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**DEPARTMENT OF TRANSPORT**
**METEOROLOGICAL BRANCH**

# THE EFFECT OF PRECIPITATION ON THE LEVEL OF LAKE MICHIGAN/HURON

by

F. B. MULLER, J. G. GERVAIS and R. W. SHAW

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CANADA - DEPARTMENT OF TRANSPORT - METEOROLOGICAL BRANCH
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## ABSTRACT

Available basin precipitation and lake level data for Lake Michigan/Huron are analysed statistically. Smoothed, grouped yearly data indicated a two to three foot decline in the level of Lake Michigan/Huron since 1863, with a superimposed 75-year fluctuation of about 1 1/4 feet. The total annual basin precipitation has declined two to three inches since 1873 and it appears to have undergone a 75-year oscillation of amplitude 3 inches.

Spectrum and cross spectrum analysis, show that 96.3% of the variance of lake level is accounted for by fluctuations of 12 months and longer, and that there is a very high coherence-squared (about 0.70) between basin precipitation and lake level in the frequency range of zero to 1 cycle/12 mo. For all cycles in this range, except for fluctuations longer than 20 years and for the yearly cycle, the lake level lags the precipitation by approximately a quarter cycle. Longer fluctuations are more nearly in phase, and the yearly cycle has an apparent lag of one year. The potential of a prediction scheme, suggested by the results of this study, is discussed.

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# L'EFFET DE LA PRÉCIPITATION DE BASSIN SUR LE NIVEAU DES LACS MICHIGAN/HURON

par

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J. G. Gervais,
et
R. W. Shaw

## RÉSUMÉ

Les auteurs présentent une analyse statistique des donnees disponibles sur la précipitation de bassin et le niveau des lacs Michigan/Huron. Les données annulles groupées et lissées indiquent qu'il y a eu depuis 1863 une diminution de deux à trois pieds du niveau des lacs Michigan/Huron, ainsi qu'une fluctuation superposée d'environ 1 1/4 pied en 75 ans. La précipitation annuelle totale de bassin a diminué de deux à trois pouces depuis 1873 et elle semble avoir été soumise à une oscillation d'amplitude de 3 pounces en 75 ans.

Les analyses spectrales et spectrales croisées montrent que 96.3 p. 100 de la variance du niveau de ces lacs s'explique par des fluctuations de 12 mois et plus, et qu'il y a une très haute cohérence carrée (environ 0.70) entre la précipitation de bassin et le niveau des lacs dans la gamme de fréquences comprises entre zéro et un cycle/12 mois. Pour tous les cycles dans cette gamme, sauf pour les fluctuations s'étendant sur plus de 20 ans et pour le cycle annuel, le niveau des lacs est en retard sur la précipitation d'environ un quart de cycle. Les fluctuations plus longues sont plus près de la concordance des phases, et le cycle annual a un retard apparent d'une année. Les auteurs commentent le potential d'un schéma de prévison, découlant des résultats de cette étude.

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## 1. INTRODUCTION

The extremely low levels of the Great Lakes in recent years have been of general concern. These levels are affected by influences in two broad categories: first, geological or man made and second, relating to hydrology and meteorology. In this latter category, precipitation clearly plays a major part. Since an eighty-year record of precipitation onto Lake Huron-Michigan watershed is available, as well as a hundred-year record of their levels, this study was undertaken to examine the response of lake level to surrounding precipitation changes using methods of power spectrum and cross-spectrum analysis and statistical filtration. Lake Huron-Lake Michigan (one body of water) was selected because it is still effectively uncontrolled and subject to large changes in level.

The scope of the study, the procedure followed and organization of this report are indicated in the table of contents. Briefly, the data are described in section 2, section 3 provides information on the methods and procedures followed, while sections 4 and 5 describe the results of applying the procedures outlined in 3. The implications of the results for a possible forecast scheme are discussed in section 6.

## 2. DATA

Mean monthly values of lake levels from 1860 to 1963 inclusive at Harbour Beach, Michigan were available from the U. S. Army Corp of Engineers. The level is given to the nearest tenth of a foot above the datum at Father Point, Quebec.

The monthly basin precipitation data in hundredths of inches from 1873 to 1963 inclusive were calculated from raingauge

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* This study was undertaken during the summer of 1964 when Mr. Gervais an undergraduate of Assumption University of Windsor was employed as a Student Assistant by the Meteorological Branch. Mr. Shaw was a graduate student of Queen's University enrolled in the School of Graduate Studies of the University of Toronto.

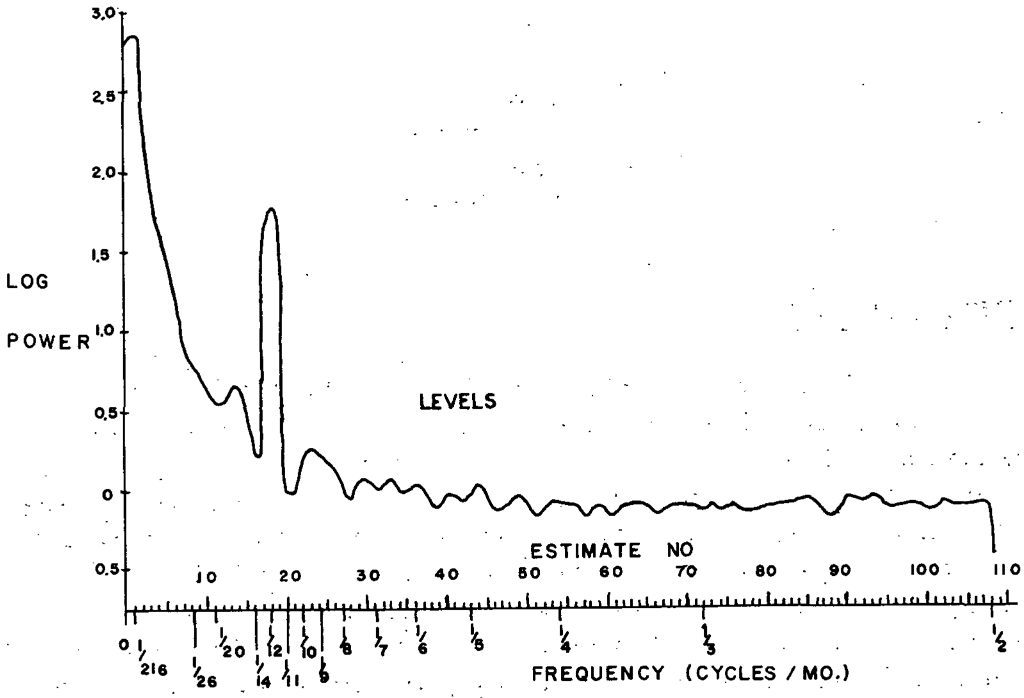


Figure 1
Logarithmic Spectrum of Raw Lake-Level Series

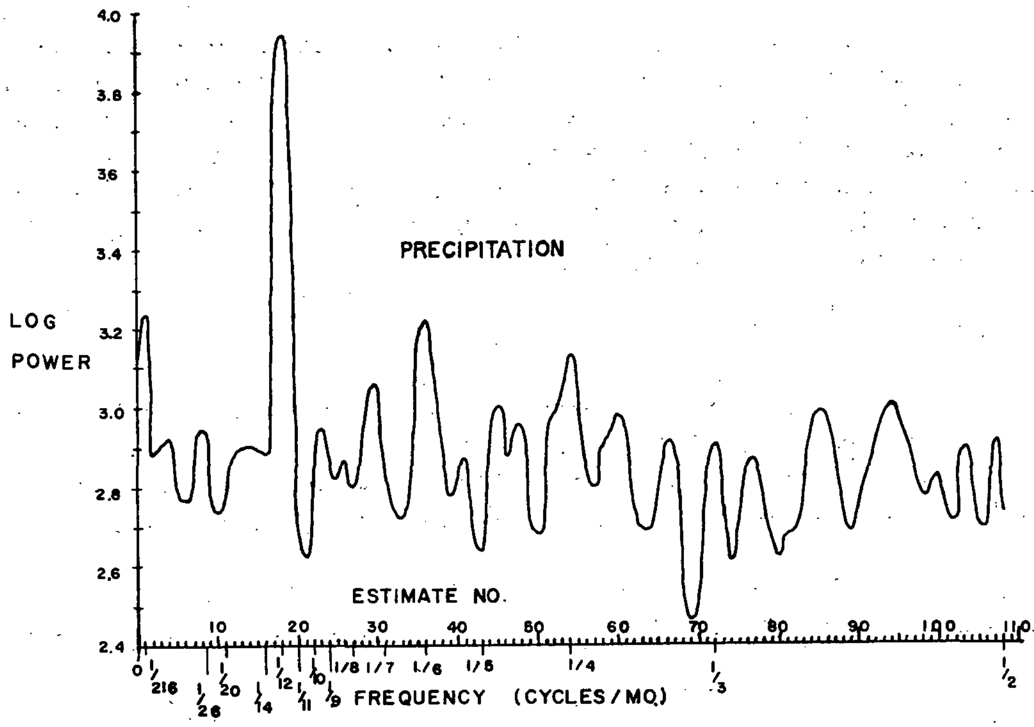


Figure 2
Logarithmic Spectrum of Raw Basin Precipitation Series

readings of the Meteorological Service of Canada and the U. S. Weather Bureau. The total number of raingauges on both sides of the border varied from 12 in 1873 to 283 in 1964, remaining more or less steady in the last 10 years. The combined data for the Michigan/Huron watershed were calculated using data from two sources. From 1873 to 1933 inclusive the Meteorological Service of Canada combined the data for the two systems using the weighting factors of 0.477 for the precipitation on the Lake Michigan basin (area 67,900 sq. mi.) and 0.523 for that on the Lake Huron basin (area 72,600 sq. mi.). From 1900 to 1963 inclusive, data for the individual watersheds were computed separately by the U. S. Army Corps of Engineers. These were combined using the above weighting factors. In the overlapping years (1900 to 1933) there were small discrepancies between the two sets of data; these were resolved by taking the mean of the two values.

Precipitation directly onto the lake was not taken into account, since very little data on this are available. Rather, the surrounding land records were examined for any information they by themselves might contain concerning lake levels.

## 3. METHOD

Single spectral analysis, using the method of Blackman and Tukey (1958) was performed on the raw series, calculating 108 spectral estimates in the frequency range of zero to 1/2 $months^{-1}$, thus giving a resolution of 1/216 $months^{-1}$. An IBM 7090 computer program available in the Atmospheric Research Section of the Meteorological Branch was used to obtain all spectral analyses in this study. For an additional description of the method, see Panofsky and Brier (1958).

The individual spectra of the raw data (Figs. 1 and 2) indicated that, for further analysis, suppression of certain prominent peaks in the spectra was desirable, as these powerful frequencies would tend to obliterate detail in and spread power to neighbouring parts of the spectra. Thus, the three statistical filters described below were designed, using the IBM 7044 computer and a program described by McCulloch (1965). Explanations of statistical filtering may be found in Panofsky and Brier (1958) and Holloway (1957).

The filters designed were:

a) A low-pass filter (Fig. 4a) with cutoff frequency (response 10%) at 1/16 $months^{-1}$.

b) A band-suppress filter for the lake level data (Fig. 5). This was to "prewhiten" the raw lake level series, i.e. to pass all frequencies in the raw series at a roughly equal amplitude level.

c) A band-suppress filter for the basin precipitation data (see Fig. 6). This was used in a manner similar to above to prewhiten the raw basin precipitation values.

The low-pass filter with cutoff frequency 1/16 $months^{-1}$ was used to smooth the monthly values of lake level and basin precipitation. The smoothed monthly values were then grouped into mean annual lake levels and total annual basin precipitation values. The pre-smoothing was done to eliminate fluctuations of period somewhat less than twelve months, variance from which might be passed by the equally-weighted grouping mean ordinarily used to find the annual values.

After applying each pre-whitening filter to its respective series, single spectrum and cross-spectral analysis was done on the resulting filtered series. The purpose of the latter was to find, as a function of frequency, the relationship between basin precipitation and lake level; in other words, how much of a given fluctuation at a given frequency in precipitation manifests itself in a similar fluctuation in lake level? Coherence (and coherence-squared) is the frequency-dependent quantity corresponding to correlation coefficient, and was chosen as the statistic to measure this relationship. The cross-spectral analysis program also calculated the phase relationship between precipitation and lake level at each frequency.

On the basis of broad maxima in the coherence spectrum (Fig. 9) four statistical band-pass filters were designed for application to the raw data. Fig. 16 shows the response chacteristics of these filters.

