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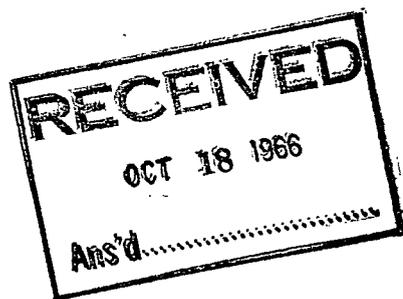
**DEPARTMENT OF TRANSPORT  
METEOROLOGICAL BRANCH**

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**EXTREME VALUES OF THE TERMS  
NEGLECTED IN THE VORTICITY EQUATION**

BY

JOHN CAMPBELL AND DAVID DAVIES



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ABSTRACT

For each of three synoptic situations, all the terms in the vorticity equation are computed at 1000, 850, 700, and 500 mb. When the results based on diagnostic vertical motion fields are studied in some detail, they show that the terms neglected in the Central Analysis Office baroclinic model attain their maximum magnitude in the neighbourhood of developing disturbances. In these regions, especially at 1000 mb, their sum may approach or exceed that of the terms retained.

VALEURS EXTRÊMES DES TERMES NÉGLIGÉS  
DANS L'ÉQUATION DE TOURBILLON

par

John Campbell  
and  
David Davies

RÉSUMÉ

Pour chacune des trois situations synoptiques, tous les termes dans l'équation de tourbillon sont calculés à 1,000, 850, 700 et 500 mb. Lorsque les résultats basés sur les champs de mouvement vertical analysés sont étudiés tant soit peu en détail, ils indiquent que les termes dont ne tient pas compte le modèle barocline du Service central d'analyse atteignent leurs grandeurs maximums dans le voisinage des perturbations en voie de formation. Dans ces régions, particulièrement à 1,000 mb, leur somme peut approcher des termes retenus ou même les dépasser.

EXTREME VALUES OF THE TERMS NEGLECTED  
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1. Introduction

Most numerical weather prediction models presently in use are based on the vorticity equation. This approach, by effectively filtering out sound and gravity waves, permits numerical integrations to be performed using time steps of the order of one hour.

In practice, several of the non-linear terms appearing in the complete vorticity equation are awkward to handle. They are, therefore, dropped completely in the baroclinic model used operationally at the Central Analysis Office (1). Although to some extent, this is a practical necessity at the present time, because of computing power considerations, the justification usually given is that, on the average, the sum of the dropped terms is an order of magnitude smaller than that of the retained terms. A subjective evaluation of the Central Analysis Office operational baroclinic forecasts indicates that there are deficiencies in the handling of short waves. It is, therefore, pertinent to inquire whether or not improved forecasts would be expected if extra terms of the vorticity equation were included in the Central Analysis Office model.

When discussing simplifications of the vorticity equations, most previous authors have confined their remarks about the relative magnitudes of dropped and retained terms to average conditions over the whole grid. No particular attention has usually been paid to extreme conditions; although Schaefer (2) recently did a study of the vorticity budget of a selected cold low. The purpose of the investigation reported here, was to compute the terms neglected in the Central Analysis Office baroclinic model - in the middle and lower troposphere for three synoptic situations - with a view to examining their extreme values rather than their average values as has usually been done in the past. By this means, it was hoped to obtain some indication of the probable worth of including extra terms in the Central Analysis Office model.

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## 2. The Vorticity Equation

The complete form of the vorticity equation is:

$$\frac{\partial Q}{\partial t} = -\underline{v}_H \cdot \nabla_H Q - \omega \frac{\partial Q}{\partial p} - \hat{\underline{k}} \cdot \nabla_H \omega \times \frac{\partial \underline{v}_H}{\partial p} + Q \frac{\partial \omega}{\partial p} \quad (2.1)$$

where

$\hat{\underline{i}}$ ,  $\hat{\underline{j}}$ , and  $\hat{\underline{k}}$  are unit vectors in the x, y and z directions:

$\underline{v}_H$  is the horizontal wind

$$\nabla_H \equiv \hat{\underline{i}} \frac{\partial}{\partial x} + \hat{\underline{j}} \frac{\partial}{\partial y};$$

$$\omega \equiv \frac{dp}{dt};$$

$$f \equiv 2 \Omega \sin \phi;$$

$$Q = \hat{\underline{k}} \cdot \nabla_H \times \underline{v}_H + f, \text{ the absolute vorticity;}$$

$\underline{\Omega}$  is the angular velocity of the Earth;

and  $\phi$  is the latitude.

