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# THE DETERMINATION OF SPOT VALUES OF VERTICAL VELOCITY AND PRECIPITATION RATE

BY

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# THE DETERMINATION OF SPOT VALUES OF VERTICAL VELOCITY AND PRECIPITATION RATE

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ABSTRACT

A graphical technique is presented for the rapid computation of spot values of vertical velocity and precipitation rate from fields of vorticity advection and 1000-500-mb thickness advection, taking into account initial unsaturation, topography, latent heat effects, and height of cloud base. The procedure can be applied on both current and predicted charts and the effect of instability is incorporated by means of two regression equations.

# LA DÉTERMINATION DES VALEURS INSTANTANÉES DE LA VITESSE VERTICALE ET DU TAUX DE PRÉCIPITATION

par

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RÉSUMÉ

Les auteurs présentent une méthode graphique permettant de calculer rapidement les valeurs instantanées de la vitesse verticale et du taux de précipitation à partir de champs d'advection de tourbillon et d'advection d'épaisseur de 1000-500 mb, en tenant compte de l'insaturation initiale, de la topographie, des effets de la chaleur latente et de la hauteur de la base des nuages. Cette technique peut être appliquée tant aux cartes courantes qu'aux cartes prévues et l'effet d'instabilité est ajouté au moyen de deux équations de régression.

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## 1. INTRODUCTION

The operational method of Q.P.F. described by one of the authors, Harley (1963a), permits the preparation of quantitative regional forecasts of precipitation from a few basic charts. The following graphical technique for obtaining spot check Q.P.F. determinations from both current and prognostic charts has been developed based on an unpublished paper by Rutherford (1964). This method enables the large scale vertical velocity ($\omega_6$) to be computed graphically from advection scale values of vorticity and thickness advection ($A_{\bar{Z}-Z}$ and $A_h$). The six-hour precipitation rate is obtained on the same graph from the $\omega_6$ values previously determined (or from the total vertical velocity, $\omega_{E_6} + \omega_m + \omega_{H_6}$) and the precipitable water values. This combined graph has been constructed for four different localities in Canada: (1) Edmonton, (2) Ottawa and Montreal, (3) Toronto, and (4) Vancouver. Each graph is suitable for application to localities within one degree latitude north or south of each of these centres, without appreciable loss of accuracy. For example: the Edmonton graph is applicable at Goose Bay; the Ottawa and Montreal graph can be used at Halifax and Moncton; the Toronto graph is applicable at London and Trenton, and the Vancouver graph can be used at Winnipeg and Seven Islands.

Graphs have also been constructed to take the following effects into account: initial unsaturation, topography and latent heat. These graphs are applicable in all regions.

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NOTE: The work done by Mr. Dragert, an undergraduate of the University of Toronto, in connection with this paper was carried out during the summer of 1964 while he was employed as a student assistant in the Research and Training Division at Meteorological Headquarters, Toronto.

Instability effects can be incorporated into the computation through the application of two regression equations. The use of these equations is discussed in section 8.

## 2. COMPUTATION OF LARGE SCALE VERTICAL VELOCITY $\omega_6$ AND PRECIPITATION RATE P

The large scale vertical velocity at the 600-mb level is given by Penner's equation (1964):

$$-\omega_6 = \frac{100}{f^2}\,(0.13\,A_{\bar{Z}-Z} + 0.12\,A_h), \qquad (1)$$

where $\omega_6$ = large scale vertical velocity at 600 mb, in $10^{-3}$ mb/sec,

$f$ = $2\Omega \sin\phi$, the Coriolis parameter, in $10^{-5}$ sec$^{-1}$,

$A_{\bar{Z}-Z}$ = advection of the 500-mb space mean ($\bar{Z}$-Z) vorticity $-\vec{V}_g \cdot \nabla(\bar{Z}-Z)$, in m/3 hr,

$A_h$ = advection of the 1000-500-mb thickness $-\vec{V}_g \cdot \nabla h$, in m/3 hr.

For application to absolute vorticity charts, equation (1) becomes:

$$-\omega_6 = \frac{100}{f^2}\left(\frac{0.13}{2.65}\,A_{\zeta_a} + 0.12\,A_h\right), \qquad (2)$$

where $A_{\zeta_a}$ is the advection of the absolute vorticity $-\vec{V}_g \cdot \nabla \zeta_a$ in $10^{-5}$ sec$^{-1}$ units and the factor 1/2.65 converts $A_{\zeta_a}$ to $A_{\bar{Z}-Z}$.

The map scale factor is required when $\omega_6$ is obtained from the values of $A_{\zeta_a}$ and $A_h$ using an advection scale.[6] This factor S is given on the Ferguson advection scale (Ferguson 1963), but it should not be used when computing $\omega_6$ from the graphs given in this paper (Figs. 1-4), since the factor S has been incorporated in the technique by introducing the map factor term into equation (2) thus:

$$-\omega_6 = \frac{100}{f^2} \cdot \frac{1.949}{(1+\sin\phi)^2\,\sin\phi}\left(\frac{0.13}{2.65}\,A_{\zeta_a} + 0.12\,A_h\right), \qquad (3)$$

where $\phi$ is the latitude of the locality for which the graph is constructed.

A graph of $\omega_6$ has been constructed from equation (3) in terms of the $\bar{Z}$-Z scale values of absolute vorticity advection $A_{\zeta_a}$ and thickness advection $A_h$, in units of m/3 hr, for each of the localities: Edmonton, Ottawa and Montreal, Toronto and Vancouver, (Figs. 1-4). As indicated in section 1, the graphs can be used at other localities within one degree latitude north or south of these centres.

The values of the coefficients of $A_{\zeta_a}$ and $A_h$ in equation (3) for the different localities are given in Table 1.

TABLE 1

| LOCALITY | LATITUDE | $\omega_6$ EQUATION |
|---|---|---|
| Edmonton | 53° 34' N | $0.027\ A_{\zeta_a} + 0.065\ A_h$ |
| Ottawa and Montreal | 45° 28' N | $0.042\ A_{\zeta_a} + 0.103\ A_h$ |
| Toronto | 43° 41' N | $0.048\ A_{\zeta_a} + 0.117\ A_h$ |
| Vancouver | 49° 11' N | $0.034\ A_{\zeta_a} + 0.082\ A_h$ |

Basic $\omega_6$ equations for localities at the indicated latitudes.

In Figs. 1 - 4, the advection scale values of $A_{\bar{Z}-Z}$ are given along the left-hand ordinates and those of $A_h$ along the abscissae, at 50 m/3 hr intervals. The corresponding vertical velocities $\omega_6$ are given by the slanting straight lines in the graph at intervals of $2\ 10^{-3}$ mb/sec. Values for the precipitable water content of the 1000-500-mb layer are given along the right hand ordinate of each graph at intervals of 0.2 in. These have been combined with the $\omega_6$ values to give the precipitation rate P in in./6 hr, using the table of precipitation rates given in (2) based on the Godson precipitation rate equation. The values of P are given by the curved dashed lines at 0.1 or 0.2 in./6 hr intervals.

In order to use Figs. 1 - 4 when the advection of absolute vorticity and 1000-500-mb thickness are of opposite sign, the scale values of absolute vorticity advection ($A_{\zeta_a}$) should be changed to relative vorticity advection ($A_{\bar{Z}-Z}$) before entering the graphs. The scale value of $A_{\zeta_a}$ is multiplied by 0.41 ($\simeq$ 0.4) since

$$A_{\overline{Z}-Z} = 0.41\, A_{\zeta_a}. \qquad (2)$$

The resultant value of $A_{\overline{Z}-Z}$ is then added to that of $A_h$ to obtain the sum $A_{\overline{Z}-Z} + A_h$. The required value of $\omega_6$ is determined by entering the abscissa of the appropriate graph at the value of $A_{\overline{Z}-Z} + A_h$ thus obtained. For example, if the scale value of the absolute vorticity advection ($A_{\zeta_a}$) at Toronto is -150 m/3 hr and $A_h$ = 250 m/3 hr, then $A_{\overline{Z}-Z}$ = 0.41 X -150 = -62 m/3 hr and $A_{\overline{Z}-Z} + A_h$ = 250 -62 = 188 m/3 hr. The abscissa of the graph for Toronto, (Fig. 3) is then entered at the value: 188 m/3 hr, which gives $\omega_6$ = -23 $10^{-3}$ mb/sec.

When $A_h$ is negative and $A_{\zeta_a}$ is positive the procedure is the same as that described above. For example, with $A_h$ = -100 m/3 hr and $A_{\zeta_a}$ = 300 m/hr, $A_{\overline{Z}-Z}$ = 123 m/3 hr and, hence, $A_{\overline{Z}-Z} + A_h$ = 23 m/3 hr. Enter the abscissa in Fig. 3 with the value: 23 m/3 hr and obtain -3 $10^{-3}$ mb/sec for $\omega_6$.

If the negative sign predominates, the procedure will be the same, but the resultant vertical velocity will be downwards. Thus, with $A_{\zeta_a}$ = -450 m/3 hr and $A_h$ = 100 m/3 hr, then $A_{\overline{Z}-Z}$ = -185 m/3 hr, (from Table 1a) and $A_{\overline{Z}-Z} + A_h$ = -85 m/3 hr. Entering the abscissa (Fig. 3) at the value: -85 m/3 hr, gives $\omega_6$ = + 10 $10^{-3}$ mb/sec.

If the scale values obtained for either $A_{\zeta_a}$ or $A_h$ are zero, Figs. 1 - 4 may be entered directly using the unaltered values of $A_{\zeta_a}$ and $A_h$. No adjustment of the original scale value of $A_{\zeta_a}$ is needed in this case.

The values of $A_{\overline{Z}-Z}$ (rounded off to the nearest 5 m/3 hr), corresponding to 0.41 $A_{\zeta_a}$ are given in Table 1a for convenient reference.

TABLE 1a

| $A_{\zeta_a}$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 5 | 5 | 10 | 10 | 10 | 15 | 15 | 20 |
| 50 | 20 | 25 | 25 | 25 | 30 | 30 | 35 | 35 | 35 | 40 |
| 100 | 40 | 45 | 45 | 45 | 50 | 50 | 55 | 55 | 55 | 60 |
| 150 | 60 | 65 | 65 | 70 | 70 | 70 | 75 | 75 | 80 | 80 |
| 200 | 80 | 85 | 85 | 90 | 90 | 90 | 95 | 95 | 100 | 100 |
| 250 | 100 | 105 | 105 | 110 | 110 | 115 | 115 | 115 | 120 | 120 |
| 300 | 125 | 125 | 125 | 130 | 130 | 135 | 135 | 135 | 140 | 140 |
| 350 | 145 | 145 | 150 | 150 | 150 | 155 | 155 | 160 | 160 | 160 |
| 400 | 165 | 165 | 170 | 170 | 170 | 175 | 175 | 180 | 180 | 180 |
| 450 | 185 | 185 | 190 | 190 | 195 | 195 | 195 | 200 | 200 | 205 |
| 500 | 205 | 205 | 210 | 210 | 215 | 215 | 215 | 220 | 220 | 225 |

Values of $A_{\bar{Z}-Z}$ as a function of 0.41 $A_{\zeta_a}$ in m/3 hr, (to the nearest 5 m/3 hr).

Figs. 1 - 4 can be used to compute the magnitude of subsiding motion by reversing the sign of the advection values on the graphs. Thus, if Fig. 3 is entered for values of $A_h$ and $A_{\bar{Z}-Z}$ = -100 and -150 m/3 hr, respectively, the value obtained for $\omega_6$ is + 19 $10^{-3}$ mb/sec.