A sample plot of the low-passed series and the series containing the yearly fluctuations is shown in Figures 25 and 26. For each band-pass filtered series, with the exception of the 0 to 1/40 $months^{-1}$ (low-pass), the following analyses were made:

a) The mean line was drawn on the graph. Since the zero frequency response of the three band-pass filters was zero (Fig. 16) none of the trend, which would be analogous to DC voltage in an electrical wave, was passed. Consequently it was possible to draw a mean line since fluctuations in the band-passed series were deviations from a mean of zero (analogous to AC voltage). A cycle was considered to be the portion of the series between successive upward crossings of the mean line, and the period of each oscillation was measured in this manner. Periods of fluctuations were plotted against real time (Fig. 18, 19, 20), the date of the oscillation being taken as the beginning of the oscillation. This plot was made as an attempt to find a pattern in the fluctuations in each frequency band throughout the years. This pattern, if existent, would be an aid in a prediction scheme.

b) Histograms were drawn for periods of oscillation, i.e. the elapsed time between successive upward crossings of the mean line (see Fig. 17).

c) The envelope of the fluctuations (see Fig. 26 for an example) was also plotted against real time in Figs. 21, 22 and 23, to see if a pattern existed. The envelope was determined by measuring the departure of positive peaks from the mean line, taking the date as the date of the peak. Such a graph would give a condensed view of the behaviour of the lake levels and precipitation in each frequency band.

d) In each band-passed series, the correlation coefficient for a lag of zero was calculated between the envelope of precipitation fluctuation and that of the

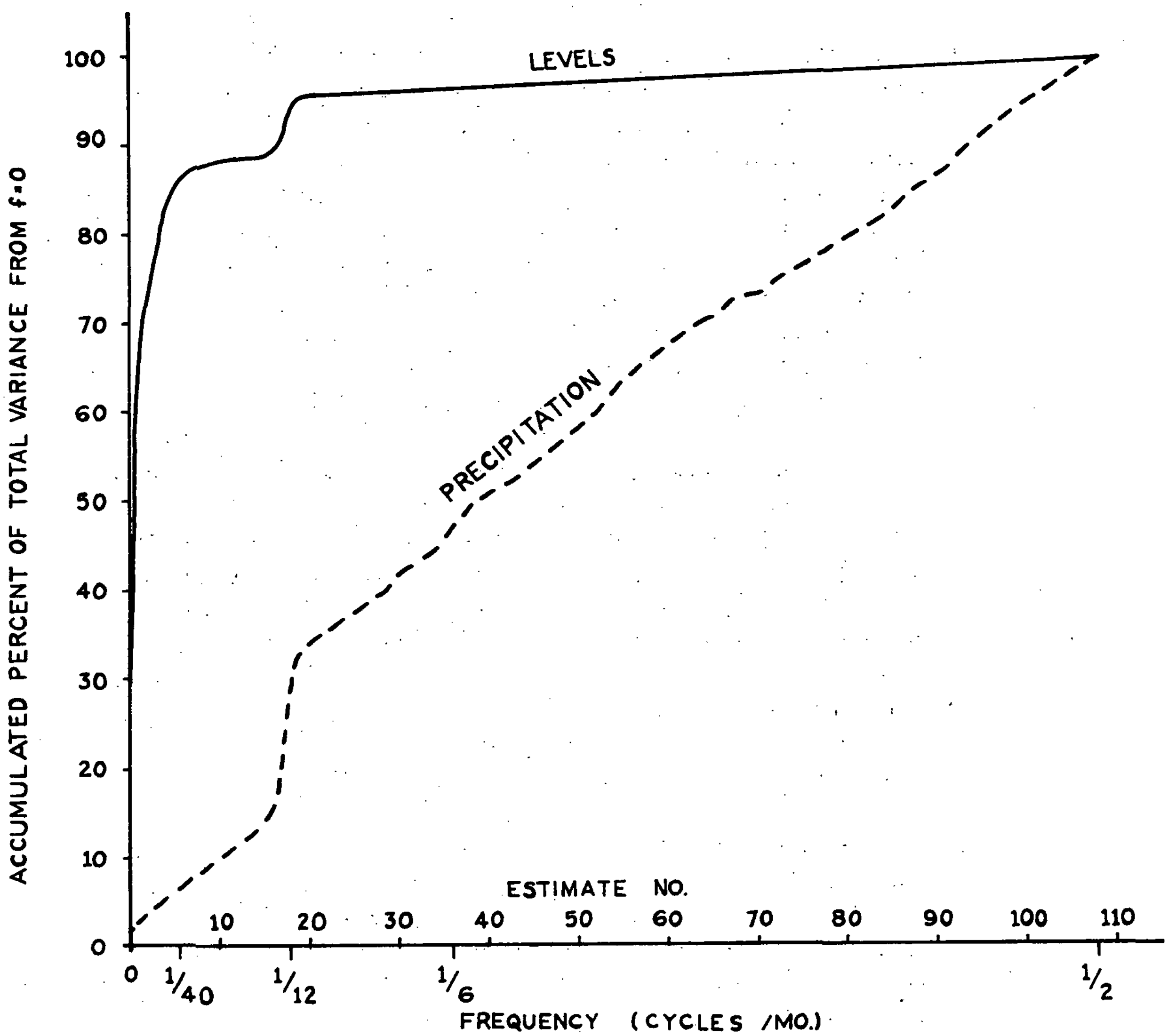


Figure 3
Accumulated Percent of Total Variance Accounted for by the Band from Zero Frequency to a Given Frequency

lake level. Similarly the correlation coefficient (lag zero) was found between the plot of period against real time for the precipitation and that of the lake level.

For all four filtered series (including the low-pass) the correlation coefficient between basin precipitation and lake level was calculated, using appropriate lags determined from the cross-spectral analysis by the following formula:

$$L = \frac{\Phi \Upsilon}{360}$$

where L is lag used (months),

$\Phi$ is phase lag in degrees of the frequency of highest coherence in the band, determined from the cross spectral analysis, and

$\Upsilon$ is period (months) of the frequency of highest coherence.

## 4. RESULTS OF SPECTRAL ANALYSES

### Spectra of raw data:

#### Lake levels

The spectrum found for the lake level (Fig. 1) is dominated by the zero frequency estimate, the long period fluctuations and the annual cycle. Fig. 3 was plotted by integrating the area under the spectral curves of lake level and of precipitation, since these curves are expressions of variance per frequency interval. Thus, Fig. 3 shows for a given frequency the accumulated percent of the total variance accounted for from zero frequency to the given frequency. From this figure it can be seen that 96.3% of the variance in lake levels is accounted for by the yearly cycle and longer period fluctuations; the yearly cycle itself accounts for 6.4% and the band from zero frequency to 1 in 9 years accounts for 77.3% of the variance. In other words, 3/4 of the variance in lake levels is explained by fluctuations lasting 9 years or longer.

The peak between the frequencies 1/20 months$^{-1}$ and 1/14 months$^{-1}$ is not considered significant at the 5% level. Panofsky and Brier (5) provide an explanation of the Chi-square test used to reach this conclusion. Briefly, the particular record chosen is a sample from a very much longer record stretching into the past and future. The spectrum measured from this sample may differ considerably from this much longer record. The Chi-square test allows one to accept or reject the hypothesis that the long-term spectral power at this frequency would correspond to a smooth spectrum with no local peak. To reject this hypothesis is to conclude that a peak is significant at for instance the 5% level. The probability that this conclusion is false is given by the figure quoted for the significance level.

The annual peak is the only one that is significant at the 5% level.

Precipitation

The spectrum for the raw precipitation data (Fig. 2) appears much less smooth than that of the raw lake levels. This is partly because of the expanded vertical scale made possible by the relatively larger variance density in higher frequencies, but it is probably also to be explained by the local effects of each station in the network. Each station has its own individual variation depending on position, topography and wind direction. The total of all stations will contain this variability, although it may not be present in the true areal total of precipitation. In particular, relatively large and spurious fluctuations may be observed in the basin totals on short time scales -- on longer time scales these might be expected to cancel out somewhat. A further result of the addition of local effects will be that spectral estimates from a given length record should be less stable compared to those from lake level measurements.

The actual basin precipitation spectrum shows long-term fluctuations to be relatively less important, but the annual cycle is still well-marked. In additon, there are four peaks which are significant at the 5% level as explained earlier. These are at: 1/216, 1/7.5, 1/6, 1/4, months$^{-1}$.

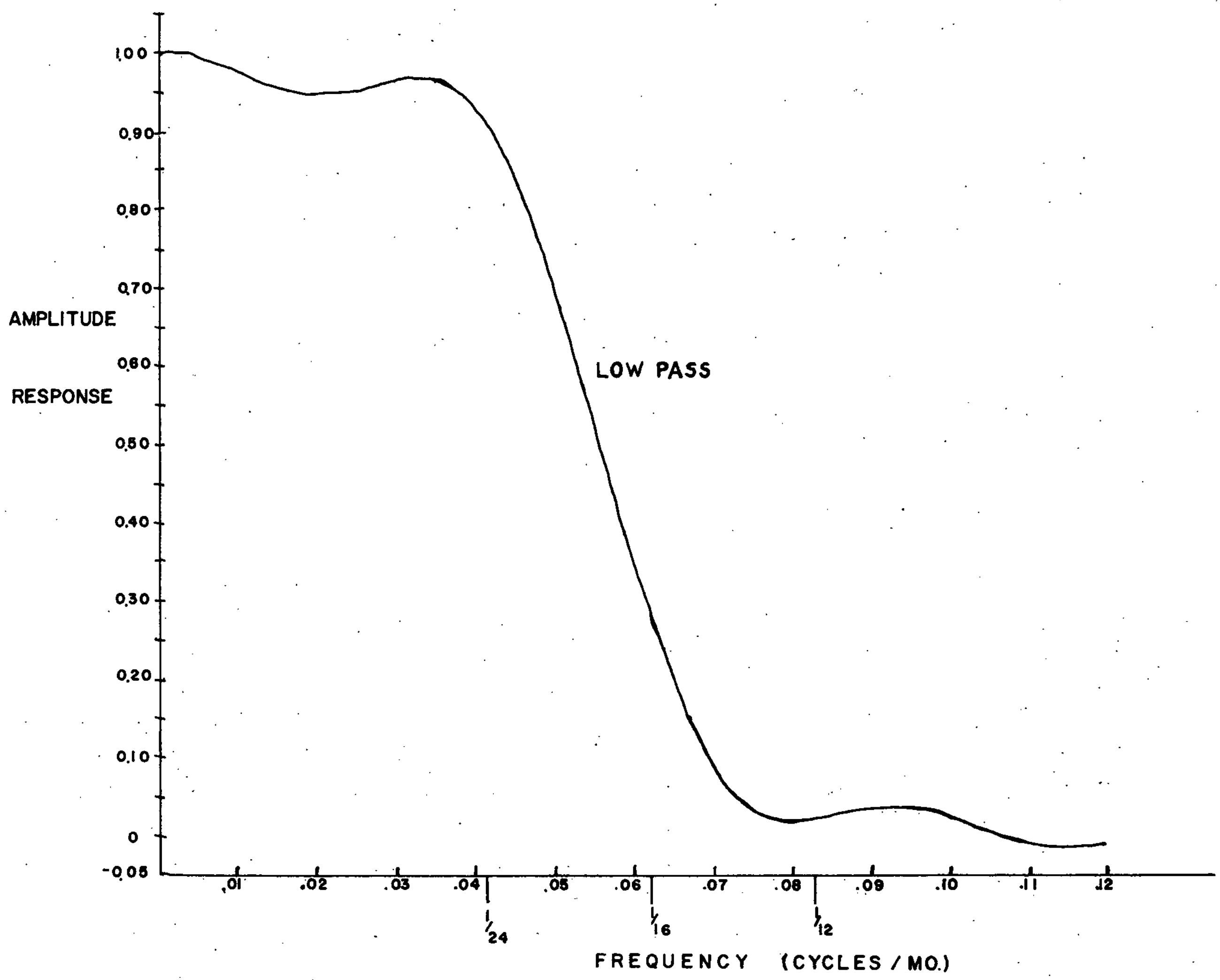


Figure 4
Frequency Response of Low Pass Filters to Smooth Monthly Values

The small peak at 1/26 $\text{months}^{-1}$ is of borderline significance. Figure 3 shows that fluctuations of 12 months and longer account for only 32.6% of the variance in the precipitation, indicating the greater prominence of fluctuations shorter than 12 months and possibly also the effects of sampling difficulties. The yearly cycle itself accounted for 20% of the total variance. Thus, while on the scale of the annual cycle and longer, 50% of the variance is in changes longer than 12 months, the corresponding figure for the lakes is 90%.

It is of some general interest that the precipitation record shows a peak near 1/26 $\text{months}^{-1}$ in view of the discovery a few years ago that a complete 26-month cycle apparently dominates wind changes in the equatorial stratosphere. Other workers (e.g. Godson 1963, Angell and Korshover 1962) have identified spectral peaks near this frequency in time series of many other meteorological variables in high latitudes as well as equatorial. In this case the variable represents an average over a sizeable geographical area, but Gargett (1965) reports a similar peak in the spectrum of precipitation recorded at the Toronto Meteorological Observatory.

## Spectra of Pre-whitened Data:

### Lake levels

The spectrum of the pre-whitened lake levels (Fig. 7) appears much less smooth than the spectrum of the raw data, since the long-term and annual fluctuations have been reduced in amplitude by the pre-whitening filter (Fig. 4) to values comparable to the remainder of the spectrum, thereby allowing the use of a more expanded vertical scale. The long-term and annual fluctuations have been greatly reduced. The peak at 1 $\text{months}^{-1}$ is now significant by the 5% Chi-square test, whereas it did not appear in the spectrum of the raw data. There is also a possible peak at 1 $\text{months}^{-1}$. The depressions at either side of the annual cycle are solely the result of the filtration, since the pre-whitening filter (Fig. 5) reduced not only the annual cycle, but the neighbouring frequencies as well. Note the decrease of power with increasing frequency.

Precipitation

The spectrum of the 'pre-whitened' basin precipitation (Fig. 8) is quite "white" (see 3); the long period fluctuations, the annual cycle and the 6-month cycle have been greatly reduced. The peak at 1/7.5 months$^{-1}$ still appears, but the filter (Fig. 5) reduced the peaks at 1 cycle/216 and 1 cycle in 26 months, and caused depressions on either side of the annual and 6-month cycle.

Since the high-frequency oscillations represented only a small variance fraction of the lake level record, they were not of interest at this time and were not studied in detail. In particular, one might inquire as to the portion of the high-frequency variance which is due to the shape of the annual cycle and the extent to which it also accounts for the details in this spectral region. This is readily done by performing a spectral analysis on the mean annual curve (Fig. 24) replicated a large number of times. For each frequency, the difference in power between this spectrum and that of the original record would represent fluctuations that are independent of the annual cycle.