It is convenient to divide the horizontal wind into a rotational component,  $\underline{v}_H^r$  and a divergent component,  $\underline{v}_H^d$ . The rotational part of the wind can then be described by a scalar stream function ( $\psi$ ) field:

$$\underline{v}_H^r \equiv \hat{\underline{k}} \times \frac{g}{f_0} \nabla_H \psi \quad (2.2)$$

The divergent part of the wind can similarly be described by a scalar velocity potential function (X) field:

$$\underline{v}_H^d \equiv \frac{g}{f_0} \nabla_H X \quad (2.3)$$

In both cases, the factor  $\frac{g}{f_0}$  is merely inserted for convenience of units, to make  $\psi$  and X measurable in decametres. In the derivation of the

vorticity equation in the above form from the equations of motion, by operating on them with  $\hat{k} \cdot \nabla_H X$ , use has been made of the continuity equation:

$$\frac{\partial \omega}{\partial p} = - \nabla_H \cdot \underline{v}_H \quad (= - \frac{g}{f_0} \nabla_H^2 X). \quad (2.4)$$

The simplified form of the vorticity equation used in the Central Analysis Office baroclinic model is:

$$\frac{\partial Q}{\partial t} = - \underline{v}_H^r \cdot \nabla_H Q + f_0 \frac{\partial \omega}{\partial p} \quad (2.5)$$

where  $f_0$  is the value of  $f$  at  $45^\circ N$ . (See Appendix 1 for the reason for replacing  $Q$  by  $f_0$  in the divergence term.)

### 3. Definitions of "Usual Terms" and "Missing Terms"

For the purposes of the present paper, the "usual terms" are defined to be:

- (a)  $-\underline{v}_H^r \cdot \nabla_H Q$ , the rotational advection term; and
- (b)  $f_0 \frac{\partial \omega}{\partial p}$ , the  $f_0$ -divergence term.

The "missing terms" are defined to be:

- (a)  $-\underline{v}_H^d \cdot \nabla_H Q$ , the divergent advection term;
- (b)  $-\omega \frac{\partial Q}{\partial p}$ , the vertical advection term;
- (c)  $-\hat{k} \cdot \nabla_H \omega \times \frac{\partial \underline{v}_H}{\partial p}$ , the twisting-tilting term, which can be

broken down into two parts due respectively to  $\underline{v}_H^r$  and  $\underline{v}_H^d$ ;

and (d)  $(Q - f_0) \frac{\partial \omega}{\partial p}$ , the  $(Q - f_0)$ -divergence term. The sum of the usual terms plus the sum of the missing terms is defined to be the complete vorticity

tendency. Note that in the present context the complete vorticity tendency refers to  $\frac{\partial \Omega}{\partial t}$  as computed from equation (2.1); it does not refer to the total time derivative  $\frac{d\Omega}{dt}$ .

#### 4. Input Fields

All computations were carried out at six pressure levels, 1000, 850, 700, 500, 300, 200 mb, over the rectangular 504 point grid shown in Fig. 1, which is a 21 x 24 subsection of the regular Central Analysis Office 1709 point octagonal grid. Only two fields,  $\psi$  and  $\omega$ , were required at each pressure level to evaluate the usual and missing terms of the vorticity equation, since  $X$  can be computed from  $\frac{\partial \omega}{\partial p}$  by relaxing the continuity equation (2.4).

Stream function ( $\psi$ ) fields were readily available from an application of the operational balance equation program to the objectively analysed height fields.