## 3. ADJUSTMENT OF $\omega_6$ FOR INITIAL UNSATURATION

Adjustment of the large scale vertical velocity $\omega_6$ for initial unsaturation can be made from the equations:

$$\omega_{E6} = \omega_6 + \omega_s, \qquad (4)$$

$$\omega_s = 0.5\,(T-Td)_7, \text{ for } (T-Td)_7 > 2^\circ\,C, \qquad (5)$$

$$\omega_s = 0, \qquad \text{for } (T-Td)_7 \leq 2^\circ\,C, \qquad (6)$$

where $\omega_{E6}$ is the effective vertical velocity at 600 mb, in $10^{-3}$ mb/sec;
$\omega_s$ = the vertical velocity equivalent of the 700-mb dew point spread, in $10^{-3}$ mb/sec;
and $(T-Td)_7$ = the 700-mb dew point spread, in °C, see reference (2).

Equation (4) shows that, in regions of ascending motion, where $\omega_6$ is negative, the value of $\omega_6$ is always reduced by $\omega_s$ (which is always positive), except when the air is initially saturated (i.e., when $(T-Td) \leq 2°C$). In the latter case $\omega_6 = \omega_{E_6}$.

Fig. 5 gives the $\omega_{E_6}$ values as slanting lines at intervals of $2\ 10^{-3}$ mb/sec, in terms of the 700-mb dew point spread $(T-Td)_7$ and the large scale vertical velocity $\omega_6$.

The prognostic 700-mb dew point spread may be obtained by making the following arbitrary assumptions:

(a) $(T-Td)_7 = 0$ when $-\omega_6 \geq 3\ 10^{-3}$ mb/sec,

(b) $(T-Td)_7 = 4$ when $-\omega_6 = 1\ 10^{-3}$ mb/sec.

In relation (a) it is assumed that the air has been ascending at $3\ 10^{-3}$ mb/sec (or more) for a 12-hour period and that the resultant ascent of 130 mb has been sufficient to produce saturation. An ascent of this magnitude to 700 mb will produce saturation in air that had originally only 50 per cent humidity.

In relation (b) it is assumed that the vertical motion of $1\ 10^{-3}$ mb/sec in 12 hr ($\equiv 43$ mb) has been sufficient to reduce the dew point spread at 700 mb to 4°C but not to zero. In this case there is no effective vertical velocity $\omega_{E_6}$, since $\omega_s$ is then $+2\ 10^{-3}$ mb/sec (equation 5), and $\omega_{E_6} = -1 + 2 = +1\ 10^{-3}$ mb/sec (equation 4).

Relations (a) and (b) thus determine the following connection between $\omega_6$ and $\omega_{E_6}$ (in $10^{-3}$ mb/sec):

$$-\omega_{E_6} = -\omega_6 \text{ when } -\omega_6 \geq 3,$$

$$-\omega_{E_6} = 1 \quad \text{when } -\omega_6 = 2,$$

$$-\omega_{E_6} = 0 \quad \text{when } -\omega_6 \leq 1.5.$$

This method has been found to work quite well in practice.

A procedure for forecasting the dew point spread has been described by Lewis (1957). In this method, the total ascent undergone by the air during the period is obtained by first computing the vertical velocity over six or twelve hour trajectories upstream from the location concerned on the appropriate 700-mb prognostic charts, (the original paper should be consulted for details). The resultant dew point change can then be obtained easily from a tephigram.

## 4. COMPUTATION OF THE OROGRAPHIC VERTICAL VELOCITY $\omega_m$

The orographic vertical velocity can be computed from the following equation:

$$-\omega_m = A\,\vec{V} \cdot \nabla H,$$

where $\omega_m$ = orographic vertical velocity, in $10^{-3}$ mb/sec;

$A = g\rho_0 \left(\frac{700}{p_0}\right)^{2.5}$

$\vec{V}$ = geostrophic wind, in knots,

$\nabla H$ = slope of ground.

For a discussion of this equation and the meaning of symbols employed refer to reference (2).

The value of $\omega_m$ is computed using a smoothed topographic map for the determination of $\nabla H$, as shown in Fig. 9 of reference (2), and a 1000-mb or 850-mb contour chart to determine $\vec{V}$. When the average elevation is less than 2500 ft, the 1000-mb contours are used, and when the average elevation is 2500 ft or more, the 850-mb contours are used. In the former case the value of A is: $4.60 \times 10^{-3}$, and in the latter case: $5.87 \times 10^{-3}$, in c.g.s. units, (assuming an average surface temperature of 10°C in both cases).

The value of $\vec{V} \cdot \nabla H$ is computed on the assumption that the topographic effect will be important only when the 1000- or 850-mb geostrophic wind is perpendicular to the topographic contours. In practice, the topographic effect is computed when the angle between the topographic and pressure contours is $\geq 60°$. The slope of the ground $\nabla H$ is expressed as $\frac{1000}{\Delta X}$ where $\Delta X$ is the distance between the 1000-ft topographic contours. If the smoothed topographic contours are at other intervals, e.g., 500 or 1500 ft, the required values of $\Delta X$ may be obtained by a suitable adjustment. Since $\vec{V}$ is assumed to be perpendicular to the topographic contours, the value of $\omega_m$, in $10^{-3}$ mb/sec, at the two levels can be obtained from the following equations:

$$-\omega_m = 4.60 \times \frac{V}{\Delta X} \text{ at the 1000-mb level,} \quad (8)$$

$$-\omega_m = 5.87 \times \frac{V}{\Delta X} \text{ at the 850-mb level,} \quad (9)$$

where V is the geostrophic wind speed, in knots (or miles per hour), and $\Delta X$ = the 1000-ft topographic contour interval, in nautical miles (or miles).

Fig. 6 is a graph of equation (8) giving the value of $\omega_m$ for the 1000-mb level in $10^{-3}$ mb/sec. Geostrophic wind speed normal to the topographic contours is given along the ordinate at intervals of 10 knots (or miles per hour), and the 1000-ft topographic contour spacing is given along the abscissa at intervals of 10 nautical miles (or miles). The corresponding values for $\omega_m$ are given by the radial lines at intervals of 0.5, 1.0, 2.0 or 4.0 $10^{-3}$ mb/sec.

Fig. 7 is a graph for equation (9), similar to Fig. 6, giving $\omega_m$ for the 850-mb level.

When $\omega_m < 0$ and $\omega_6 \geq 0$, the value of $\omega_m + \omega_6$ should be adjusted for initial unsaturation before proceeding to compute the latent heat effect, (provided $-\omega_m > \omega_6$).

## 5. THE COMPUTATION OF THE LATENT HEAT VERTICAL VELOCITY $\omega_{H_6}$

The vertical velocity due to the release of latent heat can be computed from Table 2 which is based on a finite-difference equation given by Pedersen (1963).

TABLE 2

| $T_7$ °C | -30 | -25 | -20 | -15 | -10 | -5 | 0 | 5 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $\omega_{H_6} / \omega_E$ | 0.141 | 0.237 | 0.337 | 0.520 | 0.744 | 0.997 | 1.325 | 1.738 | 2.052 |

The ratio $\omega_{H_6} / \omega_E$ as a function of the 700-mb temperature (°C).

Table 2 gives the values of the ratio $\omega_{H_6} / \omega_E$ at different values of the 700 mb temperature ( $\omega_E$ is now the sum of the vertical velocity values: $\omega_6 + \omega_m + \omega_s$).

Fig. 8 is a graph of the total vertical velocity $\omega_{E_6} + \omega_m + \omega_{H_6}$, in terms of the 700-mb temperature at intervals of 5°C, and the effective vertical velocity $\omega_{E_6} + \omega_m$ at intervals of 2 $10^{-3}$ mb/sec. The total vertical velocity is given by the curved lines at intervals of $2\ 10^{-3}$ mb/sec up to $52\ 10^{-3}$ mb/sec and $4\ 10^{-3}$ mb/sec intervals above $52\ 10^{-3}$ mb/sec. In constructing the graph, the values of $\omega_{H6}$ obtained using table 2 have been added to the corresponding values of $\omega_{E6} + \omega_m$ to give the total vertical velocity. In this way two steps are combined into one.

At 700-mb temperatures below -20°C, the relation between temperature and vertical velocity seems to become linear rather than parabolic. Some of the assumptions on which the derivation of the basic equation is founded also become somewhat less valid below -20°C. These effects account for the slight break in the curves occurring at -20°C. However, most of the important moisture bearing air masses have 700-mb temperatures above -20°C.

## 6. DETERMINATION OF THE PRECIPITABLE WATER $W_p$

The precipitable water content of the 1000-500-mb thickness layer can be obtained from the current or prognostic thickness lines of the layer (3). The precipitable water values corresponding to given thickness values are shown in Table 3.

TABLE 3

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $W_p$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| h | 509 | 529 | 542 | 552 | 559 | 566 | 571 | 576 | 580 | 585 |

Precipitable water content of the 1000-500-mb layer $W_p$ (in.), as a function of layer thickness h (in tens of gpm).

The values given in Table 3 are 70% of the saturated precipitable water content of the 1000-500-mb layer. This percentage appears to be the most realistic value to use for Q.P.F. computations.

The precipitable water values of Table 3 can be corrected for elevation by subtracting the precipitable water content of a surface layer equivalent in thickness to the height of the station. For example, suppose that a station has an elevation of 2000 ft and that the thickness of the 1000-500-mb layer at that point is 5520 m. From Table 3, the value of $W_p$ for the entire layer is 0.8 in. But the saturated $W_p$ value for the lowest 2000 ft along the same thickness line is 0.26 in. (see Table 3 in reference 3). Hence, the adjusted value of $W_p$ for the station is 0.8 - 0.26 = 0.54 in. A similar procedure is followed in constructing Fig. 9, except that, for elevations above 2500 ft, the $W_p$ content of the whole layer is reduced by less than 100% of the saturated $W_p$ content of the surface layer. For station elevations equal to or less than 2500 ft, 100% of the saturated $W_p$ content of the equivalent surface layer is subtracted from the unadjusted value of $W_p$. When the station elevation is 4500 ft or higher, 70% of the saturated $W_p$ content of the equivalent surface layer is subtracted. When the station elevation is between 2500 ft and 4500 ft, the saturated $W_p$ of the equivalent surface layer is assumed to change linearly from 100% at 2500 ft to 70% at 4500 ft. This assumption is reflected in Fig. 9 by the change in slope of the curve between 2500 ft and 4500 ft. The precipitable water values obtained in this way have been found to give computed precipitation rates in good agreement with observations.

In Fig. 9, the effective precipitable water content of the atmospheric layer below 500 mb is given by the slanting lines, which are drawn at intervals of 0.1 or 0.2 in., in terms of the 1000-500-mb thickness at intervals of 60 m, and the topographic elevation at intervals of 500 ft.

## 7. DETERMINATION OF THE PRECIPITATION RATE P

The values of $W_p$ (obtained from Fig. 9) are used in conjunction with the total vertical velocity (obtained from Fig. 8) to obtain the precipitation rate P in in./6 hr from Figs. 1, 2, 3, or 4. The slanted isolines of $\omega_6$ can also be used as isolines of $\omega_{E6} + \omega_m + \omega_{H6}$.

The precipitation rate obtained from Figs. 1, 2, 3, or 4 can be adjusted for height of cloud base by using Fig. 18 or Table 11 in reference 2.

Since the cloud base is generally below 1500 ft in most major precipitation occurrences, this effect can generally be neglected.

## 8. EFFECT OF INSTABILITY ON THE PRECIPITATION RATE

In any region where continuous precipitation is occurring, part of the precipitation can be ascribed to the release of potential instability in generally ascending air. This effect cannot at present be computed easily by any existing operational procedure. A first approximation to the magnitude of the precipitation resulting from this process may be obtained, however, by a statistical procedure based on the results obtained during a test of the operational Q.P.F. method carried out at Toronto early in 1963. A statistical analysis of the results obtained during that test produced two regression equations; the first for use when the instability released was not intense, i.e., when no thunderstorm activity was reported in the area where the precipitation was occurring, viz:

$$y = 1.06x + 0.06 \pm 0.06, \qquad (10)$$

where y is the observed precipitation rate, and x is the computed precipitation rate, in in./6 hr. The standard error of the estimate in this case was $\pm$ 0.06. The second equation can be used when intense instability is present, i.e., when thunderstorms or cumulonimbus clouds are reported in the area. This equation is:

$$y = 1.86x + 0.09 \pm 0.07, \qquad (11)$$

where y and x have the same meaning as in equation (10). Equation (10) is based on 32 occurrences, whereas equation (11) is based on only 4. The use of an equation based on such a small number of cases is not usually justified, but results obtained by its use have been reasonably good, so that it is considered suitable for application on an experimental basis.