Relationship between Lake Level and Precipitation

Coherence

Figure 9 displays the coherence-squared spectrum for the pre-whitened lake level and precipitation, giving 108 estimates in the frequency range of zero to 0.5 months$^{-1}$. Upon examination, one feature is readily apparent: there is a very high level of coherence-squared (averaging 0.69) between lake levels and precipitation in the low frequency range of zero to 1/6 months$^{-1}$, in contrast to the mean coherence-squared of 0.39 for the entire frequency range. The relatively low coherence-squared of 0.44 and 0.28 at and near zero frequency is artificial and due to the lake level pre-whitening filter which has zero response at zero frequency, preventing any determination of coherence at that frequency. However, from the cross-spectral analysis of the grouped annual data (discussed below) it can be seen that its coherence-squared at zero frequency is 0.78, as the zero frequency component is entirely present. Hence, coherence-squared values at and near zero frequency from the analysis of the annual data have been added as a correction to Figure 8 (see dashed line).

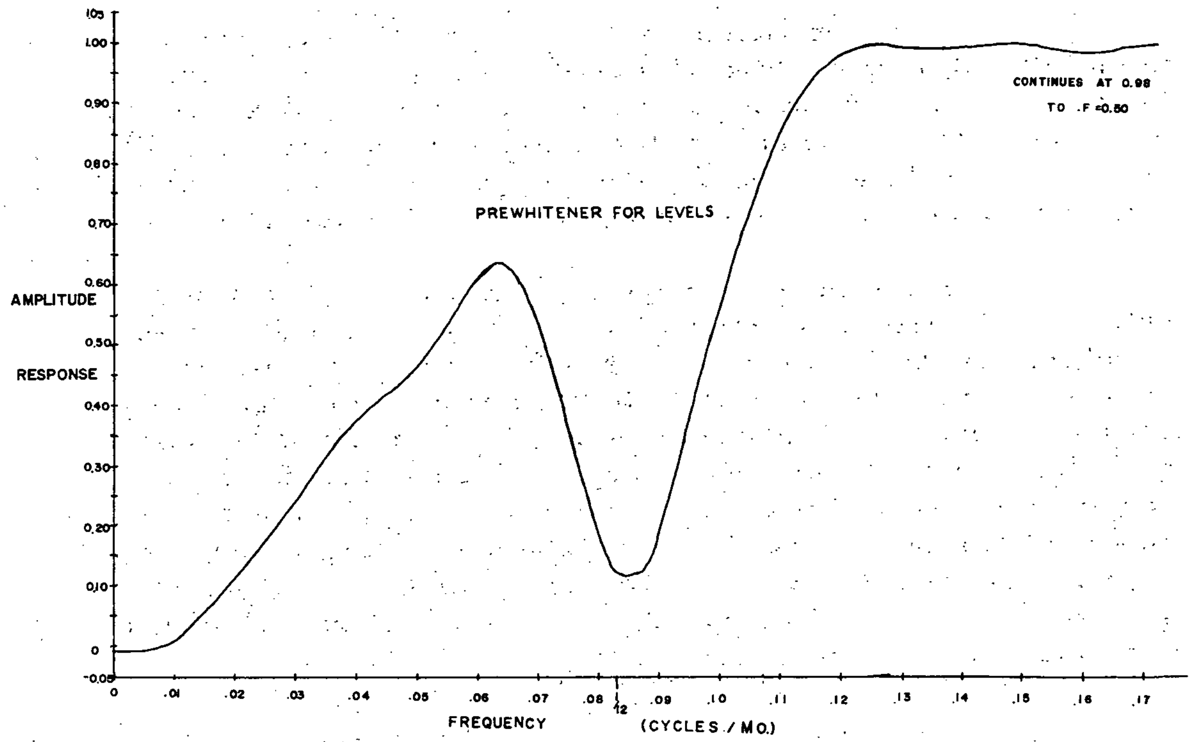


Figure 5
Frequency Response of Filter to Prewhiten the Raw Lake-Level Series

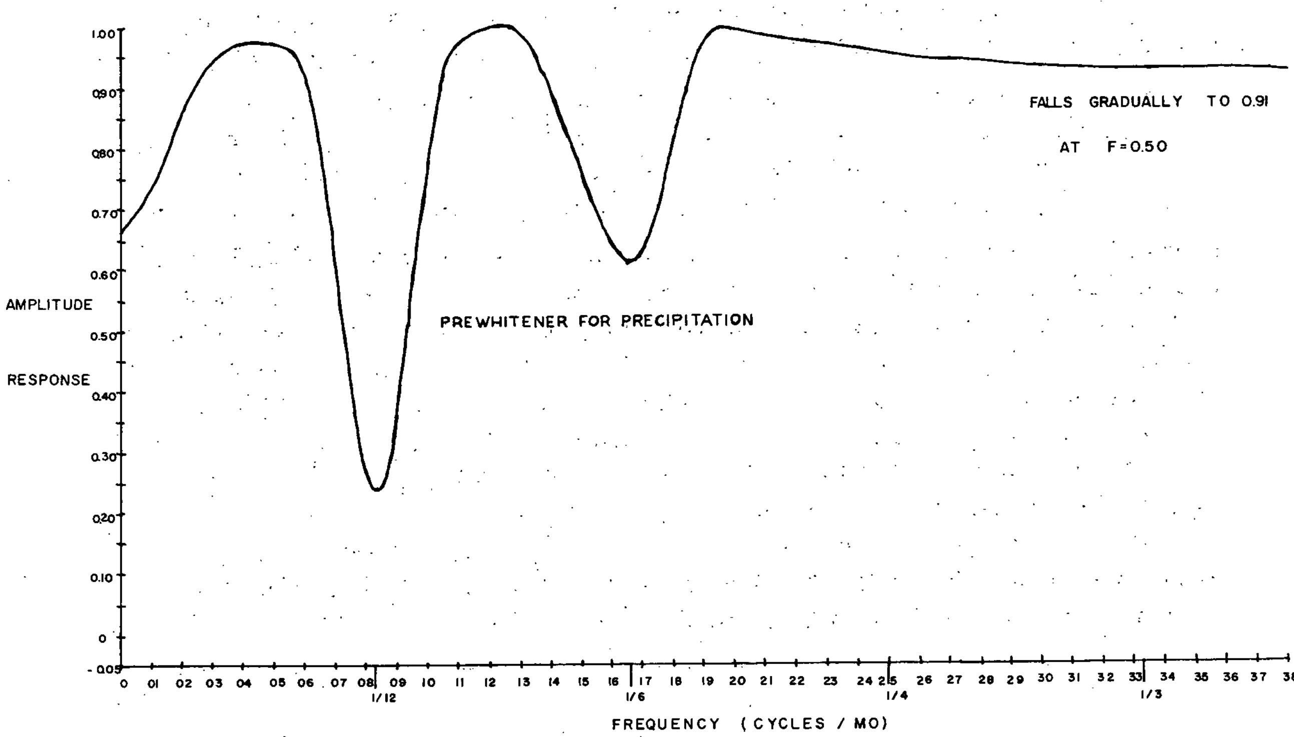


Figure 6
Frequency Response of Filter to Prewhiten the Raw Basin Precipitation Series

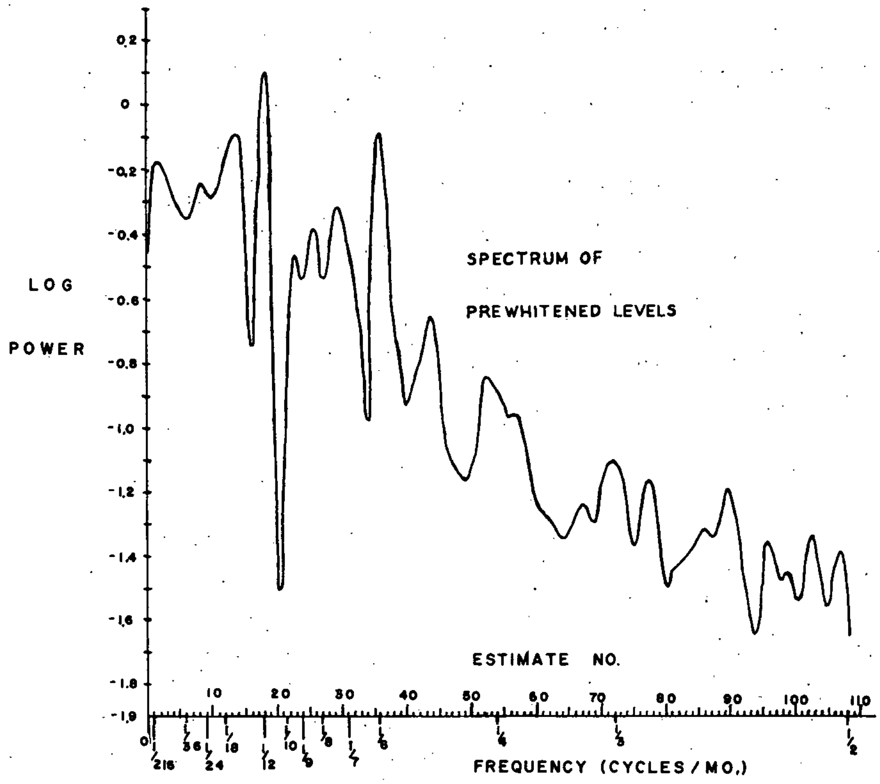


Figure 7
Logarithmic Spectrum of Prewhitened Lake-Level Series

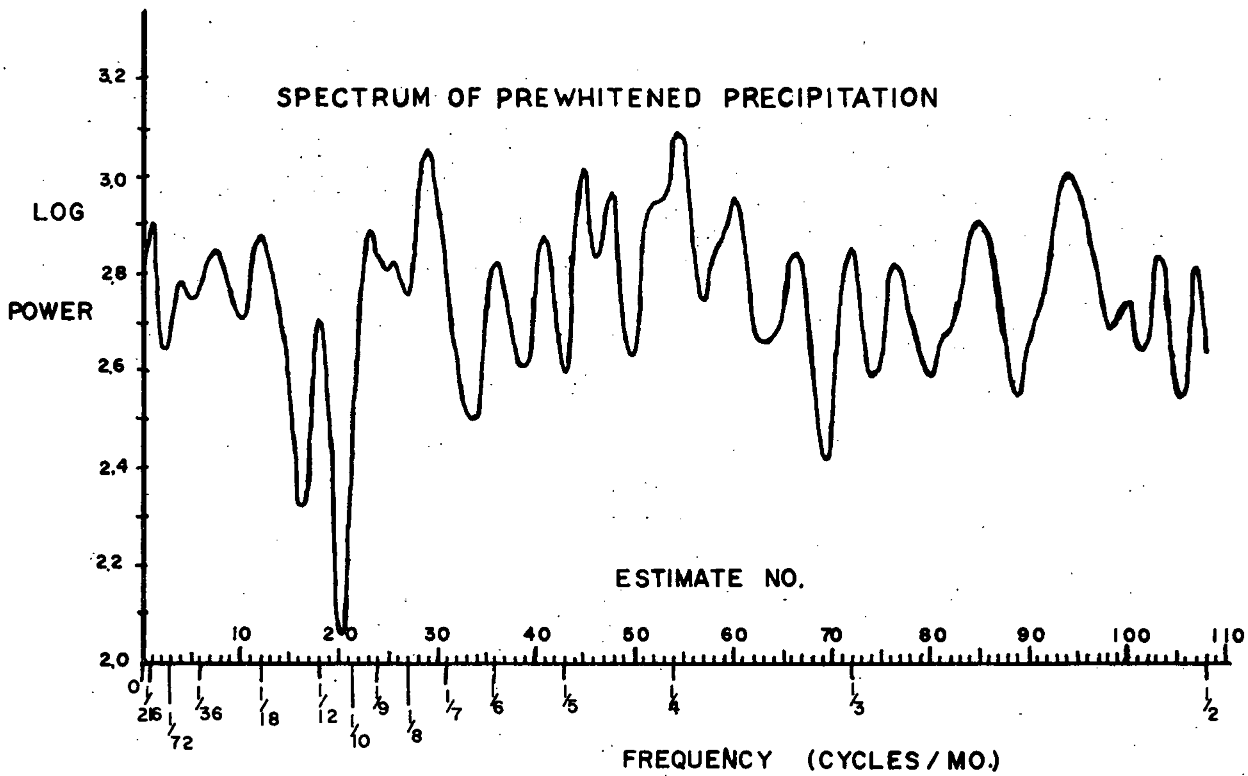


Figure 8
Logarithmic Spectrum of Prewhitened Precipitation Series

If the corrected values are taken into account, the average value of coherence-squared for the entire frequency range is now 0.40, and for the range of 0 to 1/6 months$^{-1}$ is 0.70. Figure 9 shows the average coherence-squared from zero frequency to a given frequency, using the corrected very-low-frequency values in Fig. 9. In Fig. 9 there appear to be peaks of coherence-squared at 1/26, 1/18, 1/12, and 1/6 months$^{-1}$ but it should be noted again that coherence is high for all frequencies between zero and 1/6 months$^{-1}$. Figure 10 shows that above 1/6 months$^{-1}$, the average coherence-squared from zero frequency falls steadily.

## Phase Relationship

The correlation coefficient (zero lag) between the two pre-whitened series is 0.098, and for the annual grouped values it is still lower at $1.07 \times 10^{-5}$. These correlation coefficients are very low because they do not take into account phase differences between precipitation and lake level which, as it will be seen, are very important. Thus, the two series which at first glance seem unrelated, are, in fact, highly correlated if phase lag is accounted for.

The graph of the lag angle of lake level behind precipitation for the frequency range of zero to 1/3 months$^{-1}$ (Fig. 11) shows that, with the exception of the trend (very low frequency) and the annual cycle, the lake level lags the precipitation by approximately 100 , or a quarter cycle, throughout. Thus, the time lag between lake level and precipitation is not constant, but increases with the length of the oscillation. At very low frequencies the angle becomes smaller -- at zero frequency, phase has no meaning. By contrast, the annual cycle in lake level appears to be either in phase or a multiple of cycles (possibly one year) behind the annual cycle in precipitation. Examination of the series filtered to show the annual cycle only (Fig. 26) indicates that lake levels apparently lag precipitation by one year in the annual cycle.