Vertical motion ( $\omega$ ) fields were readily available from four sources:

- (i) From a previously programmed version of the diagnostic scheme of Haltiner et al (3) for obtaining vertical motion fields from objectively analysed height and temperature fields, under the assumption that static stability is a function of pressure only.
- (ii) From the first-hour (forward time-step) stream function tendencies given by the Central Analysis Office baroclinic model, by an application of the thermodynamic equation in the form:

$$\frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial p} \right) + \frac{V_H^r}{H} \cdot \nabla_H \left( \frac{\partial \psi}{\partial p} \right) + \frac{\sigma f_0}{g} \omega = 0 \quad (4.1)$$

where  $\sigma \equiv - \frac{1}{p\theta} \frac{\partial \theta}{\partial p}$  is the static stability parameter,

$\theta \equiv T \left( \frac{p_0}{p} \right)^K$  is the potential temperature,

$P_0$  is 1000 mb,

$$K \equiv \frac{R}{C_p}$$

$R$  is the gas constant,

$C_p$  is the specific heat at constant pressure,

$\rho$  is the density,

$T$  is the temperature.

- (iii) From the second-hour (centred time-step) stream function tendencies given by the Central Analysis baroclinic model.
- (iv) From the diagnostic Haltiner program using a variable static stability. It should be noted that the effects of release of latent heat were not included in the computation of any of the  $w$ 's.

The missing and usual terms were computed four times for each of the cases studied, using vertical motion fields from each of these sources in turn. The results obtained using the Haltiner diagnostic vertical velocity, with constant static stability, will be presented and discussed in some detail; although space considerations make it impractical to include charts of the individual terms in this report. The results obtained using the vertical velocities from the three other sources will be discussed briefly, and some mention made of the differences among the four sets of output charts.

## 5. Technical Details

Simple first order finite difference approximations were used to evaluate the X- and Y- derivatives occurring in the usual and missing terms.

For the p-derivatives of all quantities the following formulae were used:

- (i) At the interior levels, the formula given by Haltiner et al (3),

$$\left(\frac{\partial F}{\partial P}\right)_K = \frac{1}{(\Delta P_u + \Delta P_l)} \left\{ \frac{\Delta P_u}{\Delta P_l} F_L - \left(\frac{\Delta P_u}{\Delta P_l} - \frac{\Delta P_L}{\Delta P_u}\right) F_K - \frac{\Delta P_L}{\Delta P_u} F_U \right\} \quad (5.1)$$

where F is any variable and the subfixes K, L, and U refer to centre, lower, and upper respectively; and  $\Delta P_u$  is the separation between the centre and upper pressure levels, and  $\Delta P_l$  is the separation between the lower and centre pressure levels.

(ii) At 1000 mb.

$$\text{if } \left| F_{1000} - F_{850} \right| < \xi \quad (F) \quad (\text{See note at end of section}),$$

$$\left. \frac{\partial F}{\partial P} \right|_{1000} = \frac{F_{1000} - F_{850}}{150}; \quad (5.2a)$$

$$\text{if } (F_{1000} - F_{850})(F_{1000} - F_{700}) < 0,$$

$$\left. \frac{\partial F}{\partial P} \right|_{1000} = \frac{3}{2} \left( \frac{F_{1000} - F_{850}}{150} \right); \quad (5.2b)$$

$$\text{if } (F_{1000} - F_{850})(F_{1000} - F_{700}) \geq 0 \text{ and } \frac{F_{1000} - F_{700}}{F_{1000} - F_{850}} \geq 4,$$

$$\left. \frac{\partial F}{\partial P} \right|_{1000} = \frac{1}{2} \left( \frac{F_{1000} - F_{850}}{150} \right); \quad (5.2c)$$

$$\text{if } (F_{1000} - F_{850})(F_{1000} - F_{700}) \geq 0 \text{ and } \frac{F_{1000} - F_{700}}{F_{1000} - F_{850}} < 4,$$

$$\left. \frac{\partial F}{\partial P} \right|_{1000} = \frac{1}{2} \left\{ 3 - \frac{(F_{1000} - F_{700})}{2(F_{1000} - F_{850})} \right\} \frac{(F_{1000} - F_{850})}{150}. \quad (5.2d)$$

