOR FOUR WAVEFORMS

TABLE A-I:

Sea Surface Elevation for
1.25 m Sea State Waveform

(WMO Sea State 3, see Figure 1)

t [s]	z [m]
0.0	1.0043830E-03
0.5	-0.2021858
1.0	-0.2471464
1.5	-0.1168220
2.0	5.7182550E-02
2.5	0.1277755
3.0	7.4657351E-02
3.5	4.4019185E-03
4.0	1.3643354E-02
4.5	7.1282476E-02
5.0	5.2440349E-02
5.5	-0.1133358
6.0	-0.3441980
6.5	-0.4677157
7.0	-0.3787713
7.5	-0.1312559
8.0	0.1218595
8.5	0.2661704
9.0	0.3001656
9.5	0.2903189
10.0	0.2796174
10.5	0.2527592
11.0	0.1777311
11.5	5.6717604E-02
12.0	-7.4159302E-02
12.5	-0.1822286
13.0	-0.2569290
13.5	-0.2910444
14.0	-0.2632747
14.5	-0.1587504
15.0	-5.9008026E-03
15.5	0.1256600
16.0	0.1799687
16.5	0.1666785
17.0	0.1467915
17.5	0.1605859
18.0	0.1762953
18.5	0.1211431
19.0	-3.3663183E-02
19.5	-0.2247063
20.0	-0.3392529

TABLE A-II:

Sea Surface Elevation for
2.5 m Sea State Waveform

(WMO Sea State 4, see Figure 2)

t [s]	z [m]
0.0	0.1897467
0.5	1.6809326E-02
1.0	-0.4379600
1.5	-0.9499672
2.0	-1.267744
2.5	-1.259495
3.0	-0.9528314
3.5	-0.4723860
4.0	4.7244102E-02
4.5	0.5066378
5.0	0.8416494
5.5	1.017678
6.0	1.040938
6.5	0.9662387
7.0	0.8690546
7.5	0.7876452
8.0	0.6855729
8.5	0.4805622
9.0	0.1268649
9.5	-0.3190971
10.0	-0.7148446
10.5	-0.9249011
11.0	-0.9150128
11.5	-0.7678952
12.0	-0.6096498
12.5	-0.5133817
13.0	-0.4610674
13.5	-0.3860542
14.0	-0.2456238
14.5	-5.4577224E-02
15.0	0.1414223
15.5	0.3128783
16.0	0.4609098
16.5	0.5892916
17.0	0.6737531
17.5	0.6704731
18.0	0.5597737
18.5	0.3778794
19.0	0.1973988
19.5	6.9530345E-02
20.0	-1.6652301E-02

TABLE A-III:

Sea Surface Elevation for
4 m Sea State Waveform

(WMO Sea State 5, see Figure 3)

t [s]	z [m]
0.0	-0.6281233
0.5	-1.405133
1.0	-2.003186
1.5	-2.255195
2.0	-2.112317
2.5	-1.650483
3.0	-1.015226
3.5	-0.3462285
4.0	0.2700941
4.5	0.7977871
5.0	1.214613
5.5	1.480290
6.0	1.540015
6.5	1.361879
7.0	0.9769419
7.5	0.4858861
8.0	2.0147026E-02
8.5	-0.3197643
9.0	-0.5070183
9.5	-0.5825239
10.0	-0.6055542
10.5	-0.5986286
11.0	-0.5295085
11.5	-0.3458775
12.0	-3.7616082E-02
12.5	0.3224839
13.0	0.5970181
13.5	0.6576331
14.0	0.4683646
14.5	0.1207688
15.0	-0.2085713
15.5	-0.3562688
16.0	-0.2651803
16.5	-1.2486114E-02
17.0	0.2450572
17.5	0.3715939
18.0	0.3310867
18.5	0.1931235
19.0	6.5305248E-02
19.5	3.3144355E-03
20.0	-3.4827083E-02

TABLE A-IV:

Sea Surface Elevation for
6 m Sea State Waveform

(WMO Sea State 6, see Figure 4)

t [s]	z [m]
0.0	9.0192519E-02
0.5	-0.2550332
1.0	-0.5398811
1.5	-0.6905800
2.0	-0.6925619
2.5	-0.5863372
3.0	-0.4375289
3.5	-0.2993416
4.0	-0.1884615
4.5	-8.6299971E-02
5.0	3.7913531E-02
5.5	0.1984822
6.0	0.3780967
6.5	0.5364289
7.0	0.6373613
7.5	0.6770691
8.0	0.6928738
8.5	0.7430924
9.0	0.8655944
9.5	1.037751
10.0	1.163651
10.5	1.102867
11.0	0.7335442
11.5	2.2355154E-02
12.0	-0.9340569
12.5	-1.921450
13.0	-2.675353
13.5	-2.977661
14.0	-2.740737
14.5	-2.040820
15.0	-1.084906
15.5	-0.1258527
16.0	0.6354703
16.5	1.116718
17.0	1.360946
17.5	1.484997
18.0	1.598973
18.5	1.739471
19.0	1.850662
19.5	1.821532
20.0	1.558010

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