The constancy of phase angle, apart from the annual cycle would have been unexpected on the basis of a constant run-off time for precipitation; it is, however, suggestive of a model in which long-term storage with longer transit time comes more into play as the duration of a fluctuation in precipitation becomes longer.

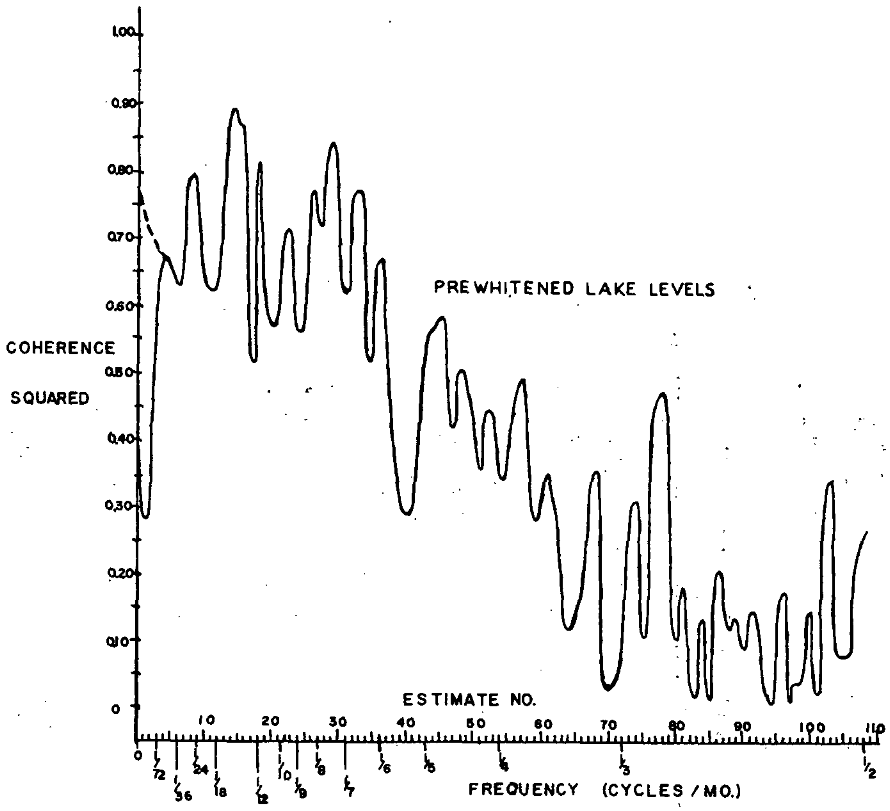


Figure 9
Coherence-Squared Between Prewhitened Lake-Levels and Basin Precipitation at Each Frequency

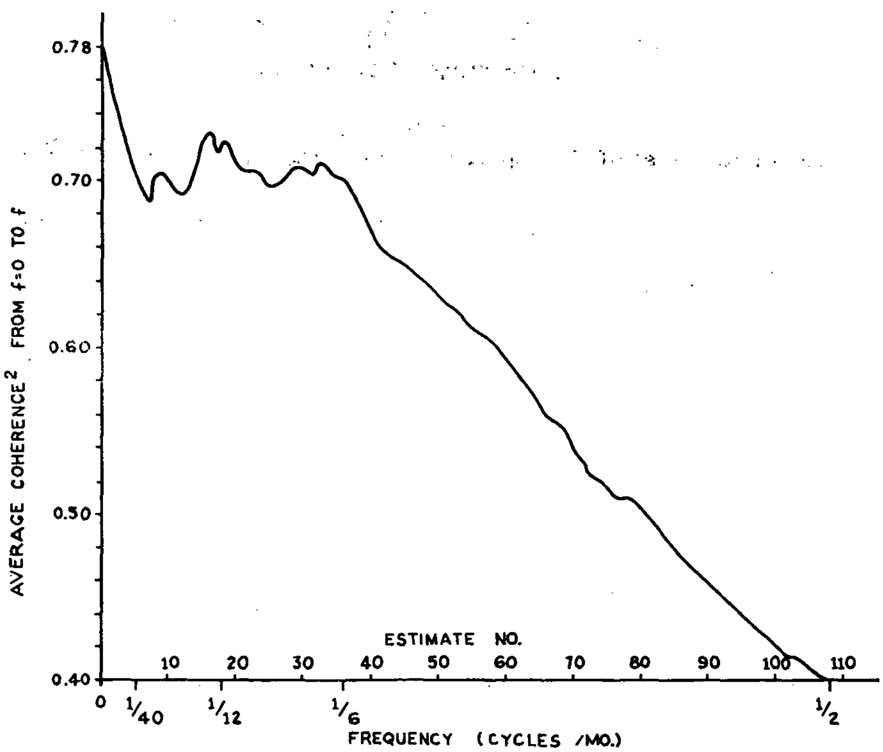


Figure 10
Average Coherence-Squared Accounted for by the Band from Zero Frequency to a Given Frequency

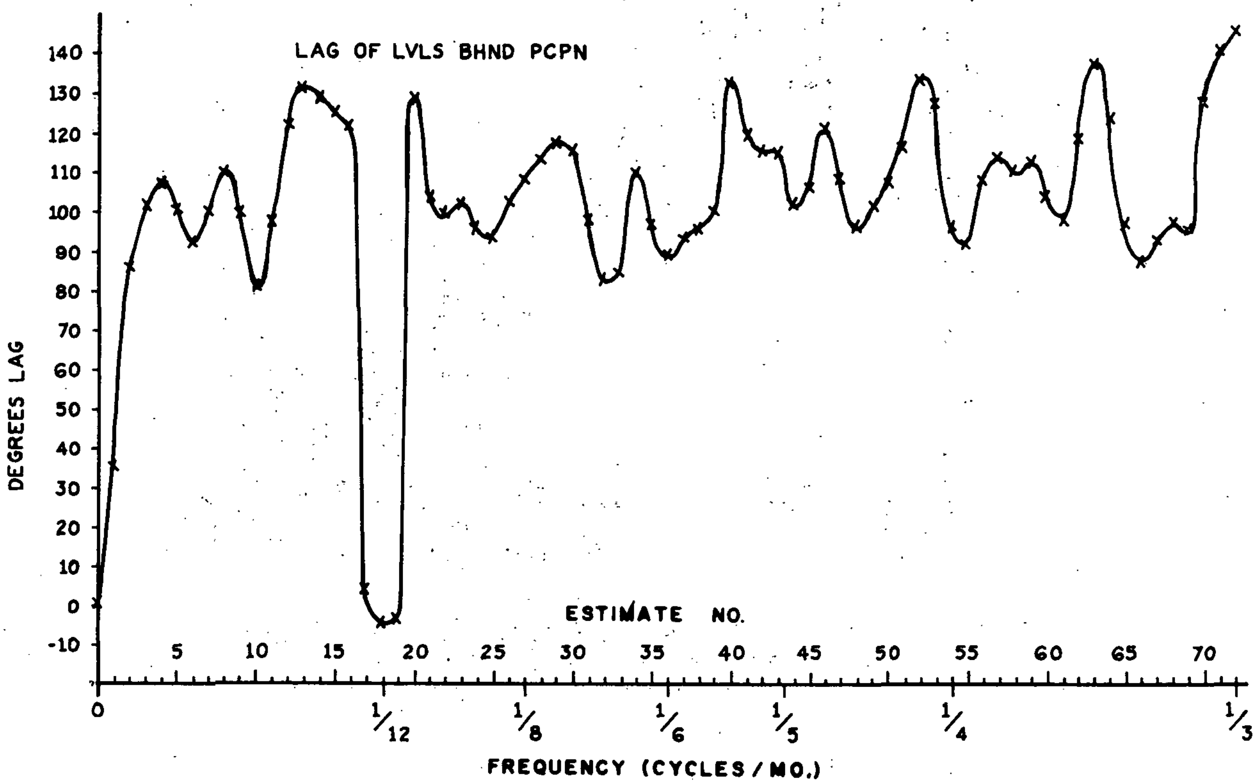


Figure 11
Phase Lead (Degrees) of Precipitation over Lake Levels at Each Frequency

The annual cycle behaves very differently for a number of reasons which deserve further study; however, there are some obvious influences which may be considered qualitively. Firstly, one is reminded that estimates of lake evaporation (e. g. Richards 1965) suggest it to be comparable in magnitude to basin precipitation and, of course, opposite in sign. It may well be that much of the power (variance) of evaporation, which acts without lag, is concentrated in the annual cycle, and that the component in the lake-level cycle due to evaporation acts to erode what would otherwise be a lagged crest in the annual march of lake level. Secondly, the annual cycle and shorter accounts for 67% of the precipitation variance. In this time-scale precipitation may well be quite strongly negatively correlated with evaporation, so that again the effect is to increase the inphase peak in lake levels. Thirdly, spring snow melt clearly precedes the annual precipitation maximum and again the effect is to advance the phase of the lake levels. Finally, influence of inflow from Lake Superior would have to be expressed mainly in the annual cycle since longer time scale fluctuations would require manipulation of the annual cycle of storage in Lake Superior.

## Relationships as shown by Annual Data

Fig. 12 shows the results of smoothing the monthly data with the low pass filter (Fig. 4) and then forming non-overlapping 12-month totals and averages. The preliminary smoothing was performed in order to give a better overall picture of the behaviour of the lake levels and basin precipitation. The reason for this is that an equally-weighted moving mean usually used to find yearly values will let through some power of frequency greater than once in 12 months. Thus, if higher frequency components are filtered out of the monthly values to begin with, this cannot occur and the yearly values represent only fluctuations on time scales longer than once per year. For purposes of comparison, yearly values from unsmoothed data are also displayed in Fig. 12. In the case of lake level data, the smoothing of the monthly values reduced the variance by as little as 1% since 96.4% of the variance in lake levels is caused by fluctuations of 12 months or longer; thus, the smoothing filter could not affect the variance appreciably. In precipitation, however, the smoothing of monthly data reduced the variance of yearly totals by 2.3%, a somewhat larger value because of the greater importance of shorter period fluctuations in precipitation.

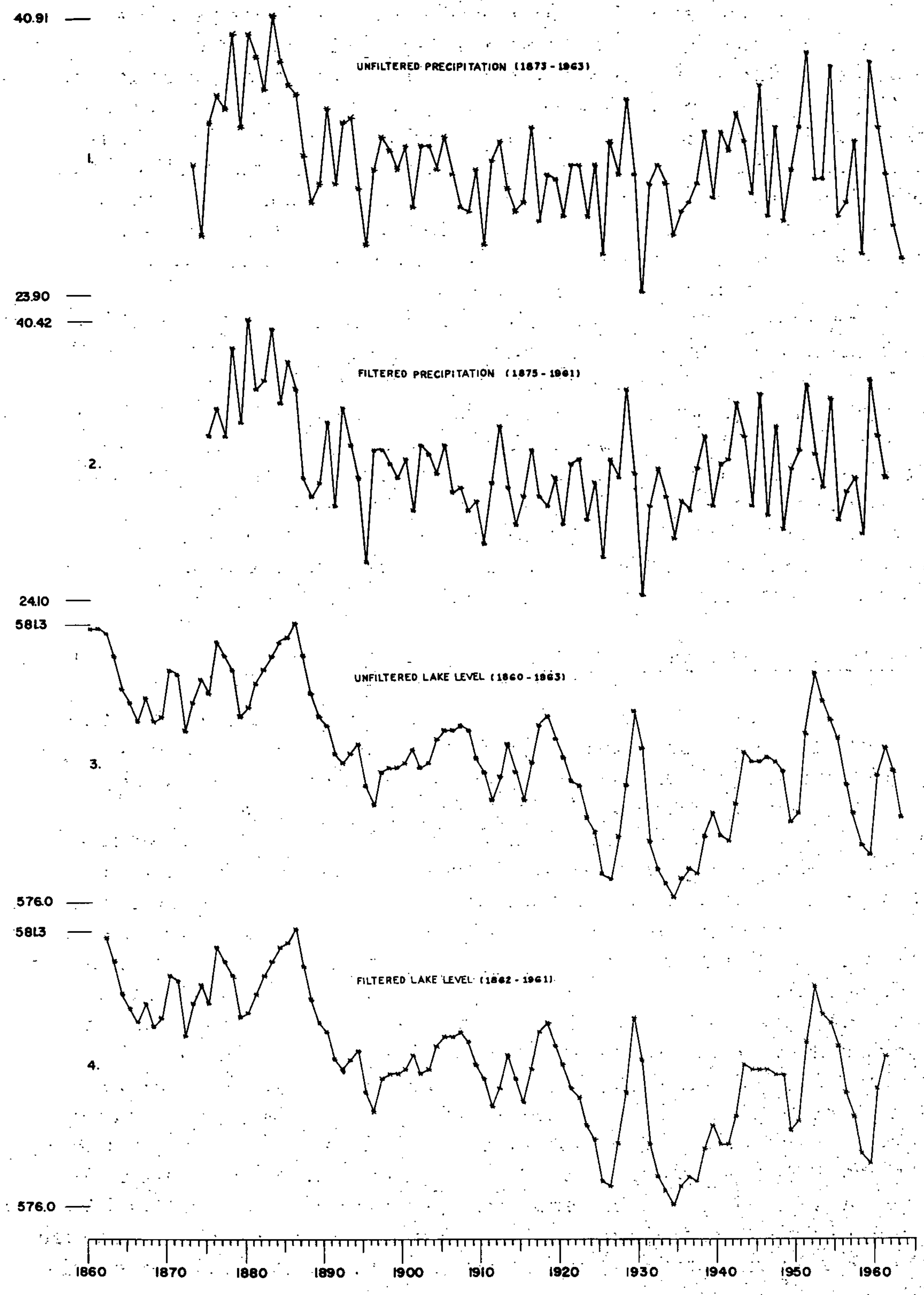


Figure 12
Graph of Mean Annual Lake Level and Total Annual Precipitation
1. - Unfiltered Precipitation (1873 - 1963)
2. - Filtered Precipitation (1875 - 1961)
3. - Unfiltered Lake Level (1860 - 1963)
4. - Filtered Lake Level (1862 - 1961)