(iii) At 200 mb,

$$\text{if } \left| f_{300} - f_{200} \right| < \xi \quad (F) \quad (\text{See note at end of section}),$$

$$\left. \frac{\partial F}{\partial p} \right|_{200} = \frac{F_{300} - F_{200}}{100}; \quad (5.3a)$$

if  $(F_{300} - F_{200})(F_{500} - F_{200}) < 0,$

$$\left. \frac{\partial F}{\partial p} \right|_{200} = \frac{3}{2} \left( \frac{F_{300} - F_{200}}{100} \right); \quad (5.3b)$$

if  $(F_{300} - F_{200})(F_{500} - F_{200}) \geq 0$  and  $\frac{F_{500} - F_{200}}{F_{300} - F_{200}} \geq 8,$

$$\left. \frac{\partial F}{\partial p} \right|_{200} = \frac{1}{2} \left( \frac{F_{300} - F_{200}}{100} \right); \quad (5.3c)$$

if  $(F_{300} - F_{200})(F_{500} - F_{200}) \geq 0$  and  $\frac{F_{500} - F_{200}}{F_{300} - F_{200}} < 8,$

$$\left. \frac{\partial F}{\partial p} \right|_{200} = \frac{1}{2} \left\{ 3 - \frac{(F_{500} - F_{200})}{(F_{300} - F_{200})} \right\} \frac{(F_{300} - F_{200})}{100}. \quad (5.3d)$$

(Note:  $\epsilon$  (F) is some small positive number, pre-chosen for each particular F. When F is  $\omega$ ,  $\epsilon$  is taken as  $10^{-10}$  mb/hr; when F is  $\psi$ ,  $\epsilon$  is taken as  $10^{-10}$  dkm.)

## 5. Results

Three synoptic situations were studied: 12Z February 24th, 1965; 00Z March 18th, 1965; and 00Z January 8th, 1965. Charts of the height fields at 850 and 500 mb are presented here, together with the 850 and 500-mb charts of the complete vorticity tendency and the sum of the missing terms, both as computed from the diagnostic vertical motions based on constant static stabilities. For each case, four grid-points are chosen at which the missing terms contributed appreciably to the complete vorticity tendency, at least at some of the levels. For these twelve selected points, Tables are given in which the values of all the individual terms are listed separately.

The charts of Figs. 2-7 are those for the case of 12Z February 24th, 1965. The main feature of interest was the disturbance in southern central United States. This low subsequently developed into a major storm as it moved north-eastwards, although the rapid deepening phase did not commence until twelve hours after map time. Two of the selected points, A and B, were taken near the centre of the 850-mb low. The other two points, C and D, were taken in the Atlantic trough near Newfoundland which was approaching its maximum intensity at map time. Tables 1-4 list the values of the individual terms at these points.

The charts of Figs. 8-13 are those for the case of 00Z March 18, 1965. The main feature on the map at 850 mb is the low over the Great Lakes which had been deepening for the previous thirty-six hours, and which continued to deepen for the next twelve hours. All four selected points, E, F, G, and H, were taken in the neighbourhood of this low. Tables 5-8 list the values of the individual terms at these points.

The charts of Figs. 14-19 are those for the case of 00Z January 8th, 1965. They show a typical North American winter situation. Three short waves were moving around the long wave trough centred over the continent. Point I was selected in the short wave trough over the South-western United States, which was not very active because it was moving into the long wave trough position. Points J and K were selected in the intensifying low over the Gulf of St. Lawrence, which entered a rapidly deepening phase twelve hours after map time. The fourth point, L, was chosen in the low south-east of Greenland, which was nearing the end of its deepening phase at 00Z and began filling about twelve hours later. Tables 9-12 list the values of the individual terms at these points.