In using these equations, the computed precipitation rate is substituted for x in equation (10) if thunderstorm activity has not occurred or is not expected, and in equation (11) if thunderstorm activity has occurred or is expected.

Equations (10) and (11) should be used only when the computed value of the precipitation rate is greater than 0.07 and 0.1 in., respectively, since these two equations are based on data from which computed precipitation rates equal to or below these values were excluded.

## 9. EXAMPLE OF A Q.P.F. COMPUTATION

A computation was carried out for Edmonton, Alberta, (XD) and Revelstoke, B. C., (RV) for the synoptic situation of March 17th, 1964 at 0000 GMT. The charts on which the computations were based are shown in Figs. 10 to 15. The relevant portion of the topographic and surface charts are shown in Figs. 10 and 11 respectively. The corresponding absolute vorticity, 1000-500-mb thickness, and 850-mb contour charts are shown in Figs. 12 to 14, respectively, while Fig. 15 gives the observed 6-hr precipitation for the periods: 1800 GMT March 16th to 0000 GMT March 17th and 0000-0600 GMT March 17th, 1964.

### Case 1 - Edmonton

### Computation of $\omega_6$

From Figs. 12 and 13, using the Ferguson advection scale, a value of 80 m/3 hr was obtained for the vorticity advection $A_{\overline{Z}-Z}$, and a value of 45 m/3 hr for the 1000-500-mb thickness advection $A_h$. Entering

Fig. 1 with these values of $A_{Z-Z}$ (left-hand ordinate) and $A_h$ (abscissa), gives a vertical velocity $\omega_6$, of -5.0 $10^{-3}$ mb/sec.

Computation of $\omega_{E_6}$

The 700-mb dew point spread (obtained from the 700-mb chart, which is not shown) is 1°C. Entering Fig. 5 with the value $(T-Td)_7 = 1°C$ for the ordinate and $\omega_6 = -5\ 10^{-3}$ mb/sec for the abscissa gives the effective vertical velocity $\omega_{E_6} = \omega_6 = -5\ 10^{-3}$ mb/sec. Alternatively, using equations (5) and (6) instead of Fig. 5, we see that, since $(T-Td)_7 < 2°C$, $\omega_s = 0$, and hence, by equation (4), $\omega_6 = \omega_{E_6}$.

Computation of $\omega_m$ and $\omega_{E_6} + \omega_m$

To determine the orographic vertical velocity $\omega_m$, the altitude of Edmonton is first obtained, i.e., 2219 ft. Since the average elevation of the local area is less than 2500 ft, the 1000-mb geostrophic wind and Fig. 6 are used. However, in this case, the 1000-mb contours are approximately parallel to the topographic contours (see Figs. 10 and 11). Hence, $\omega_m$ at Edmonton is zero and $\omega_{E_6} + \omega_m = -5\ 10^{-3}$ mb/sec.

Computation of $\omega_{H_6}$ and $\omega_{E_6} + \omega_m + \omega_{H_6}$

For the latent heat computation, the value of the 700-mb temperature at Edmonton, i.e., -8°C, and the value just obtained for $\omega_{E_6} + \omega_m$, i.e., -5 $10^{-3}$ mb/sec, are required. Entering Fig. 8 with these values for the ordinate and abscissa respectively, yields a value of -9.0 $10^{-3}$ mb/sec for the total vertical velocity ($\omega_{E_6} + \omega_m + \omega_{H_6}$).

Computation of $W_p$

The value of the precipitable water $W_p$, is obtained from the 1000-500-mb thickness and the elevation at Edmonton. The elevation of Edmonton is 2219 ft and the thickness is 5340 m (Fig. 13). Using these values in Fig. 9 gives an adjusted precipitable water content for Edmonton of 0.25 in.

Computation of P

Returning to Fig. 1 with the value of $-9 \; 10^{-3}$ mb/sec for the total vertical velocity (abscissa), and 0.25 for $W_p$ (right-hand ordinate) a precipitation rate of 0.16 in./6 hr is obtained for the period 2100 GMT March 16 to 0300 GMT March 17, 1964. The interpolated observed maximum precipitation in the area for this period was 0.14 in./6 hr (obtained by taking the mean of the 2 observed maxima in Fig. 15 i.e. $(0.21 + 0.06)/2 = 0.14$ in.). The interpolated position of the observed maximum is seen from Fig. 15 to be approximately 40 nautical miles NE of Edmonton.

Computation of the instability effect

Use can be made of equation (10) in Section 8 to estimate the effect of instability. Substitution of the value 0.16 in./6 hr for x in this equation, and using twice the standard error of the estimate ($\pm 2 \times 0.06$), gives an expected precipitation rate range of 0.11 to 0.35 in./6 hr.

Case 2 - Revelstoke

The procedure followed is the same as for case 1.

Computation of $\omega_6$

The values obtained for $A_{\bar{Z}-Z}$ and $A_h$ (from Figs. 12 and 13) are 50 m/3 hr and zero, respectively. Since the latitude of Revelstoke is almost midway between that of Edmonton and Vancouver, the mean of the $\omega_6$ values given by Figs. 1 and 4 is used as the value of $\omega_6$ at Revelstoke. From Fig. 1, the value of $\omega_6 = -1.3 \; 10^{-3}$ mb/sec and, from Fig. 4, $\omega_6 = -1.7 \; 10^{-3}$ mb/sec. Hence, the value of $\omega_6$ for Revelstoke is $-1.5 \; 10^{-3}$ mb/sec.

Computation of $\omega_{E_6}$

At Revelstoke $(T-Td)_7 = 7°C$; hence $\omega_{E_6} = 0$ (Fig. 5).

Note that, whenever the point arrived at on this graph falls anywhere to the left of the $\omega_{E_6} = 0$ line, $\omega_{E_6}$ should always be taken to be zero and the correction for initial unsaturation should be applied to $\omega_6 + \omega_m$.

Computation of $\omega_m$ and $\omega_{E_6} + \omega_m$

Although the elevation of Revelstoke is 1497 ft, the average elevation in the area is greater than 2500 ft. Therefore, when computing $\omega_m$, use is made of the 850-mb contour chart (Fig. 14). From Figs. 10 and 14, it will be seen that the 850-mb contours at Revelstoke are nearly at right-angles to the smoothed topographic contours. The measured values of X and V (see equation 8), are found to be 26 nautical miles and 35 knots respectively. From Fig. 7, the value of $\omega_m$ is found to be $-8.0\ 10^{-3}$ mb/sec. Since $\omega_{E_6}$ is, in this case, less than zero, the correction for initial unsaturation is applied to the sum of $\omega_6 + \omega_m$ i.e., $(-1.5) + (-8.0) = -9.5\ 10^{-3}$ mb/sec. From Fig. 5, with $(T\text{-}Td)_7 =$ 7°C and $\omega_6 + \omega_m = -9.5\ 10^{-3}$ mb/sec, an effective vertical velocity $\omega_E = -6.0\ 10^{-3}$ mb/sec is obtained.

Computation of $\omega_{H_6}$ and $\omega_{E_6} + \omega_m + \omega_{H_6}$

With $T_7 = -7$°C and $\omega_E = -6.0\ 10^{-3}$ mb/sec ( $\omega_E$ is the same as $\omega_{E_6} + \omega_m$), Fig. 8 gives a value of $-11.0\ 10^{-3}$ mb/sec. for the total vertical velocity, $\omega_{E_6} + \omega_m + \omega_{H_6}$.

Computation of $W_p$

The 1000-500-mb thickness at Revelstoke is 5440m (Fig. 13). With this thickness and Revelstoke's elevation of 1497 ft, Fig. 9 gives an adjusted value for $W_p$ of 0.45 in.

Computation of P

Using Fig. 1 (or Fig. 4), with the total vertical velocity = $-11.0\ 10^{-3}$ mb/sec and $W_p = 0.45$ in., a precipitation rate P = 0.29 in./6 hr is obtained. This value of P compares with a mean observed precipitation at Revelstoke of $(0.29 + 0.14)/2 = 0.22$ in.

Computation of the instability effect

Application of the regression equation for instability gives a computed precipitation range of 0.25 to 0.49 in./6 hr.

## 10. CONCLUSION

A graphical method for obtaining spot values of vertical velocity and precipitation rate has been presented, taking into account initial unsaturation, topography, latent heat, height of cloud base and instability and two examples of its use are given. The procedure can be applied to any locality in Canada and the estimated time required for the complete computation is around 10 minutes.

APPROVED,

J. R. H. Noble,
Director.

REFERENCES

(1) Ferguson, H. L., 1963: A Geostrophic Advection Scale for Polar Stereographic Charts. Canada, Department of Transport, Meteorological Branch. Technical Circular Series. CIR-3857, TEC-473.

(2) Harley, W. S., 1963a: An Operational Method for Quantitative Precipitation Forecasting. Canada, Department of Transport, Meteorological Branch. Technical Circular Series. CIR-3852, TEC-471, and CIR-4040, TEC-520.

(3) Harley, W. S., 1963b: Prognosis of the 700-mb Temperature Field and the 1000-500-mb Precipitable Water Field. Canada, Department of Transport, Meteorological Branch. Technical Circular Series. CIR-3916, TEC-487.

(4) Lewis, W., 1957: Forecasting 700-mb Dewpoint Depression by a 3-Dimensional Trajectory Technique. Mon. Wea. Rev. 85, 297-301.

(5) Pedersen, K., 1963: On Quantitative Precipitation Forecasting with a Quasi-geostrophic Model. Geof. Publik., 25, 1-25.

(6) Penner, C. M., 1963: An Operational Method for the Determination of Vertical Velocities. J. Applied Met., 2, 235-241.

(7) Rutherford, I. D., 1964: A graph for Forecasting Precipitation for Goose Bay by Harley's Technique, (unpublished).

## APPENDIX 1

## SUMMARY OF GRAPHICAL PROCEDURE

Computation of $\omega_6$

1. Obtain scale values of $A_{\zeta_a}$ and $A_h$.
2. Obtain $\omega_6$ from Figs. 1 - 4 according to latitude of station using Table (1a) if necessary.

Computation of $\omega_{E_6}$

3. Obtain $(T\text{-}Td)_7$ from the 700-mb chart or, in the prognostic case, using equation (4) and the procedure given in Section 3.
4. Obtain $\omega_{E_6}$ from Fig. 5 using $\omega_6$ and $(T\text{-}Td)_7$ values from steps 2 and 3.

Computation of $\omega_m$ and $\omega_{E_6} + \omega_m$

5. Obtain $V_g$ from the 1000 or 850-mb contours (when contour angle $\geq 60°$).
6. Obtain spacing of smoothed 1000-ft topographic contours, $\Delta X$.
7. Obtain $\omega_m$ from Figs. 6 or 7 using $V_g$ and $\Delta X$ values from steps 5 and 6.
8. Obtain sum of $\omega_{E_6}$ and $\omega_m$.

8a. If $\omega_6 \geq 0$ add $\omega_6$ and $\omega_m$ values and repeat steps 3 and 4 to get value of $\omega_{E_6} + \omega_m$.

Computation of $\omega_{H_6}$ and $\omega_{E_6} + \omega_m + \omega_{H_6}$

9. Obtain $T_7$ from the 700-mb chart, or from the 1000-500-mb thickness prognostic and Table 1 of Reference 3.
10. Using result of steps 8 or 8a, and 9, obtain the total vertical velocity $\omega_{E_6} + \omega_m + \omega_{H_6}$ using Fig. 8.