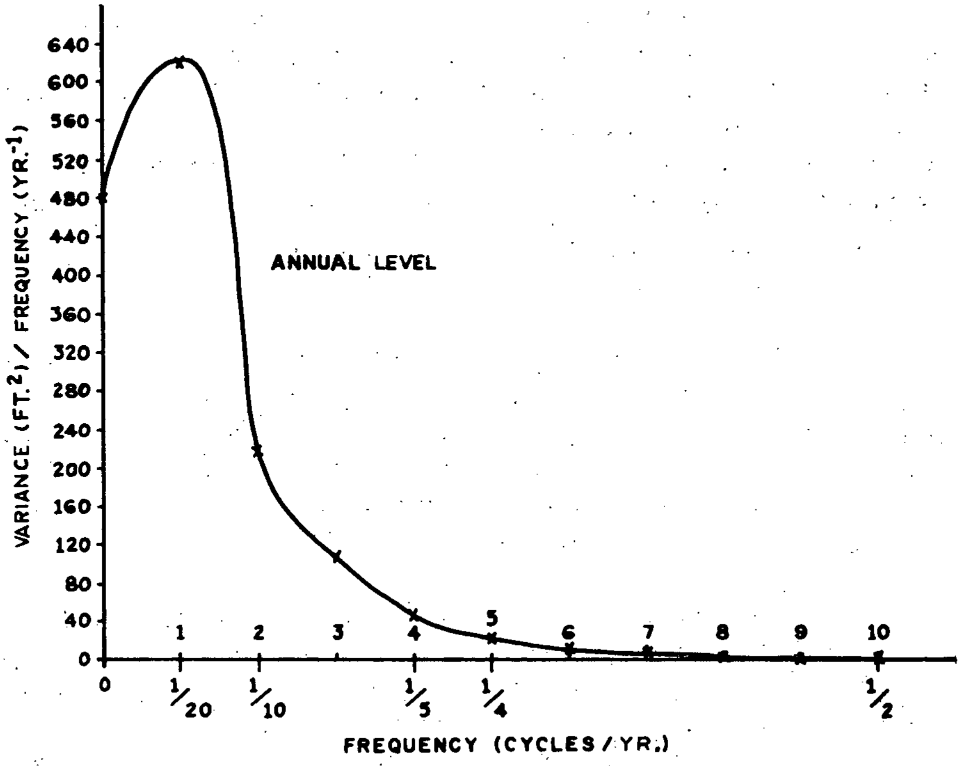


Figure 13
Spectrum (Linear Scale) of Mean Annual Lake Levels

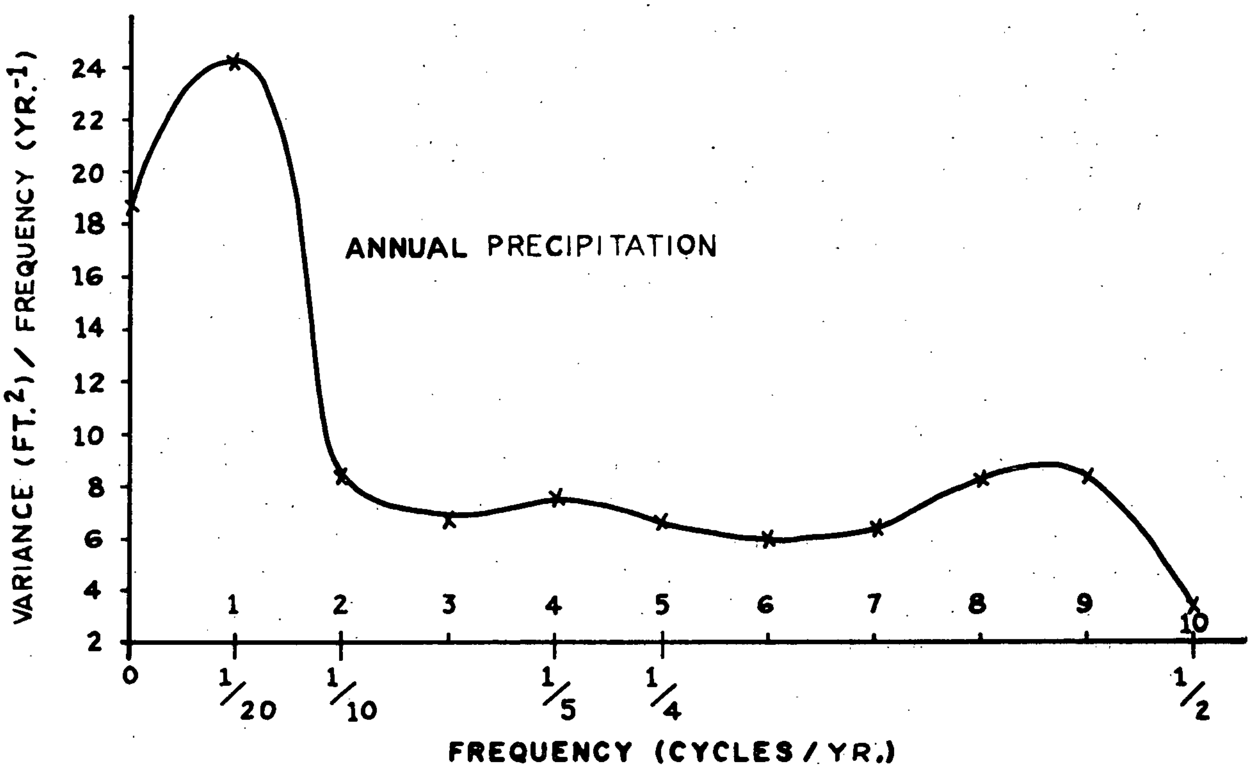


Figure 14
Spectrum (Linear Scale) of Total Annual Basin Precipitation

Figure 13 shows the spectrum for the annual grouped lake levels with a linear scale for variance. With 10 estimates in a frequency range of 0 to 1/2 $years^{-1}$, it provides more resolution at the low end of the spectrum than may be obtained with the monthly values (Fig. 1). Most of the power is in the low end of the spectrum, with a broad peak in the band from 1/40 $years^{-1}$ to 1/13 $years^{-1}$. The power then falls off rapidly with increasing frequency.

The spectrum of the total annual precipitation in Fig. 14, which covers the same frequency range as that for the annual lake levels, also displays a peak in the band from 1 cycle in 40 to 1 in 13 years. However, the power does not fall quite so rapidly with increasing frequency, and there is a smaller broad maximum at 9/20 years, i.e., about 1 cycle in 26 months.

The coherence spectrum between annual mean lake levels and total annual precipitation (Fig. 15) shows very high coherence from zero frequency up to 9 cycles in 20 years, where there is a peak. As mentioned above, this supplements the coherence findings based on monthly data which were limited in their resolution at very low frequency. This high coherence is obvious from the graphs of the mean annual lake levels (Fig. 12) one may note that the level of Lake Michigan/Huron has gradually fallen from two to three feet over the period of the record. There also appears to have been a superimposed fluctuation of period 75 years determined from the peaks at approximately 1878 and 1953. The amplitude of this very long period fluctuation is about 1.25 ft. The plot of the smoothed total annual precipitation in Fig. 12 shows a decline as well, the magnitude of which is estimated at two to three inches since 1873. Gargett (1965) found a 2.6 inch drop in annual precipitation at the Toronto City Observatory over a period of 72 years from 1869 to 1940. This is in spite of the fact that changes in instrumentation should have tended to increase the precipitation amounts recorded. That a similar decline was observed over an area as large as a Great Lake basin increases the significance of the Toronto finding. In addition, around 1885 there was a relatively sharp decline of 5 inches per year in precipitation which was also found by Gargett in her study. Precipitation also appears to have traced out a 75 year fluctuation of amplitude amounting to 3 inches in annual precipitation. The ratio of trend to the amplitude of the 75 year cycle is greater for the lake levels than for the precipitation. For a better look at the long term behaviour of lake levels and precipitation, it would be profitable to apply an extreme low-pass filter to the raw data to get a very smooth curve with only linear trend and very long term fluctuations in it.

## 5. EXAMINATION OF THE DATA AFTER BAND-PASS FILTERING

Since 96.4% of the variance in lake levels is found in the frequency band of 0 to 1 cycle/12 months, it was believed that filtration in the bands:

a) 0 - 1/40 months$^{-1}$ - low pass

b) 1/40 - 1/20 months$^{-1}$ - band-pass

c) 1/20 - 1/14 months$^{-1}$ - band-pass

would represent fairly completely the behaviour of precipitation and lake level, and their relationships, in this frequency range. See Figure 16 for the frequency responses of the filters. The meaning of the high coherence found in the cross spectra is illustrated by correlation computations performed between the filtered series. The resulting correlation coefficients together with the lag used for each frequency band is listed in Table 1.

TABLE 1

| Freq. Band | Lag | Correlation Coeff. |
|---|---|---|
| 0 to 1/40 months$^{-1}$ | 20 months | 0.748 |
| 1/40 - 1/20 months$^{-1}$ | 8 months | 0.620 |
| 1/20 to 1/14 months$^{-1}$ | 6 months | 0.755 |
| 1/14 to 1/10 months$^{-1}$ | 12 months | 0.816 |

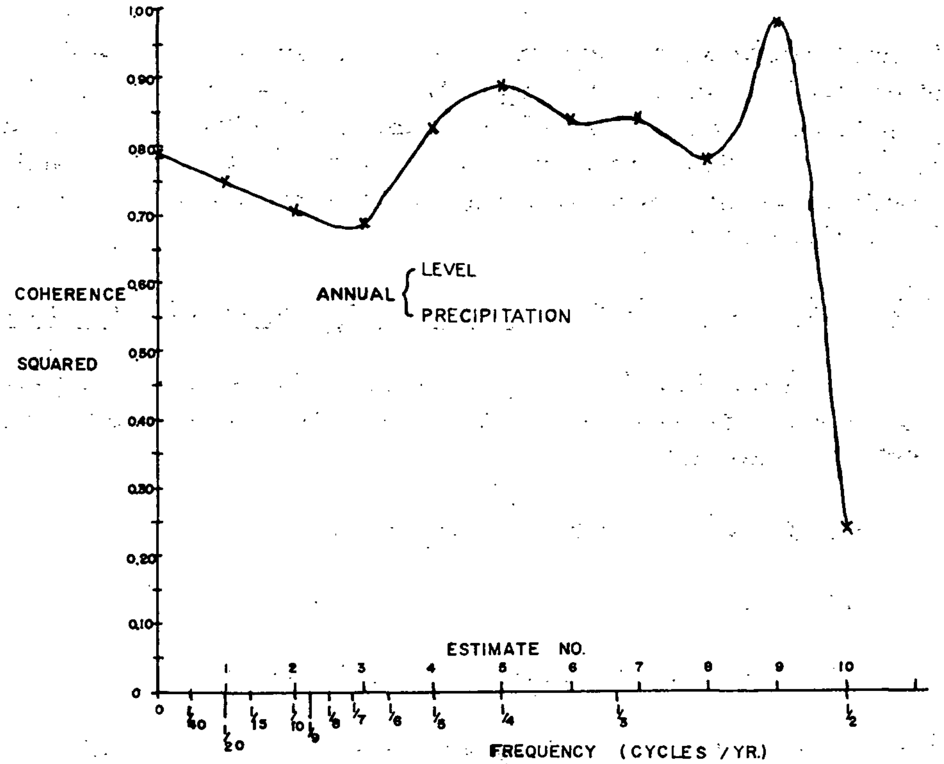


Figure 15
Coherence-Squared Between Mean Annual Lake-Levels and Total Annual Precipitation

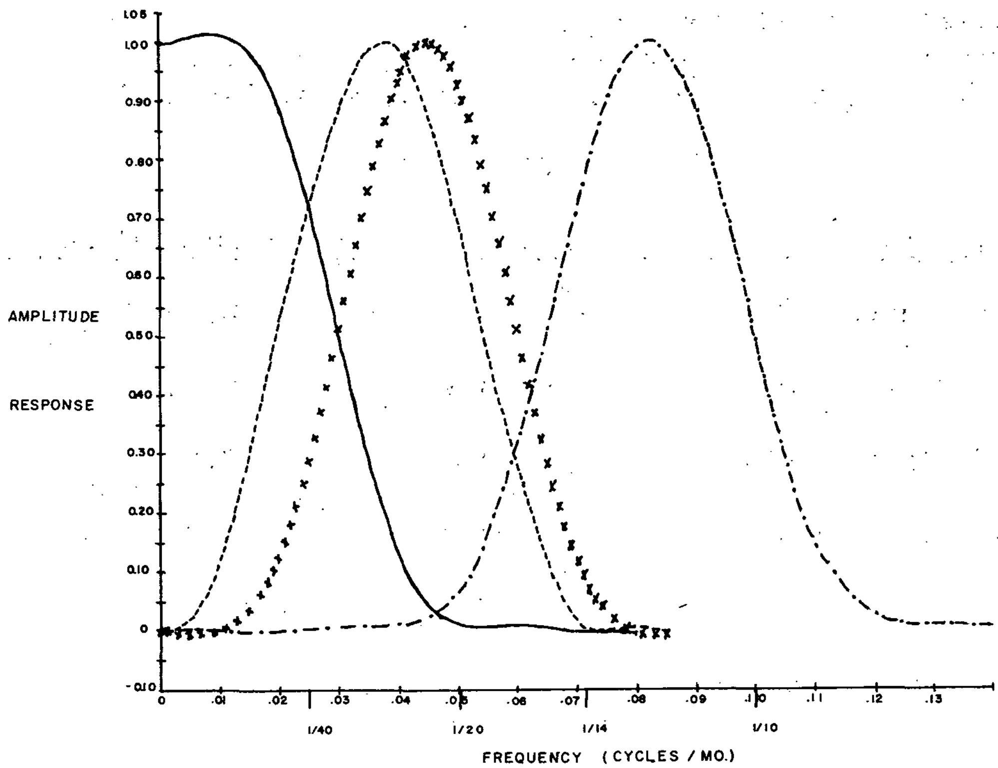


Figure 16
Frequency Response of Bandpass Filters

## Sample Plots of the Filtered Series

Samples of the plot of the output from the low-pass and the band-pass near 1/12 $months^{-1}$ are shown in Figs. 25 and 26. The general downward drift of the lake levels would be apparent if the plot of the whole low pass series were made from the listed data. In the complete low-passed series, the lake levels generally follow the fluctuations of precipitation with a lag of about a quarter of the currently dominant cycle, in accordance with the lag shown by the cross spectrum in Fig. 10. The same lag appeared consistently in the output of the band-pass filters near 1/26 $months^{-1}$, and near 1/16.5 $months^{-1}$. Thus, the average lag calculated in the cross-spectral analysis represents a rather consistent phenomenon rather than the mean of widely scattered values.