## 6. Discussion

The charts of Figs 2-19 show that at both 850 and 500 mb the sum of the missing terms does indeed tend to be relatively much smaller in magnitude than the sum of the usual terms. The corresponding series of charts for 1000 and 700 mb, which are not included here, confirm this finding, though marginally so at 1000 mb. When all the charts are considered together, it is evident that the missing terms are relatively rather more important at 1000 mb than they are at 500 mb.

In the vicinity of developing short wave disturbances, however, this general rule breaks down and the magnitude of the sum of the missing terms can approach and even exceed the magnitude of the sum of the usual terms.

An inspection of the computer print-outs of charts of the individual terms reveals that their relative importance varies from level to level. At 500 mb, on the average, the rotational advection term is the dominant one, the  $Q - f_0 \frac{\partial \omega}{\partial p}$  and  $f_0$  - divergence terms the next dominant, and the other terms much smaller. At 1000 mb, on the average, the  $Q - f_0 \frac{\partial \omega}{\partial p}$  and  $f_0$  - divergence terms are the dominant ones, followed by the  $(Q - f_0)$  - divergence term, the rotational advection term, and the divergent advection term, which all appear to be roughly comparable with each other and bigger than the remaining terms. Another interesting general property which shows up in the print-outs of the individual terms is that the portion of the twisting-tilting term arising from the divergent part of the wind is, on the average, an order of magnitude less than the other portion of the twisting-tilting term arising from the rotational part of the wind. These findings about the relative magnitude of terms making up the vorticity equation are similar to those obtained by Schaefer (2).

Tables 1-12 show that the magnitude of the sum of the missing terms exceeds the magnitude of the sum of the usual terms at seven of the twelve points studied at 1000 mb, at six of them at 850 mb, at three at 700 mb, and at one at 500 mb. At many of the twelve points the sum of the missing terms at 1000 mb is an order of magnitude greater than the sum of the missing terms at 500 mb.

The sets of charts and tables for the  $\omega$ 's from the other three sources exhibit similar features. In the case of the diagnostic  $\omega$ 's incorporating variable static stability all the affected terms generally show slightly increased magnitudes, but the actual grid-point values tend to be closely correlated with those for constant static stability. For the two sets of thermodynamic  $\omega$ 's all terms computed from the first hour stream function tendencies are about the same magnitude as, and closely correlated to, their counterparts computed from the second hour tendencies. A comparison between terms computed from the diagnostic  $\omega$ 's with constant stability and the corresponding ones computed from the thermodynamic  $\omega$ 's show varying degrees of correlation at the different levels. At 700 and 500 mb the correlation is quite good, although in both cases the thermodynamic  $\omega$  terms tend to be slightly bigger than the diagnostic  $\omega$  terms.

At 850 and 1000 mb the correlation is poor or very poor, although again, the thermodynamic  $\omega$  terms tend to be larger than their diagnostic counterparts.

There are three reasons for these low-level discrepancies. Firstly, the 850-mb thermodynamic vertical motion fields were based partly on the values for the 1000-mb stream function tendencies given by the Central Analysis Office baroclinic model, and these are known to be in need of improvement. Secondly, although in both approaches the mountains and friction vertical motion at the ground was computed from the formula given by Cressman (4), in the diagnostic case the interpolated terrain-level winds are geostrophic, whereas, in the thermodynamic case they are obtained from the stream functions. Thirdly, the problem of assigning vertical motions inside mountains was handled slightly differently in the two approaches. In the diagnostic approach, the value of the mountains and friction vertical motion at the ground was simply used for all levels below the height of the terrain. For instance, if at the (i, j) grid-point the mountain pressure-height was 600 mb and the mountains and friction vertical motion at the ground was  $\omega_g^{ij}$ , then the following assumptions were made:

$$\omega_{700}^{ij} = \omega_{850}^{ij} = \omega_{1000}^{ij} = \omega_g^{ij} \quad (6.1)$$