## APPENDIX 1 (con't)

### Computation of $W_p$

11. Obtain 1000-500-mb thickness (current or prognostic) and the elevation of the station.

12. Obtain adjusted precipitable water content $W_p$ using result of step 11 and Fig. 9.

### Computation of P

13. Obtain P from values of $\omega$ and $W_p$ obtained in steps 10 and 12 using Figs. 1, 2, 3, or 4, (the precipitation rate graphs in Figs. 1 - 4 are identical).

### Computation of the Instability effect

14. Obtain the computed precipitation range from equation (10) if no thunderstorm activity has occurred or is expected and if $P > 0.08$ in. Use equation (11) if thunderstorm activity has occurred or is expected and if $P > 0.1$ in.

Note: FIGURES 1 TO 4 ARE AVAILABLE IN LARGE OPERATING SIZES (22" x 17") FROM METEOROLOGICAL HEADQUARTERS ON REQUEST.

FIGURES 5 TO 9 ARE ALSO AVAILABLE IN LARGE SIZES (12" x 9") ON REQUEST.

# A GRAPH FOR COMPUTING $\omega_6$ AND P FOR EDMONTON

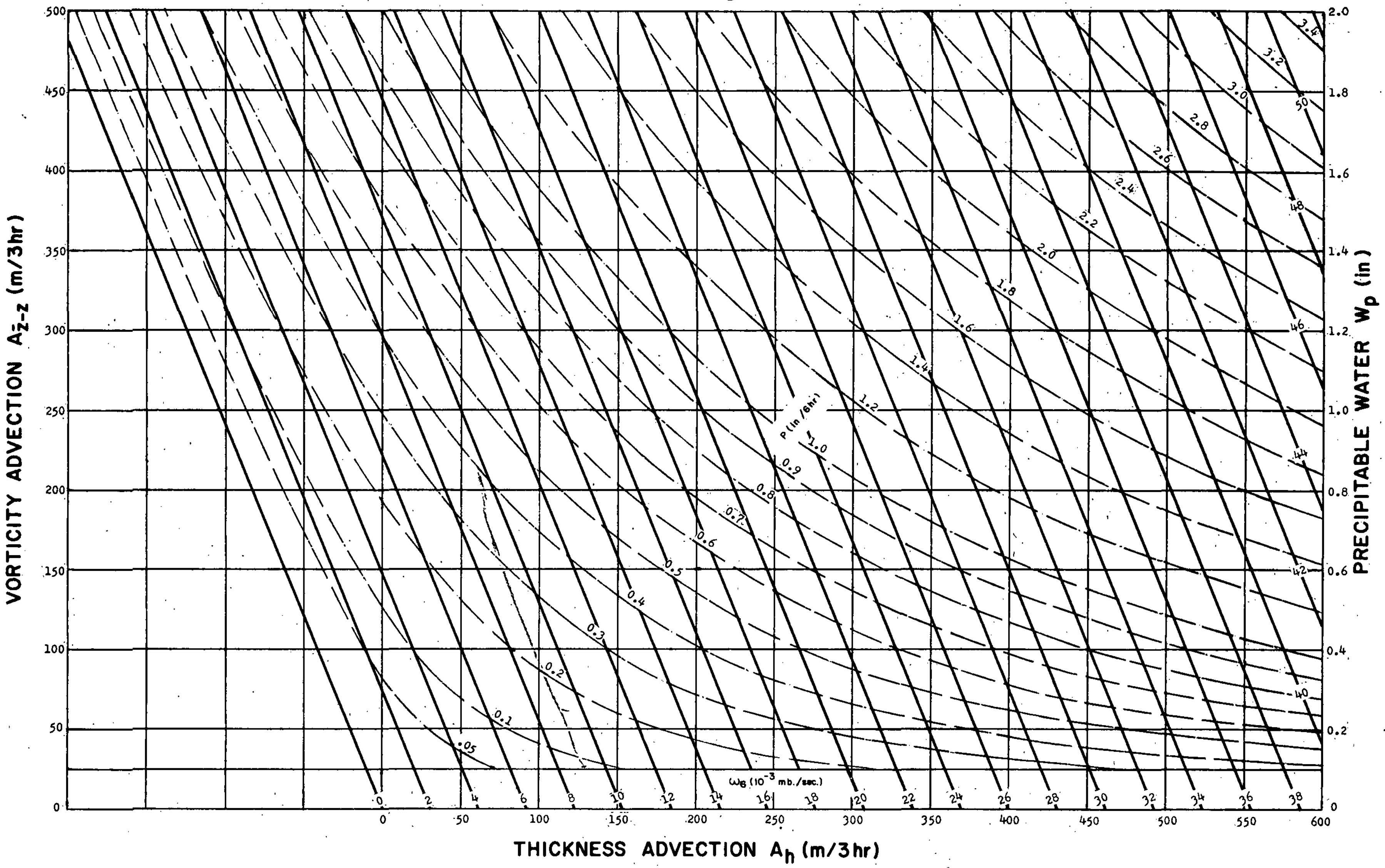



Figure 1

# A GRAPH FOR COMPUTING $\omega_6$ AND P FOR OTTAWA AND MONTREAL

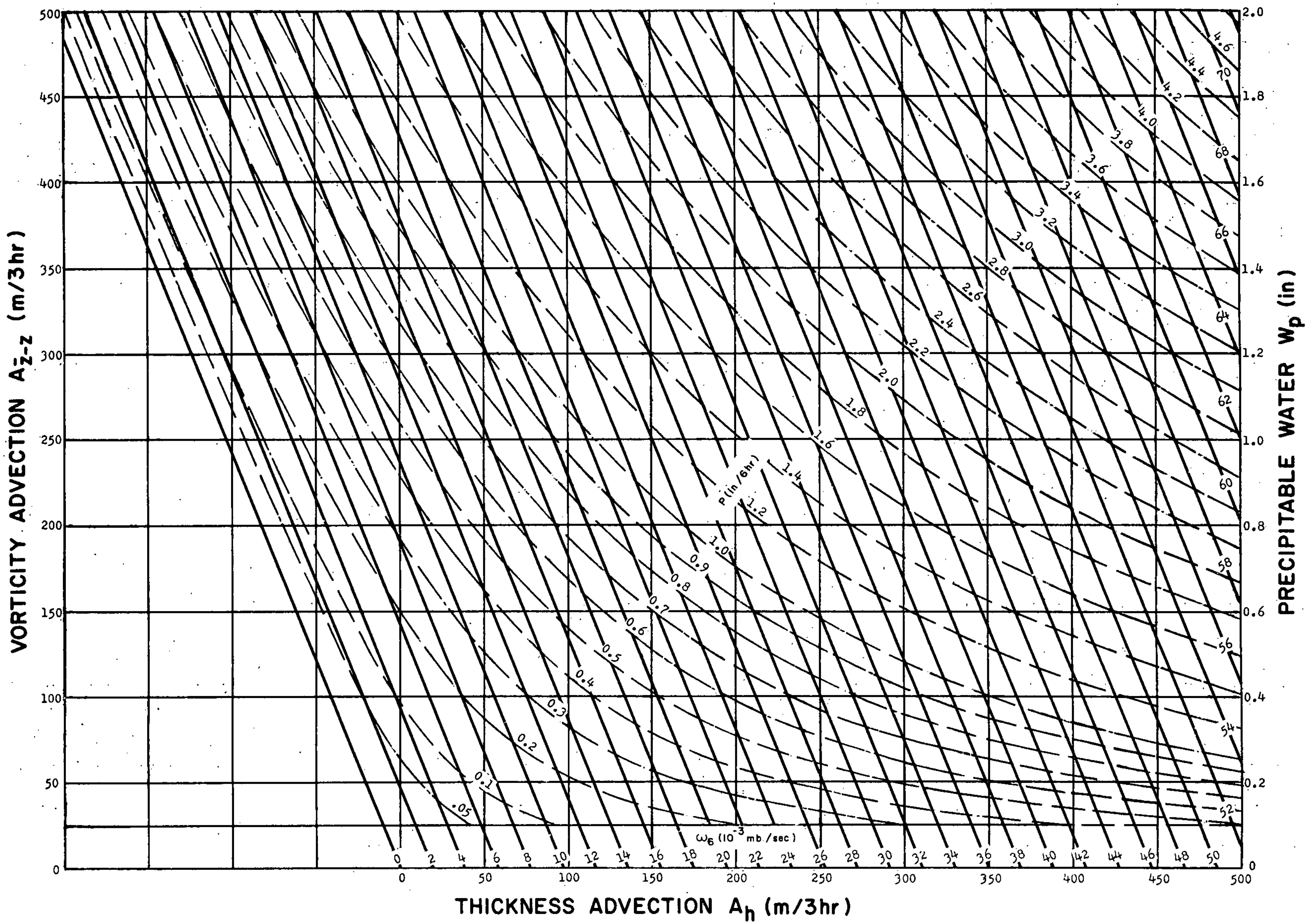



Figure 2

A GRAPH FOR COMPUTING $\omega_6$ AND P FOR TORONTO

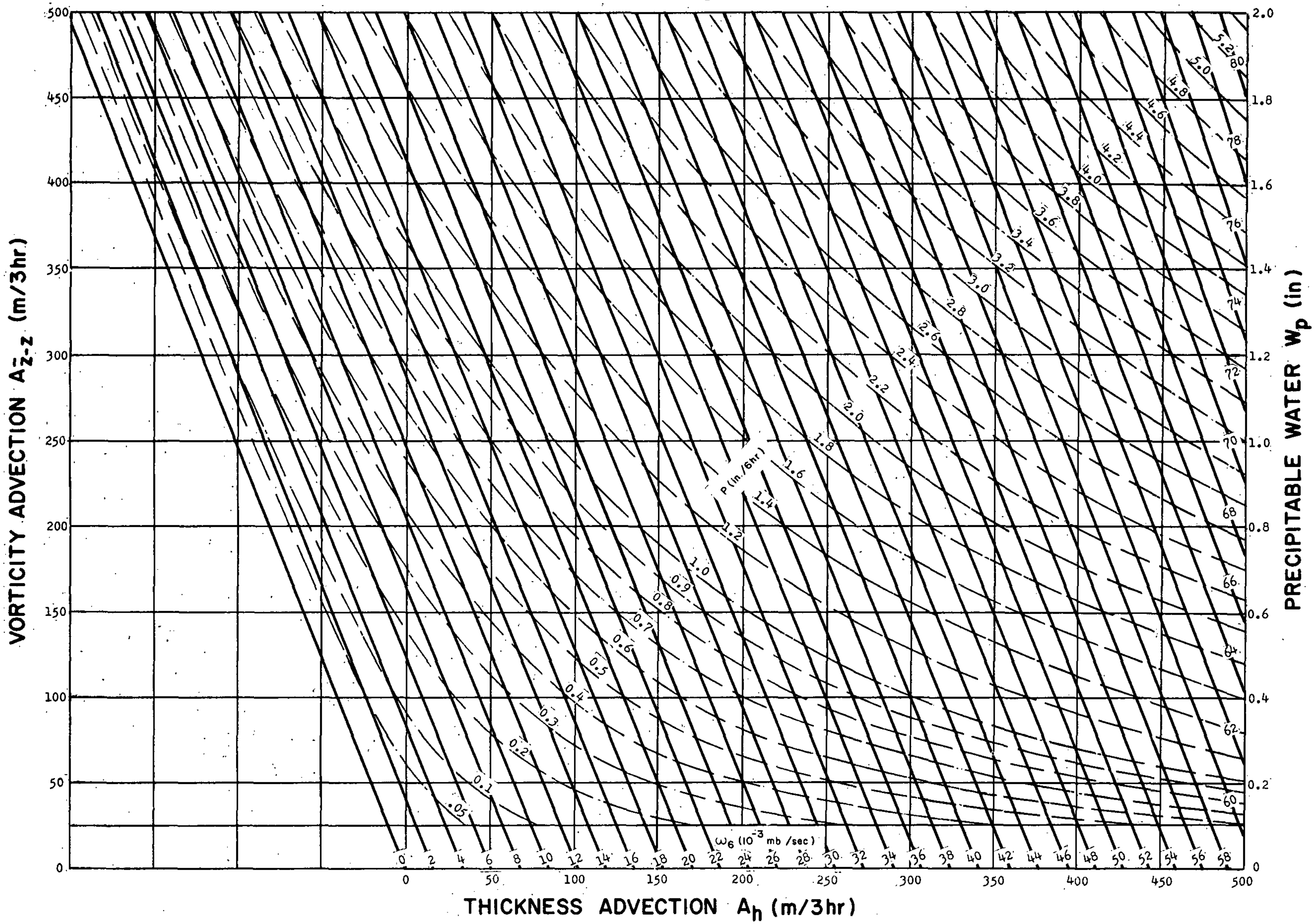



Figure 3

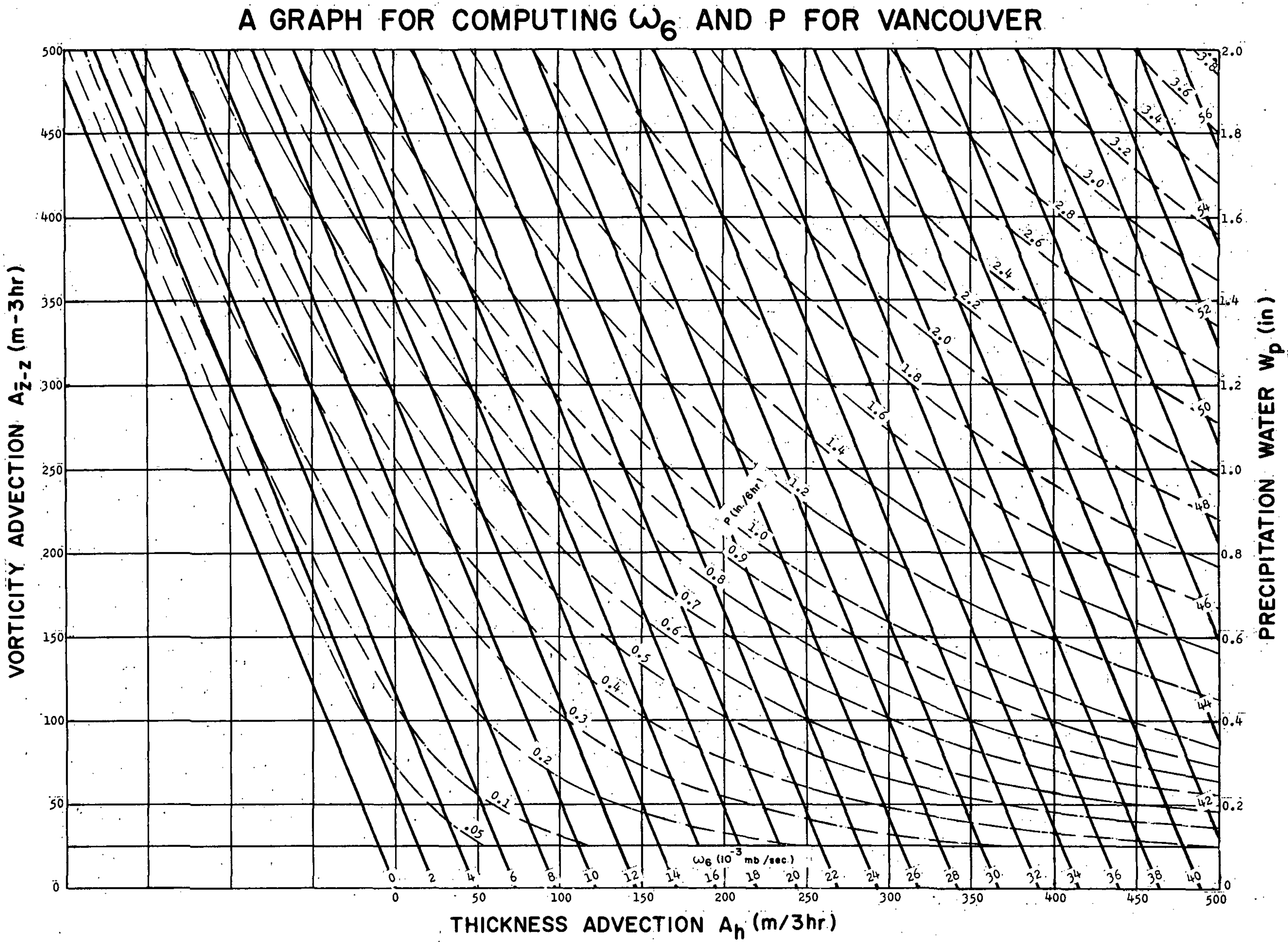

A GRAPH FOR COMPUTING $\omega_6$ AND P FOR VANCOUVER
VORTICITY ADVECTION $A_{z-z}$ (m-3hr)
PRECIPITATION WATER $W_p$ (in)
THICKNESS ADVECTION $A_h$ (m/3hr)
P (in./6hr)
$\omega_6$ ($10^{-3}$ mb/sec.)

Figure 4

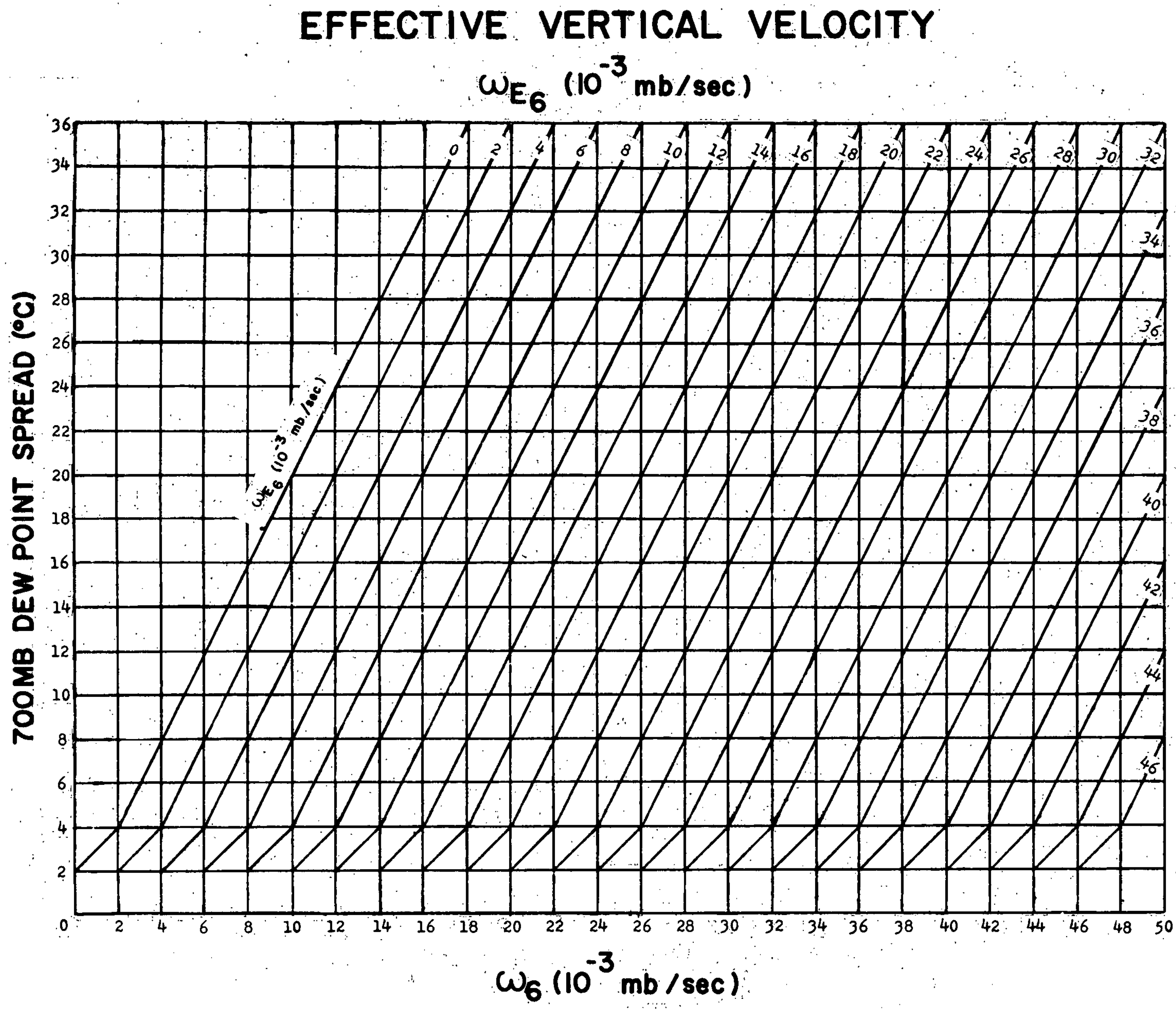

EFFECTIVE VERTICAL VELOCITY
ωE6 (10-3 mb/sec)
700MB DEW POINT SPREAD (°C)
ωE6 (10-3 mb/sec)
ω6 (10-3 mb/sec)