## Phase consistency of the Band-passed Series

The histograms for occurrences of fluctuations of specified length (Fig. 17) in both series shows that, with the exception of the band containing the annual cycle, there is no dominant frequency in any band. A histogram was not drawn for 0 - 1/40 $months^{-1}$ band because the number of cycles occurring over the length of record was so small that such a diagram would have little meaning.

For the band from 1/14 to 1/10 $months^{-1}$ the 12-month cycle in lake levels was very sharply-tuned, indicating that there was little phase shifting and that the minima and maxima occur at almost the same time each year. This is illustrated in the sample plot of the band-passed record containing the 12-month cycle in Fig. 26. The minimum occurs in February just before the spring thaw while the maximum is in August. It should be noted that the band-passed series near 1/12 $months^{-1}$ does not show all of the annual cycle but only the 12-month component of it. The total annual cycle contains components of shorter period as mentioned before; a portion of the high-frequency variance found in the spectra of the raw data could probably be accounted for by the shape of the annual cycle. Fig. 24 displays the mean annual cycle in lake level and precipitation, which was calculated by taking the month by month mean of the raw data listed in Appendix A. For precipitation the mean annual cycle is obviously non-sinusoidal, with two maxima (May and September), and a minimum in February. For lake level the mean annual cycle is slightly non-sinusoidal; this is evident when compared with a superimposed sine wave (dashed line) in Fig. 24.

The histogram in Fig. 17 shows that the 12-month precipitation cycle is also sharply-tuned, but to a lesser extent than in the lake levels. This may be partly due to the random fluctuations present in precipitation measurements, and partly due to the fact that each of the many stations that contribute to the mean basin precipitation would have its own individual yearly precipitation cycle. Certain stations, due to high precipitation in their locality will come into prominence in a certain year and will influence the areal-average annual cycle with their individual station cycle, thus causing phase and period shifting in the total basin precipitation.

## Time Variations of Frequency and Amplitude within a Frequency Band -- Comparison between Level and Precipitation

The results of the previous section make it evident that except for the annual cycle, amplitude and phase vary almost continuously over frequency. Nevertheless, there is interest in examining whether the changes in amplitude and phase that occur in one series are associated with corresponding changes in the other.

Table 2 shows the correlations found between:

a) the real - time plots of period for precipitation and levels and

b) the real-time plots of amplitude. The calculations were done for each of the frequency bands.

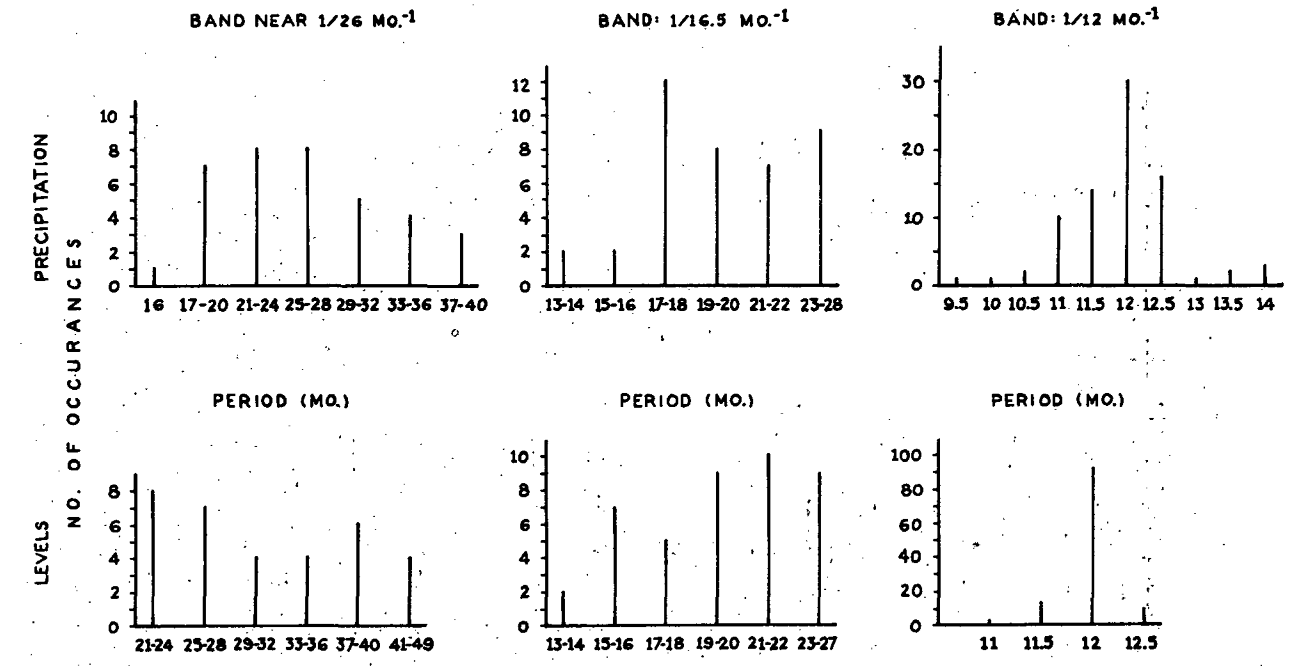


Figure 17
Histogram of Cycles Occuring in Each Bandpass Series
A. 1 Cycle/40 mo. to 1 Cycle/20 mo. Band
B. 1 Cycle/20 mo. to 1 Cycle/14 mo. Band
C. 1 Cycle/14 mo. to 1 Cycle/10 mo. Band

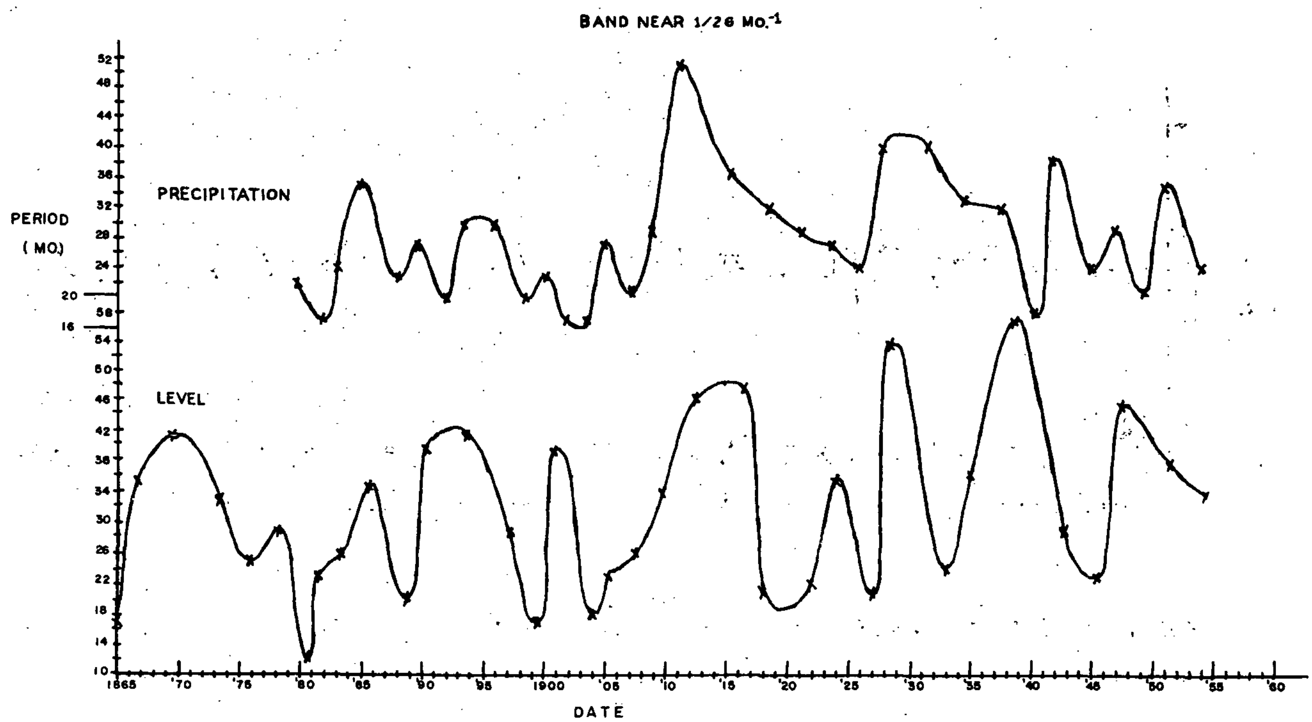


Figure 18
Period of Fluctuation against Real Time for the 1 Cycle/40 mo. to 1 Cycle/20 mo. Band

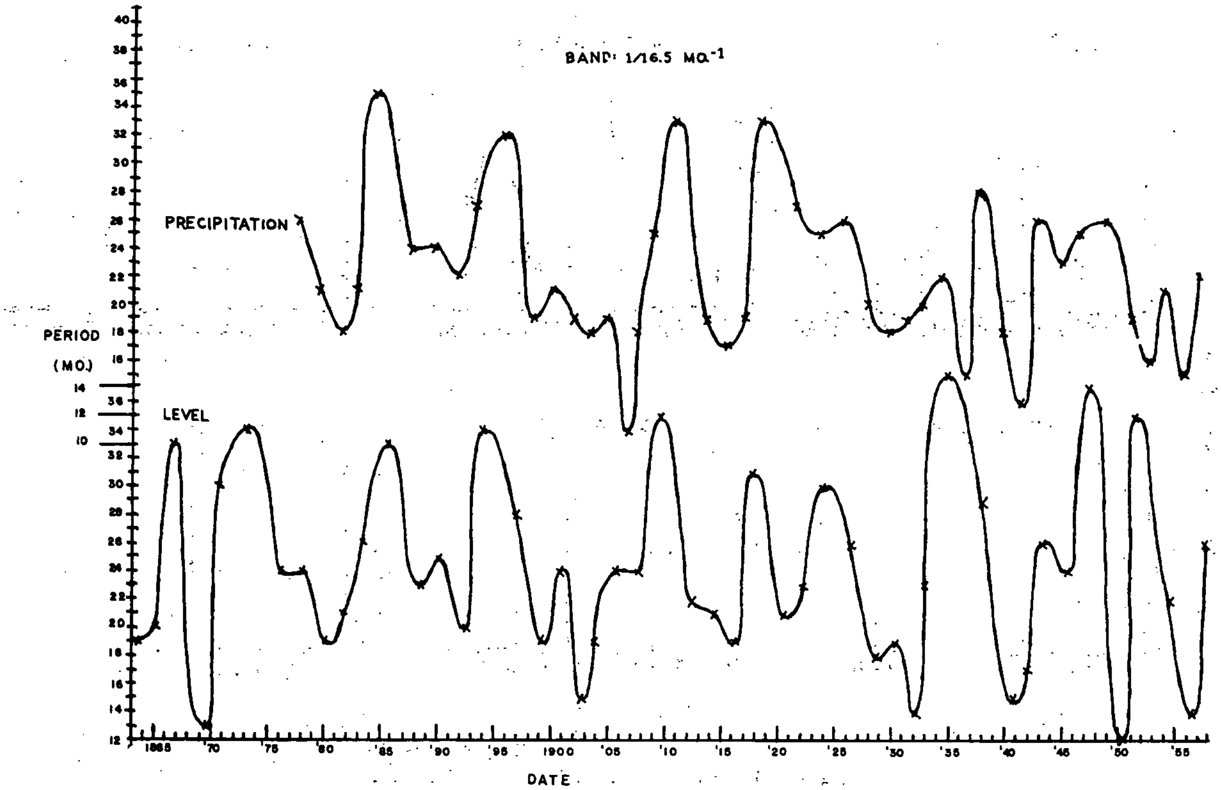


Figure 19
Period of Fluctuation against Real Time for the 1 Cycle/20 mo. to 1 Cycle/14 mo. Band