Figure 5

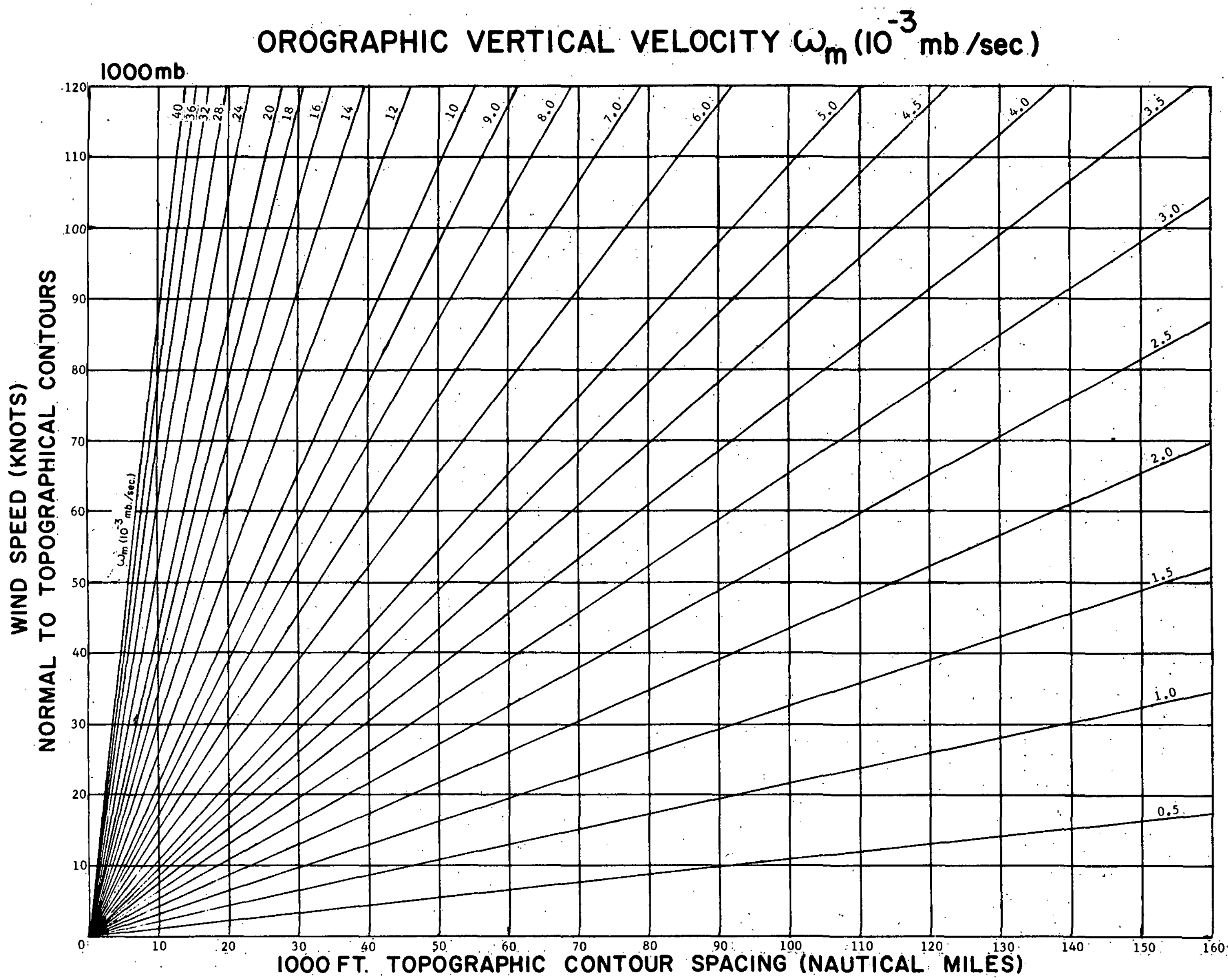
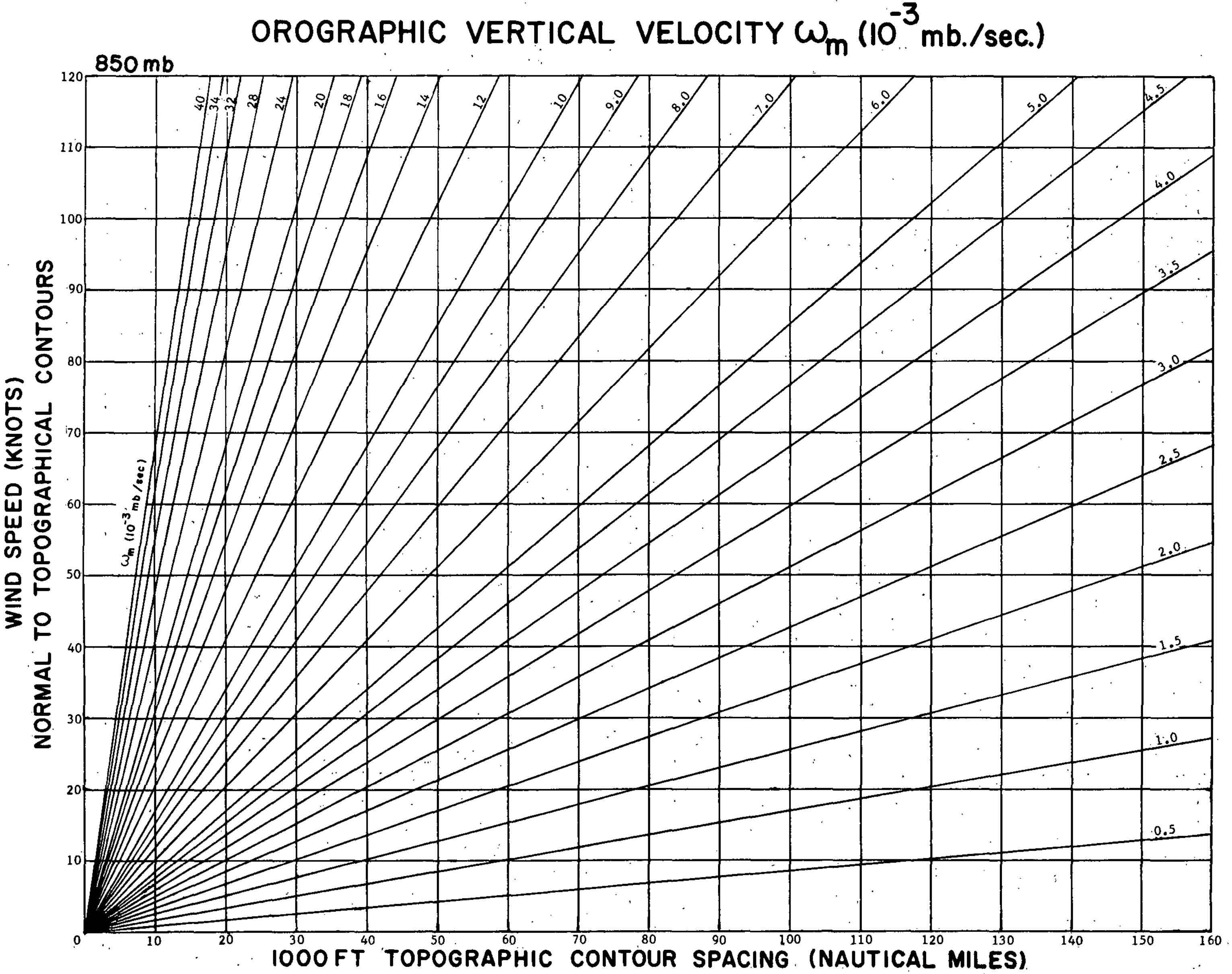

OROGRAPHIC VERTICAL VELOCITY $\omega_m$ ($10^{-3}$ mb/sec)
1000mb
WIND SPEED (KNOTS) NORMAL TO TOPOGRAPHICAL CONTOURS
1000 FT. TOPOGRAPHIC CONTOUR SPACING (NAUTICAL MILES)
$\omega_m$ ($10^{-3}$ mb./sec.)
40 36 32 28 24 20 18 16 14 12 10 9.0 8.0 7.0 6.0 5.0 4.5 4.0 3.5 3.0 2.5 2.0 1.5 1.0 0.5
0 10 20 30 40 50 60 70 80 90 100 110 120
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160

Figure 6



OROGRAPHIC VERTICAL VELOCITY $\omega_m$ ($10^{-3}$ mb./sec.)
850 mb
WIND SPEED (KNOTS) NORMAL TO TOPOGRAPHICAL CONTOURS
1000 FT TOPOGRAPHIC CONTOUR SPACING (NAUTICAL MILES)
$\omega_m$ ($10^{-3}$ mb/sec)
40
34
32
28
24
20
18
16
14
12
10
9.0
8.0
7.0
6.0
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5


Figure 7

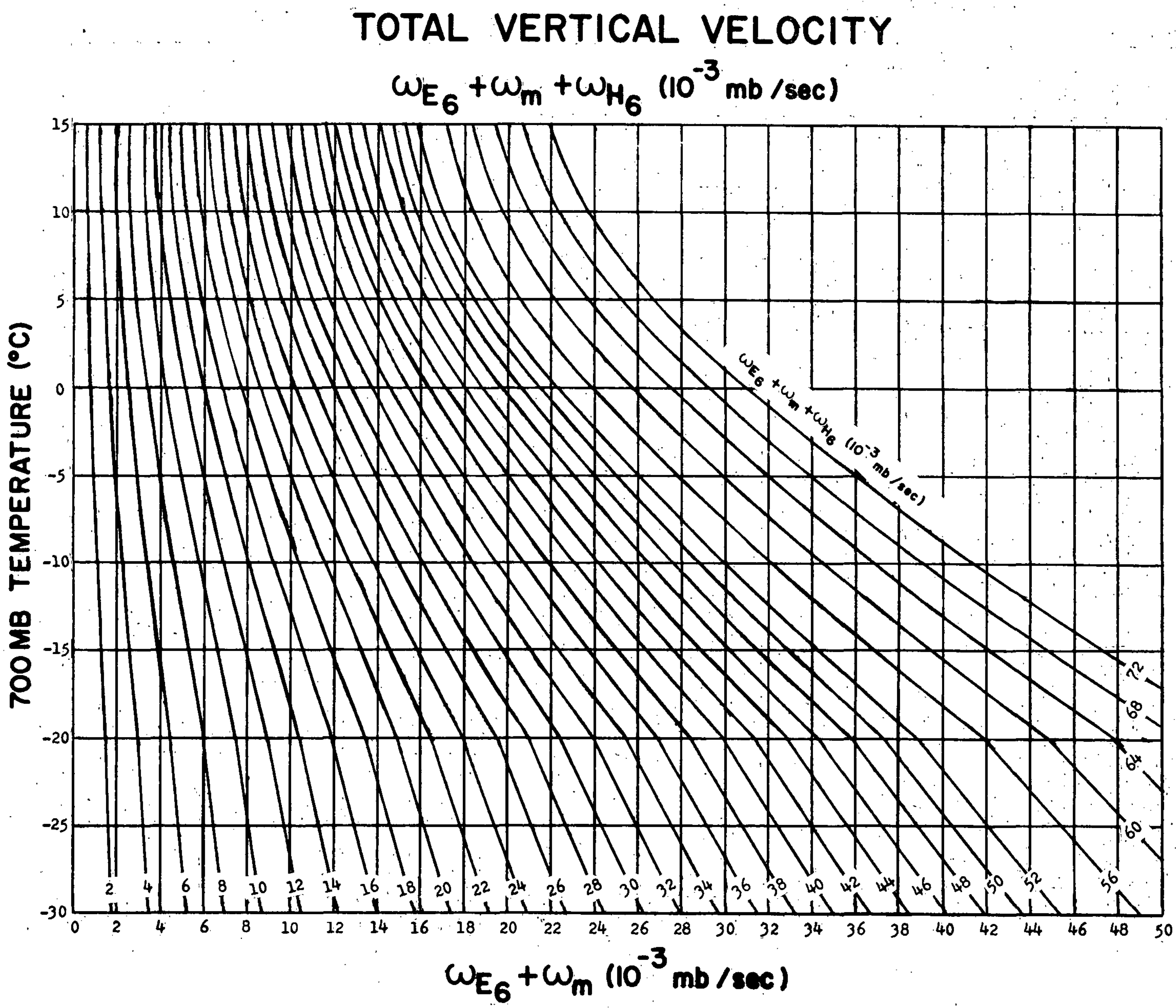

TOTAL VERTICAL VELOCITY
$\omega_{E_6} + \omega_m + \omega_{H_6}$ ($10^{-3}$ mb/sec)
700MB TEMPERATURE (°C)
$\omega_{E_6} + \omega_m$ ($10^{-3}$ mb/sec)
$\omega_{E_6} + \omega_m + \omega_{H_6}$ ($10^{-3}$ mb/sec)
15
10
5
0
-5
-10
-15
-20
-25
-30
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 56 60 64 68 72