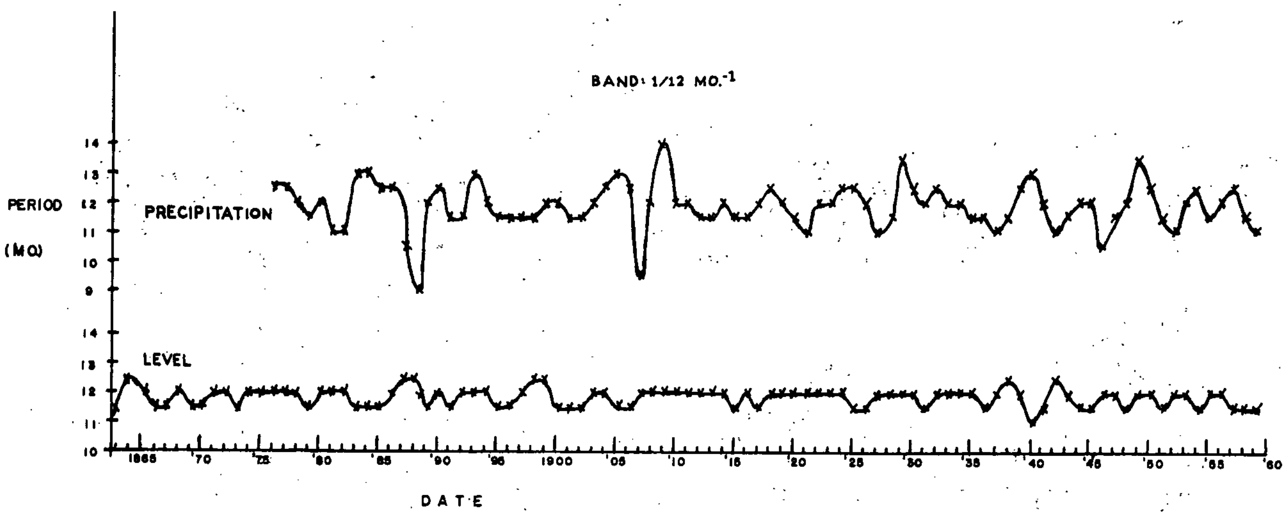


Figure 20
Period of Fluctuation against Real Time for the 1 Cycle/14 mo. to 1 Cycle/10 mo. Band

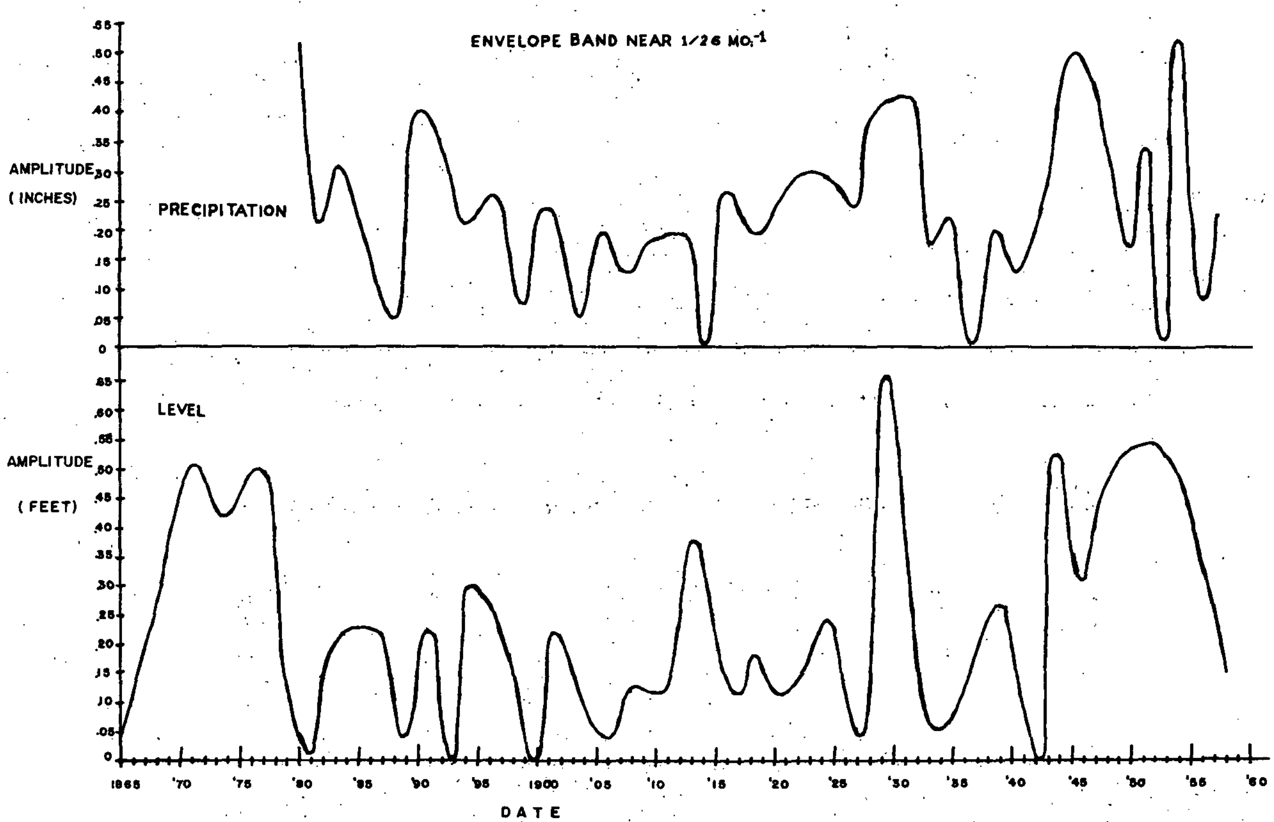


Figure 21
Envelope of Fluctuations of 1 Cycle/40 mo. to 1 Cycle/20 mo. Band

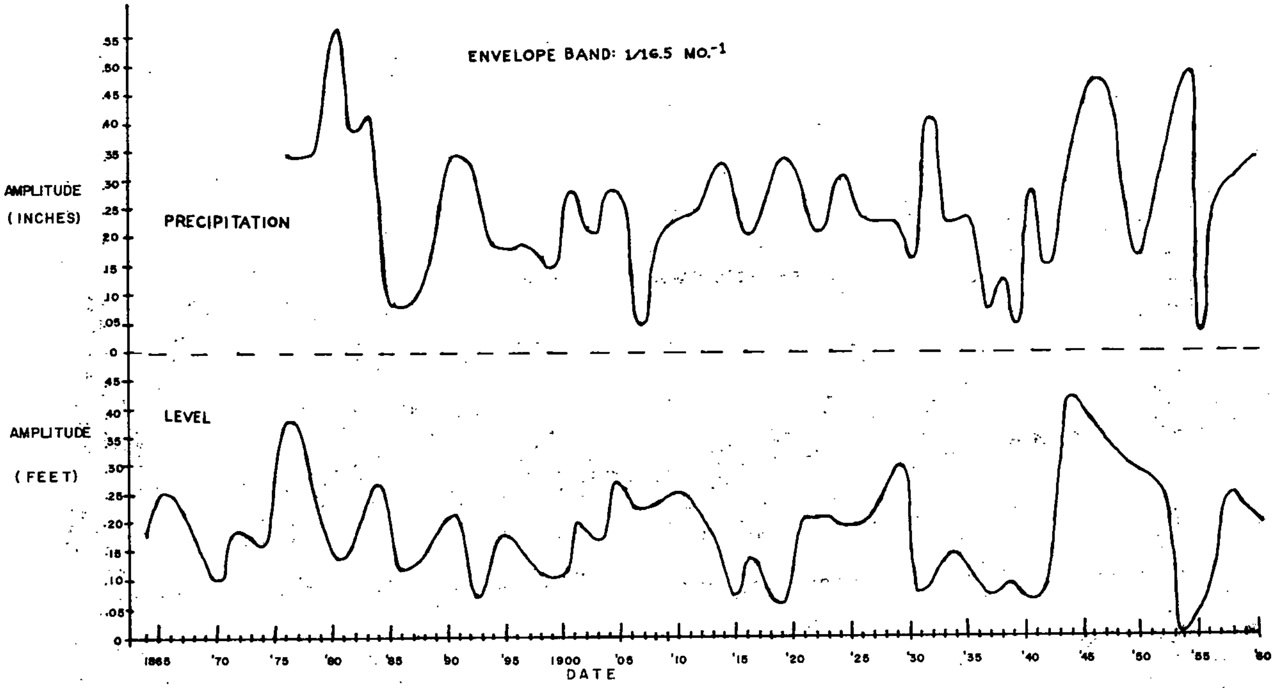


Figure 22
Envelope of Fluctuations of 1 Cycle/20 mo. to 1 Cycle/14 mo. Band

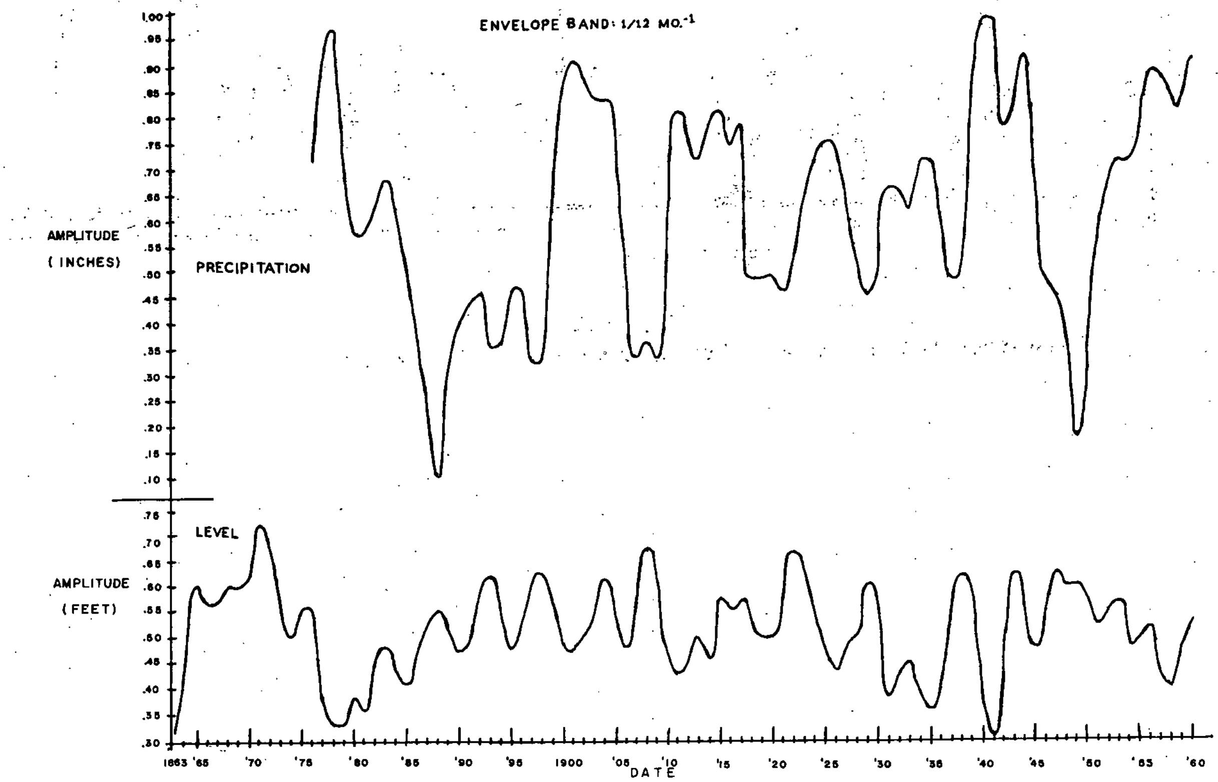


Figure 23
Envelope of Fluctuations of 1 Cycle/14 mo. to 1 Cycle/10 mo. Band

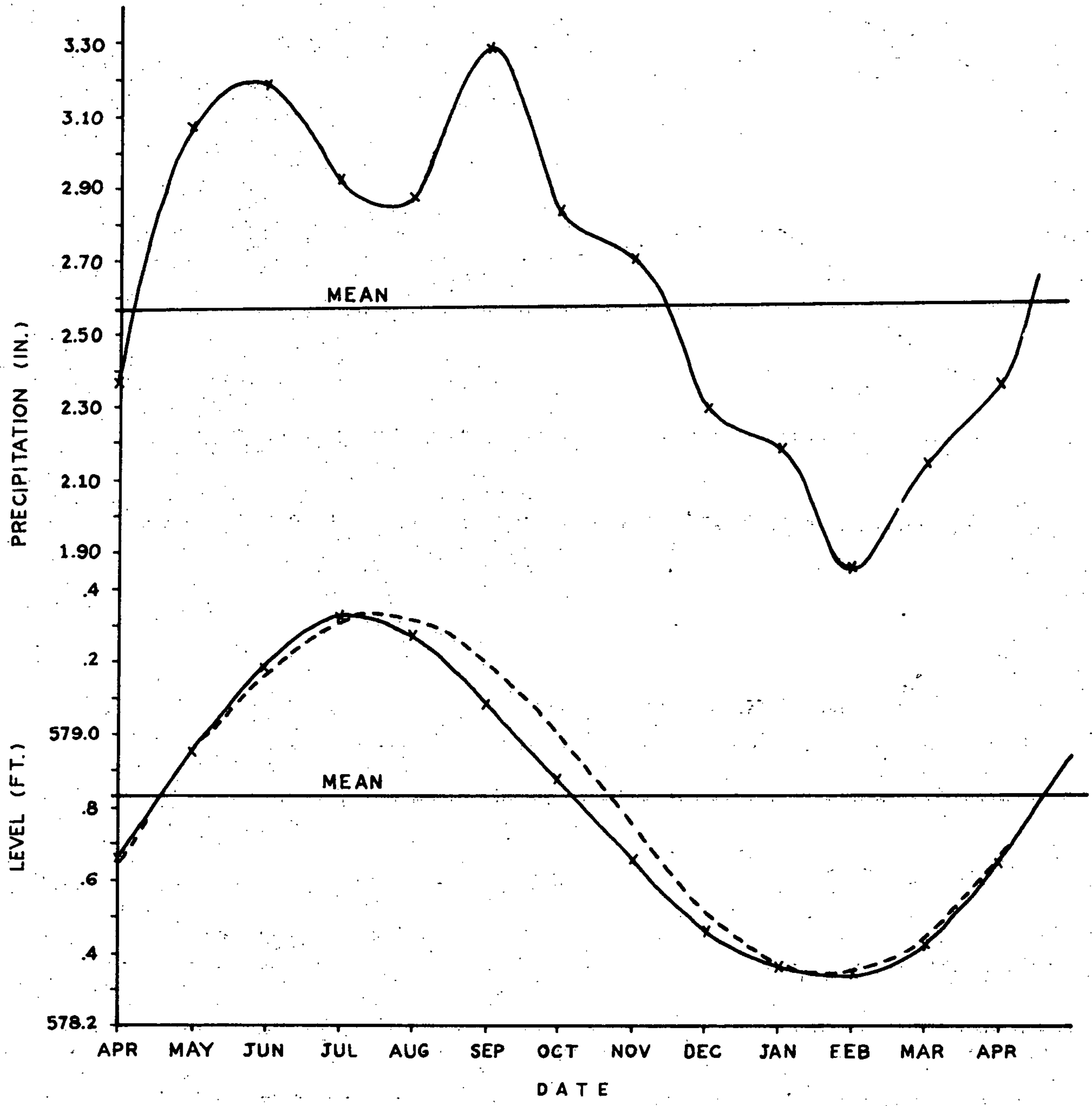


Figure 24
Mean Annual Cycle Calculated from Raw Data
A. Basin Precipitation
B. Lake Level