Figure 8

# PRECIPITABLE WATER CONTENT ADJUSTED FOR ELEVATION

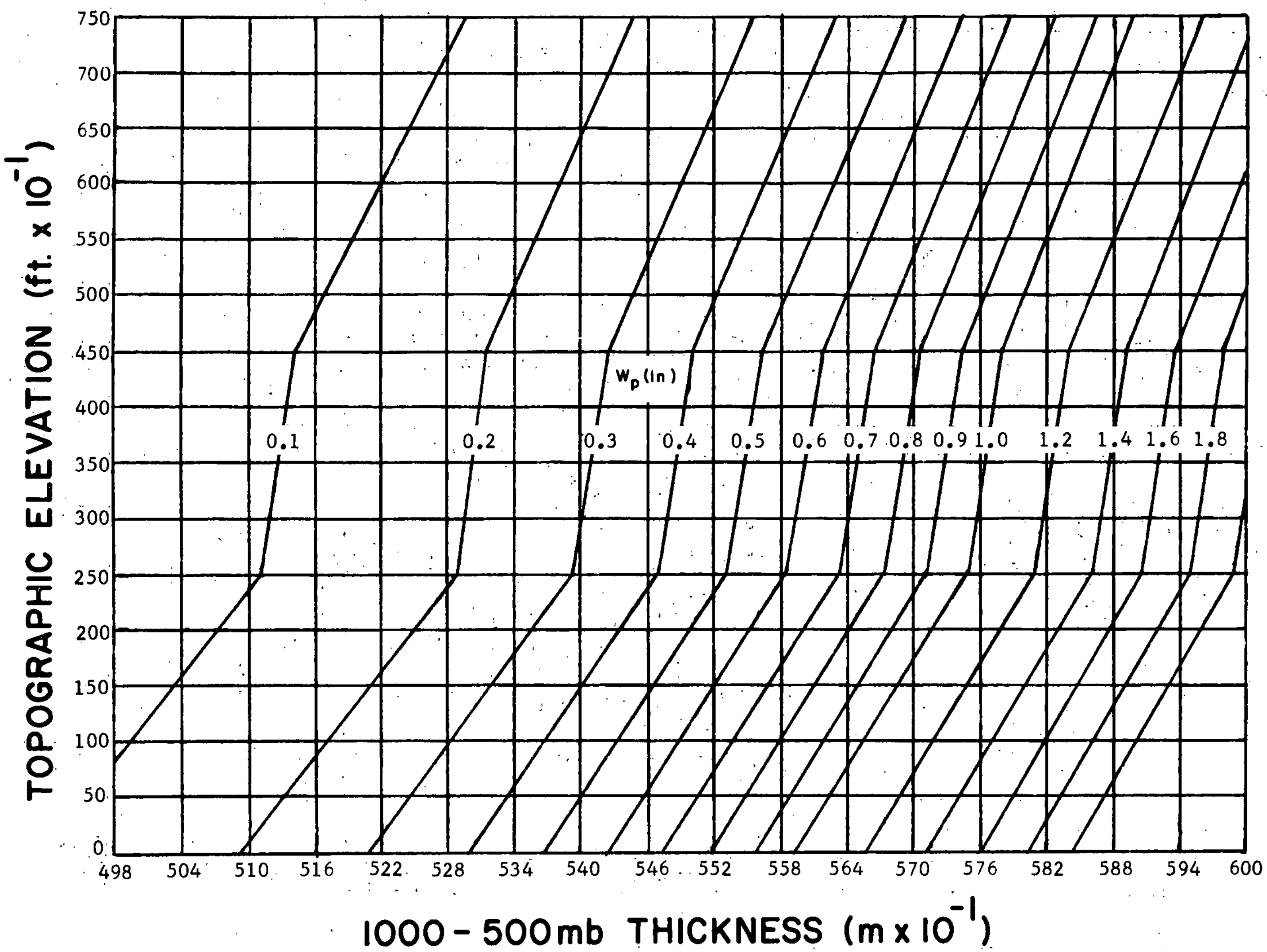



Figure 9

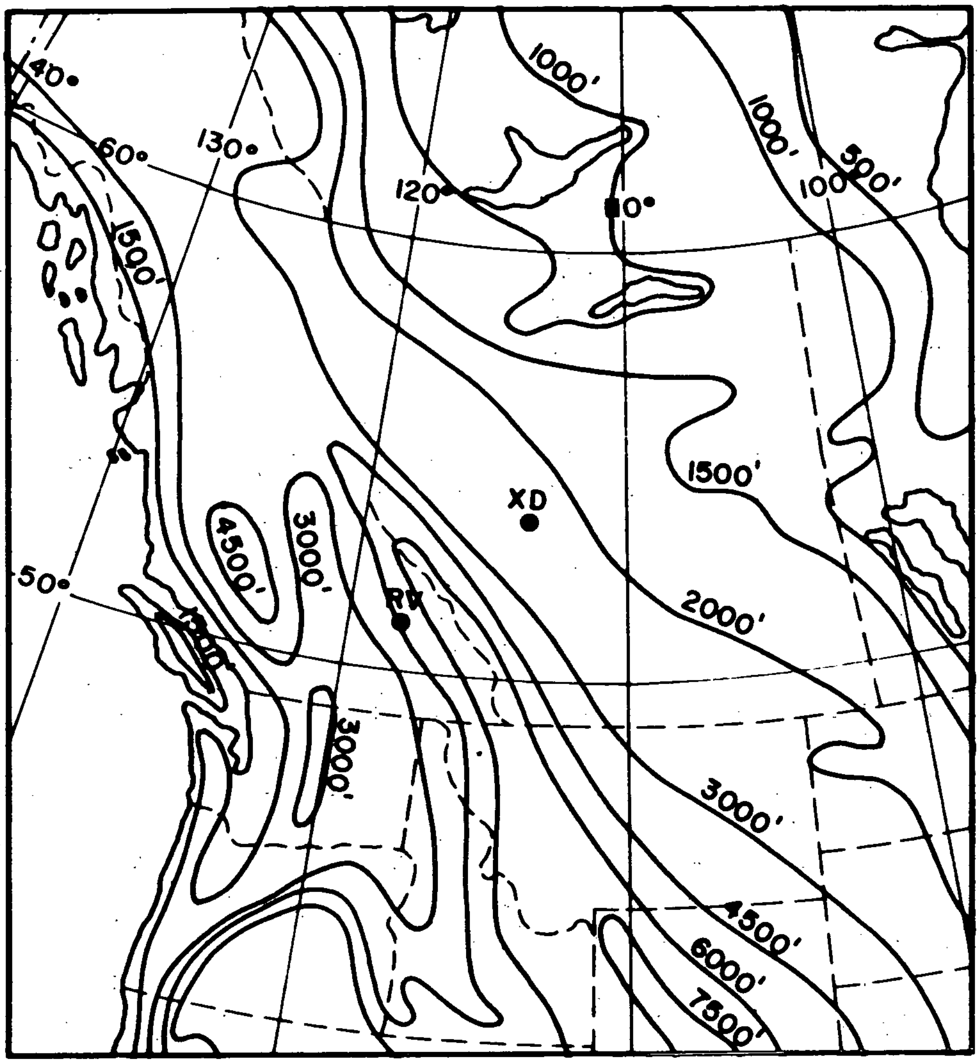



Figure 10
Smoothed Topographic Contour Chart

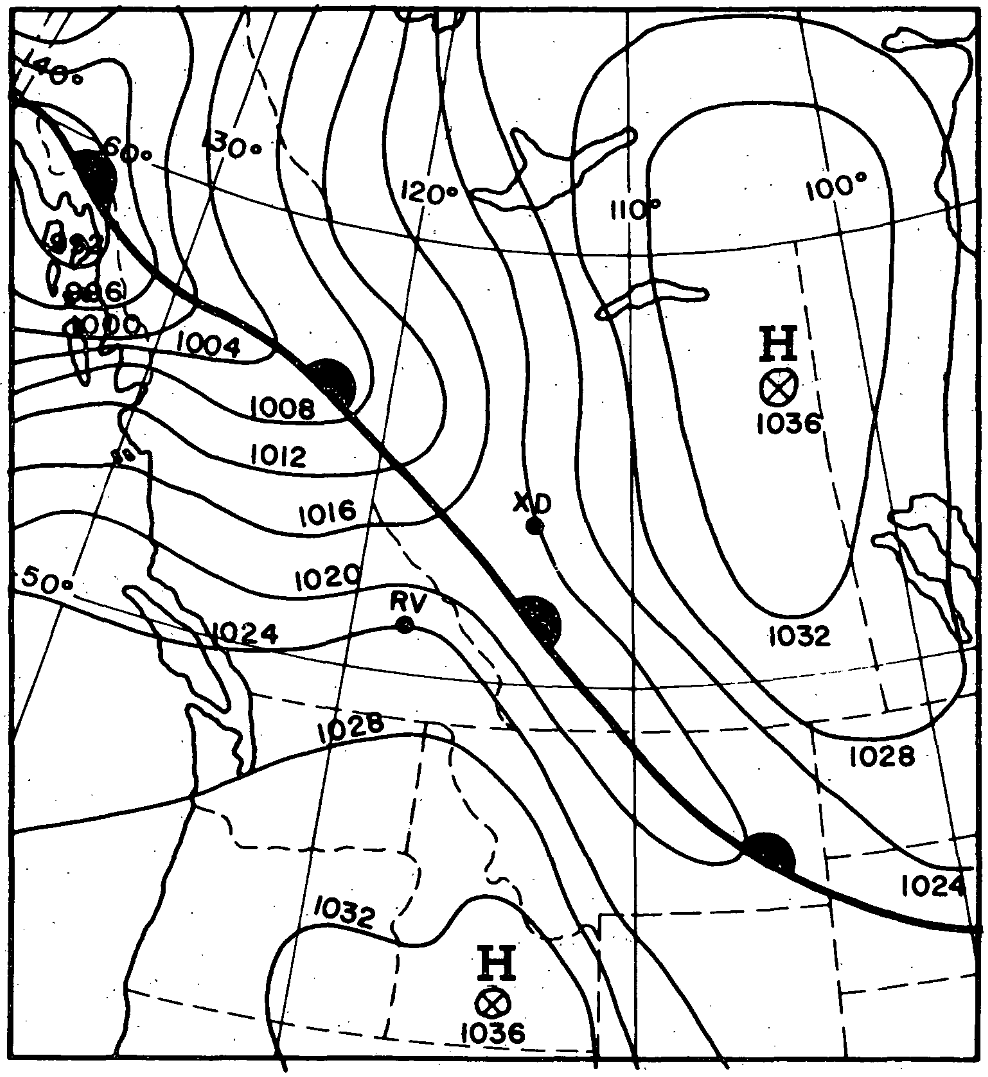



Figure 11
Surface Chart for 17 March 1964, 0000 GMT

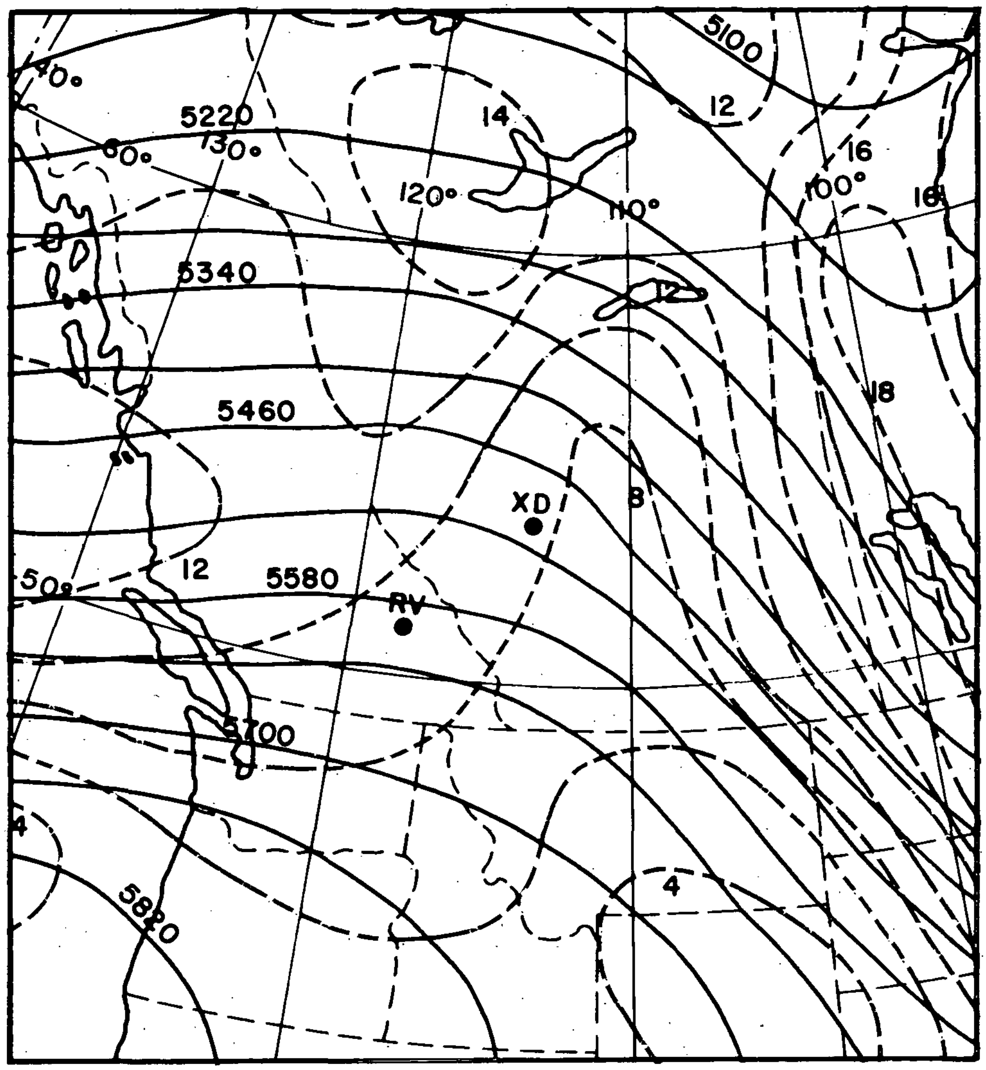



Figure 12

500-MB Absolute Vorticity Chart for 17 March 1964, 0000 GMT

500-MB Contours (m) ———

Isopleths of Absolute Vorticity ($10^{-5}$ $sec^{-1}$)

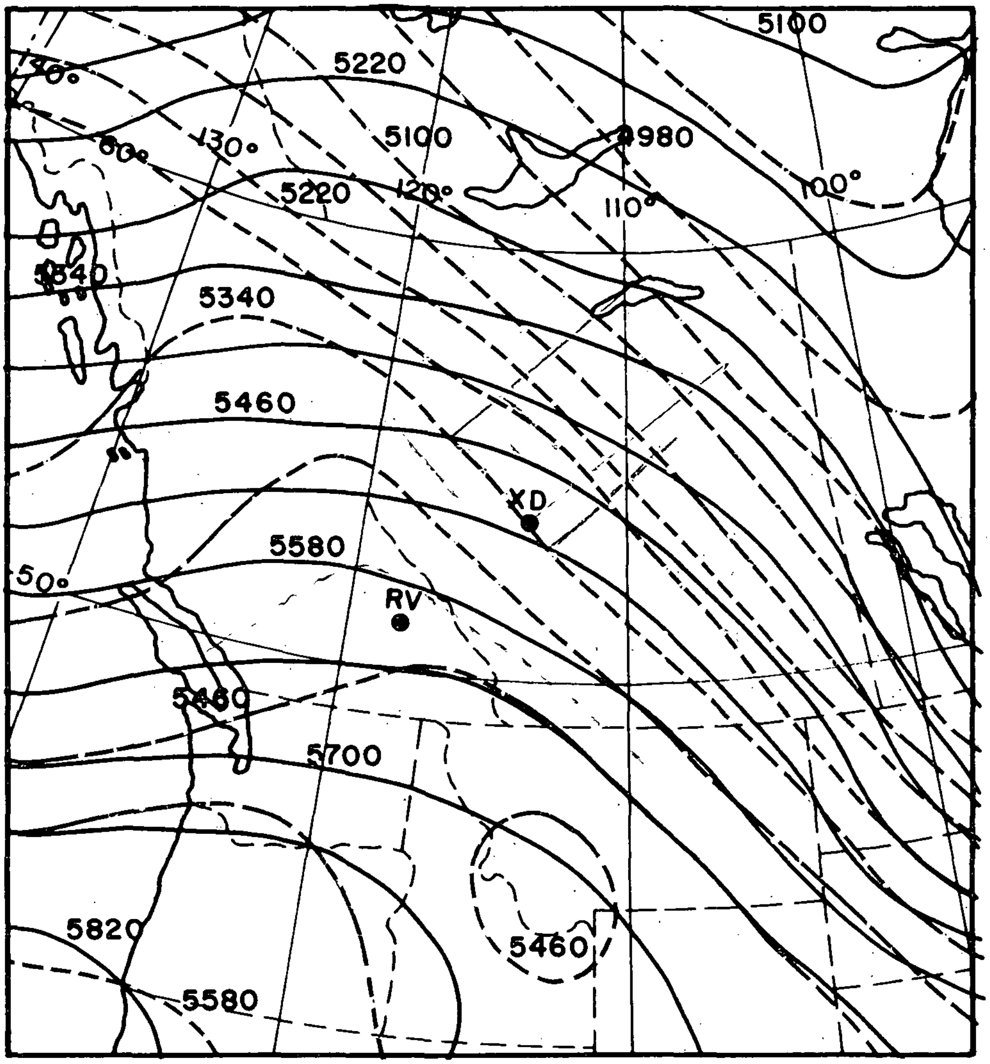



Figure 13
1000-500-MB Thickness Chart for 17 March 1964, 0000 GMT
500-MB Contours (m) ———
1000-500-MB Thickness (m) -----

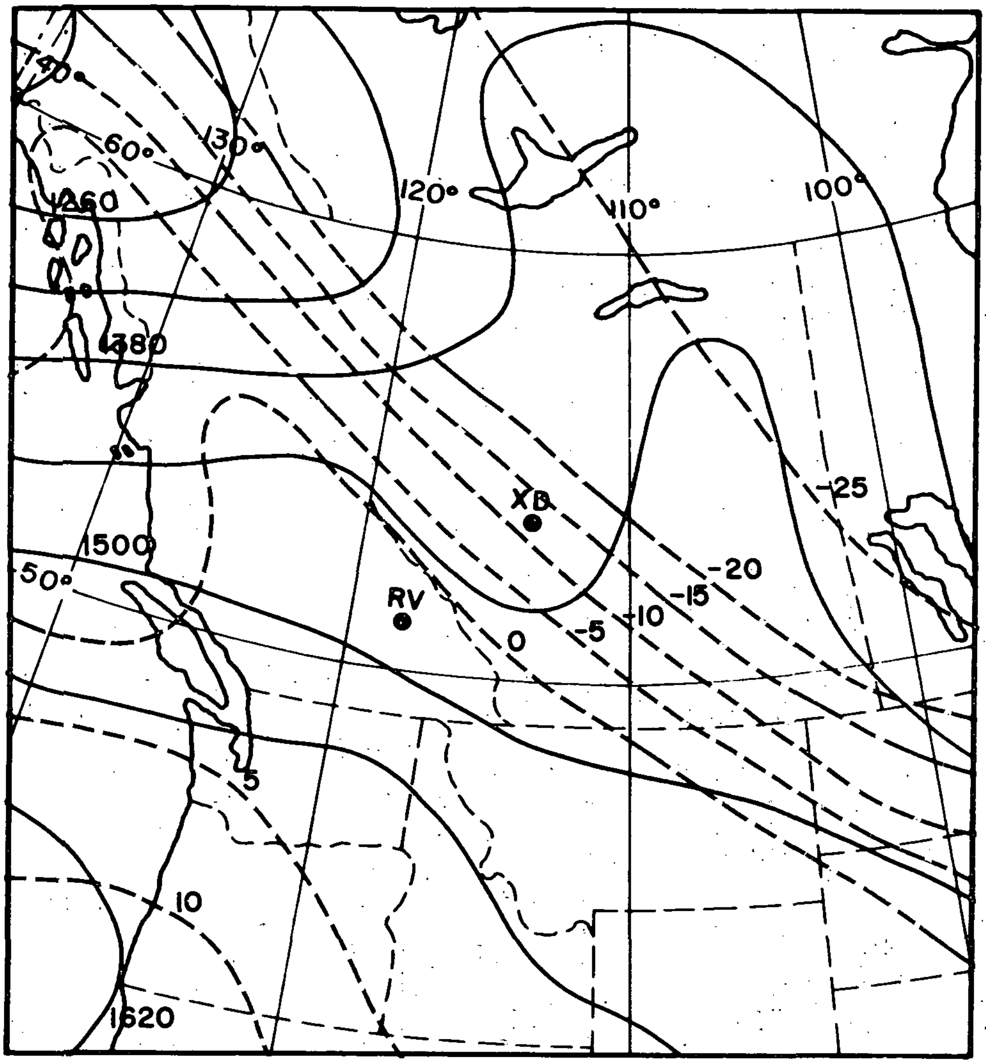



Figure 14
850-MB Chart for 17 March 1964, 0000 GMT
850-MB Contours (m) ———
850-MB Isotherms ( C)-----

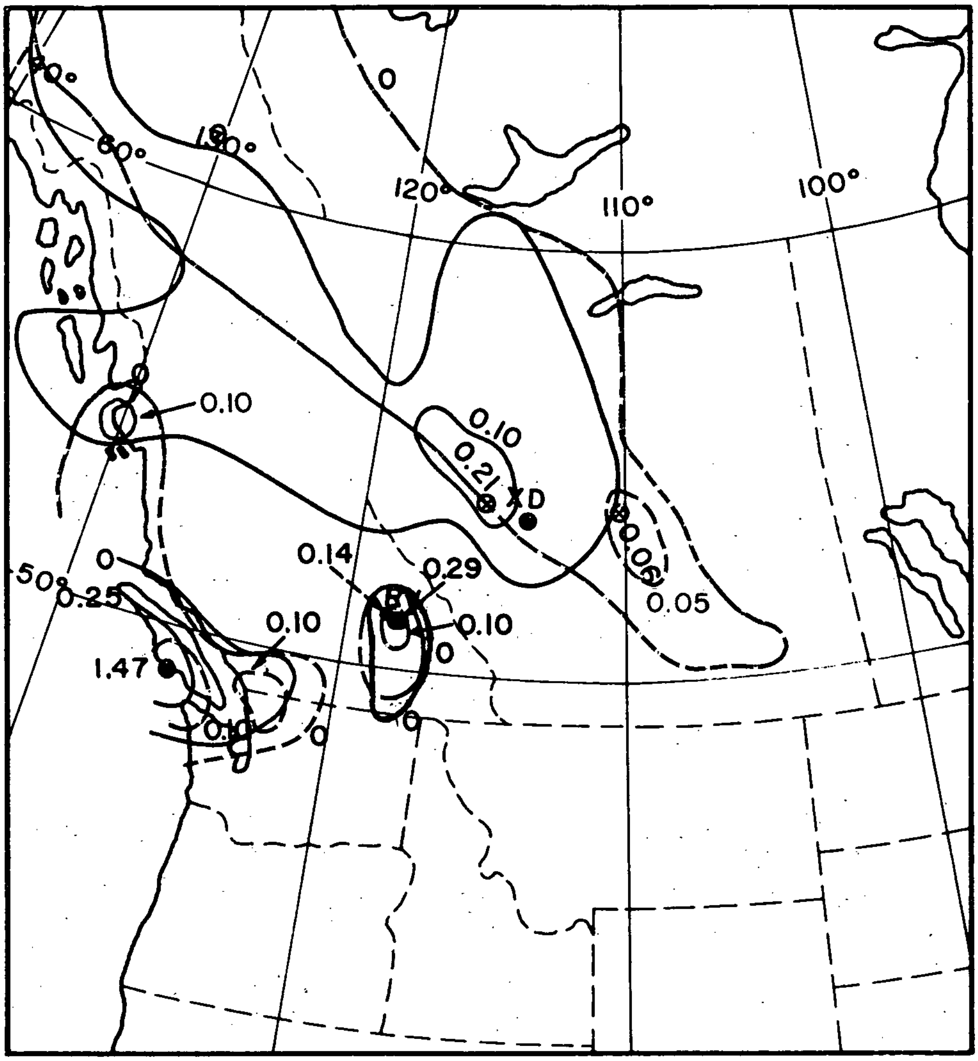



Figure 15
Observed 6 Hr Precipitation (in.)
For period 16 March 1800 GMT to 17 March 0000 GMT ———
For period 17 March 0000 GMT to 17 March 0600 GMT -----

CIR-4139 UDC: 551.558.29:
TEC-544 551.511.3
2 Nov. 64. 551.509.324.2

CANADA
DEPARTMENT OF TRANSPORT - METEOROLOGICAL BRANCH
315 Bloor St. W. - Toronto 5, Ontario.

"The Determination of Spot Values of Vertical Velocity and Precipitation Rate"
by
W. S. Harley, H. Dragert and I. D. Rutherford

17 pps. 15 figs. 6 refs.

Subject reference: 1. Vertical Velocity,
2. Precipitation Rate,
3. Regression Equations,
4. Vorticity Advection.

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