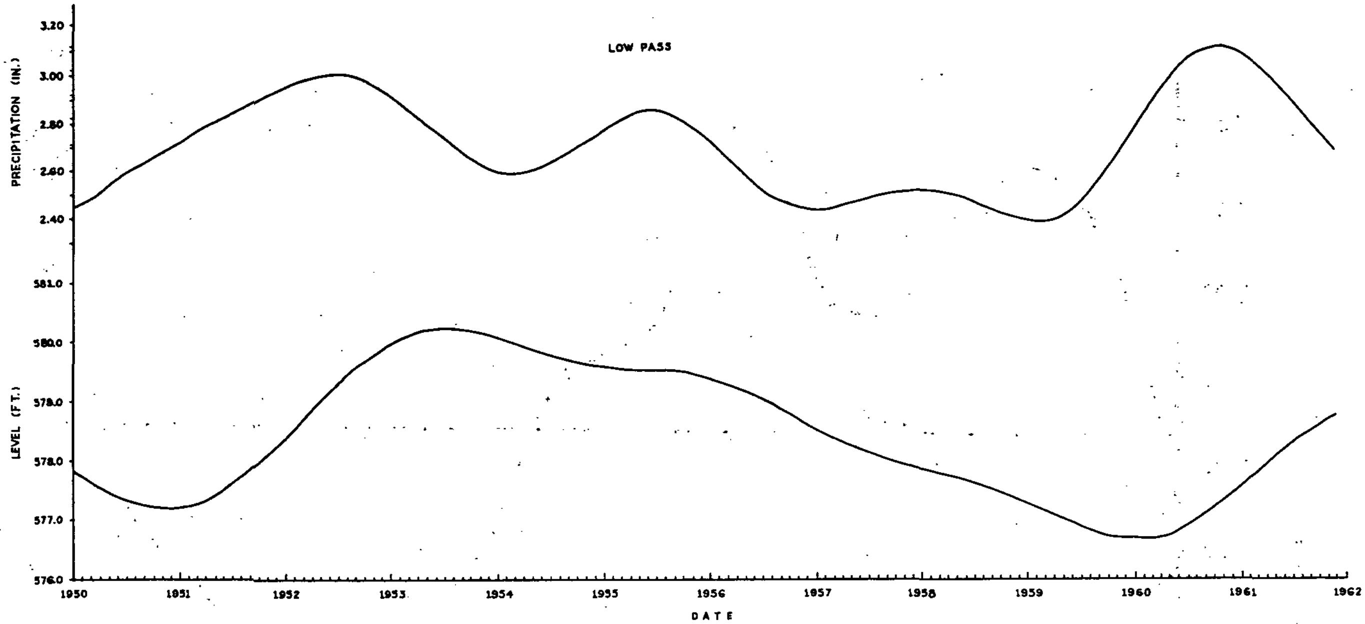


Figure 25
Sample Plot of Low Pass (Zero to 1 Cycle/40 mo.) Series from Appendix B
A. Basin Precipitation
B. Lake Level

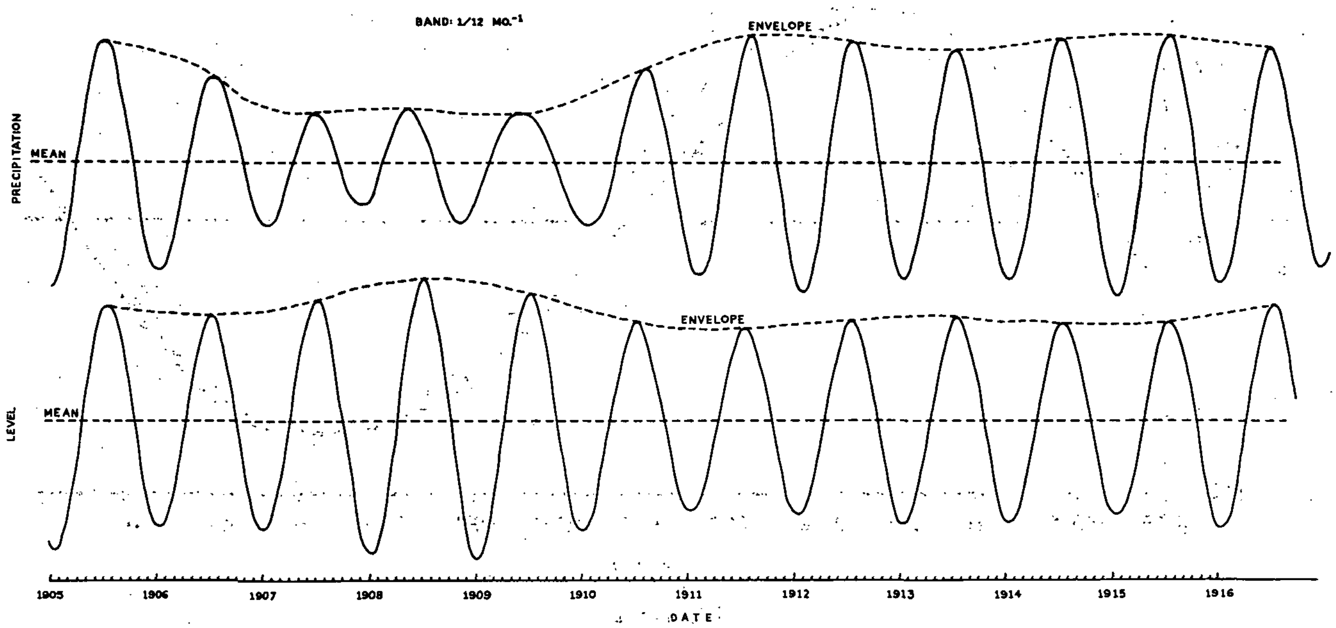


Figure 26
Sample Plot of 1 Cycle/14 mo. to 1 Cycle/10 mo. Bandpass Series from Appendix B
A. Basin Precipitation
B. Lake Level

TABLE 2

| Band Nominal Frequency | Correlation Coefficients for (1) Period | (11) Amplitude |
|---|---|---|
| near 1/26 months$^{-1}$ | 0.37 | 0.10 |
| near 1/16 months$^{-1}$ | 0.43 | 0.07 |
| near 1/12 months$^{-1}$ | 0.07 | -.54 |

Generally these values imply no relationship or only weak relationship, which may at first appear surprising. However, one must note again that with suitable constant lag the correlation of the band-passed series itself is high. This means:

a) there is a tendency to maintain a constant phase angle between the series in the frequency band considered, and

b) that there is sufficient power in both series at times when the average lag angle prevails that there is an important contribution to the sum of lagged cross-products.

Comparison of phase and power parameters as performed above deals with departures from the average made comparable by use of the frequency domain or by normalization of the amplitude. Any relationship that might appear from this procedure would be thus a higher order effect. Its absence could not weaken the overall conclusion: (i) that power exists in these frequency bands in both series and

(ii) that, particularly when it is high, there is a characteristic phase difference.

Correlation is moderate for the periods of the 1/16.5 and 1/26 months$^{-1}$ bands and virtually absent for the annual cycle. For amplitude the reverse situation obtains.

Dealing first with period, note that the definition of period depends upon the application of arbitrary criteria. In a filtered series, for instance, how small must a fluctuation be to be disregarded, if its 'period' is markedly different from the average? That wide limits were used is evident from the histogram of periods. In spite of this in the bands lacking any obvious forcing mechanism (1/16.5 and 1/26 $\text{months}^{-1}$) there remained a noticeable correlation of the periods. Here the longer fluctuations carry greater weight in the period correlation than do the shorter. Since the longer ones tend also to have larger amplitude (where a real relation exists), the correlation in period departures is understandable. However, in the annual frequency band a deviation in the time of annual maximum or minimum of precipitation estimated over the basin would not be expected to have a simple effect on the timing of the maximum or minimum of lake level. This is apparent in view of the more complicated phase relationship in the annual cycle discussed in section 4. In particular a deviation in the phase of basin precipitation would not necessarily be accompanied by a corresponding shift in the other major forcing functions of the annual lake level cycle -- namely evaporation and spring run in.

Dealing, secondly, with amplitude, the problem is to interpret the negative correlation observed for the annual cycle (with forcing present) and the lack of correlation in the bands where forcing is absent. Again one notes that the definition of amplitude involves arbitrary criteria which are similar to those for the definition of phase; for example, how much does a fluctuation in the 1/16.5 and 1/26 $\text{months}^{-1}$ frequency bands have to depart in period from the average in order that a crest in the series be disregarded in drawing the envelope? Duration alone is the criterion, and small fluctuations have equal weight with large in determining the envelope; thus, a weak relationship results. For the annual cycle there is no such arbitrariness since there is always a clearly definable fluctuation of large amplitude in every 9-15 months. Thus, the difference in the degree of relationship observed is understandable.

There remains the negative sign for the correlation of variations in amplitude in the annual cycle. To interpret this finding one must recall that the amplitude of the annual cycle represents qualitatively only the difference between summer and winter precipitation, or between summer and winter lake level. Thus, when the precipitation is relatively steady throughout the year levels have their largest fluctuation; when there is an abnormal summer winter variation in precipitation the lake levels are relatively constant. The steady precipitation case corresponds to a relatively high accumulation of snow and a relatively high peak in the early summer maximum of level resulting therefrom. Conversely, a large summer-winter difference represents a reduction in the spring run-in contribution to the summer maximum in level and a corresponding reduction in the amplitude of the annual cycle.

## 6. IMPLICATIONS FOR FORECASTING LEVELS FROM PRECIPITATION

This study makes it clear that most of the information concerning levels for this lake system is contained in the record of basin precipitation. The phase lag of approximately one quarter cycle in any frequency band (except for very low frequency and the annual cycle) has the following implication. Future lake level changes with steadily decreasing detail appropriate to increasing time scales, are mostly decided by the precipitation up to the present; the quarter cycle into the future of any time scale provides the constraint on the variation of the next smaller time scale. Thus, for example, the end of a well-marked positive half cycle in the range 20 to 40 months must be followed almost inevitably by an overall decline in levels lasting about 5 to 10 months. Smaller times-scale changes will tend to remain within the framework set by the larger time scale. An attempt to work out a scheme, based on this principle, for the forecast of the meteorological-hydrological component of level changes will encounter two main problems. Firstly, there is the difficulty that in any time scale one may come no closer to the present than about one time unit of perhaps half a time unit. For instance, on the time scale of a year and somewhat longer one may only know at best the phase and amplitude of an oscillation applicable at a point in time 6 months ago. Thus, to specify the present on a range of time scales involves an extrapolation of at least half a time unit on each time scale. The possibility of doing this successfully on the average requires special investigation but the relative continuity of features of the band-passed series give some basis for optimism. The second difficulty is that there are clearly long term changes in lake level which have nothing to do with long term changes in precipitation. These include the effects of diversions, changes in the capacity of the outflow channels, and geological changes in attitude. The effects of these one may hope to estimate, and to superimpose them upon a forecast based on precipitation alone.

## 7. SUMMARY AND CONCLUSIONS

The level of Lake Huron-Michigan over the last 100 years has been examined in relation to the record of basin precipitation. The spectra of both, and the cross-spectrum of coherence and phase have been estimated; the results of applying statistical band-pass filters in frequencies of maximum relationship were studied, and the potential discussed for a forecast scheme based on precipitation up to the present. The main findings are:

1) 96.3% of the variance of the lake level is in frequencies up to and including the annual cycle. The annual cycle accounts for only 6.4% of the variance.

2) Fluctuations in level lag behind those of precipitation by a nearly constant angle of about 100 degrees for all frequency bands except that of the annual cycle and between 0 and 1/18 $\text{years}^{-1}$.

3) The relationship is high on all time scales longer than once in 6 months; for these time scales, an average of 70% of the variance of lake level is to be explained by variation in precipitation. This corresponds to a correlation coefficient of 0.84.

4) Both level and precipitation records show downward linear trend as well as a fluctuation of approximately 75 years' duration. The level of the lake has dropped about 3 ft.

in 100 years and precipitation has declined about 2-3 inches in annual precipitation. The 75 year fluctuation has an amplitude of about 1 1/4 ft. in lake level and about 3 inches in precipitation.

5) The high information content of precipitation for future lake levels would justify experiments to attempt the development of a forecast procedure based solely on the precipitation record up to the present.

## 8. ACKNOWLEDGEMENTS

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APPROVED,

J. R. H. Noble,
Director,
Meteorological Branch.

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