In the thermodynamic approach vertical motions due to mountains and friction had to be computed separately, since lower boundary effects had not then been incorporated into the baroclinic model, and these had to be added to the values computed from the stream function tendencies to give the total vertical motion fields. Above the terrain, the mountains and friction contribution to the vertical motion was assumed to drop off linearly from the computed value at ground level to zero at 200 mb. Instead of the above section, again for a mountain pressure-height at 600 mb at the (i, j) grid-point, a continuation of the linear increase with pressure was used below ground level:

$$\frac{(600-200)}{(700-200)} \omega_{700}^{ij} = \frac{(600-200)}{(850-200)} \omega_{850}^{ij} = \frac{(600-200)}{(1000-200)} \omega_{1000}^{ij} = \omega_g^{ij} \quad (6.2)$$

At 1000 and 850 mb in mountainous regions these seemingly trivial differences in the handling of the lower boundary condition will play havoc with the correlations between the results from the two approaches.

The differences among the four sets of results would seem to indicate that the lower boundary complications do tend to make the 1000 and 850-mb computations unreliable. However, since the comments made at the beginning of this discussion apply just as forcibly in low terrain and ocean areas as they do elsewhere, and since they are just as appropriate for the diagnostic  $\omega$  results as they are for the others, their qualitative validity still stands even if their quantitative validity does not.

It might be argued that it still has not been proven conclusively that the missing terms are important in the vicinity of short wave disturbances, because approximations were made in the computation of both the diagnostic and the thermodynamic vertical motions, and conceivably these could lead to gross errors in the computation of the missing terms. This is unlikely to be the case because from a study of the results, and also from physical considerations, it appears evident that for most of the missing terms large magnitudes are closely associated with large magnitudes of  $\frac{\partial \omega}{\partial p}$ , and so necessarily with large magnitudes of mid-troposphere vertical motions. In general, particularly in the neighbourhood of developing short waves, the inclusion of extra terms in either the diagnostic  $\omega$ - equation or the thermodynamic equation would serve to boost mid-troposphere vertical motions, not to dampen them; and most certainly not to dampen them by an order of magnitude or more, as would be required to make the missing terms insignificant.

The discussion so far would seem to indicate that beneficial effects would be expected if the missing terms could be incorporated into the Central Analysis Office baroclinic model. These effects would probably show up in the form of an increased linkage between the levels in the neighbourhood of short waves; and this is precisely the type of improvement which seems to be needed.

There remains the discussion of partial inclusion of the missing terms. If only the vertical advection and twisting-tilting terms were neglected, then some simple computations based on Tables 1-12 lead to some interesting figures. At 1000 mb the number of points at which the magnitude of the neglected terms exceeds that of the retained terms drops from seven right down to nil; at 850 mb from six to one; and at 700 mb from three to one. But at 500 mb the number increases from one to two. As a closer inspection of the computer output charts of the individual terms shows, these figures are quite typical of the type of

thing encountered at grid-points located in and around short waves. It would, therefore, seem likely that the inclusion of the divergent advection term and the full  $(Q - f_0)$ -divergence term into the Central Analysis Office model would lead to improved short-wave forecasts.

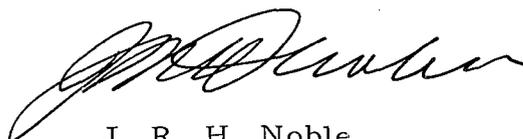
However, the experience elsewhere, for instance, at the National Meteorological Centre in Washington (5), has been that the inclusion of extra terms into the vorticity equation only leads to limited improvements in the quality of the forecasts. One possible reason for this is that there is a numerical problem in computing the missing terms with sufficient accuracy. Certainly, the results of the present study do indicate that the missing terms in the lower levels are moderately sensitive to changes in the vertical motion fields. Another possibility is that the problem of scale becomes important for the missing terms. It is well-known that interactions between superimposed flows of different wave numbers takes place via the non-linear terms in the vorticity equation. It could be that in the real atmosphere some highly significant exchanges take place between synoptic scale disturbances and smaller scale phenomena of dimensions of less than one grid-length.

As has been demonstrated by Danard (6)(7), it is also very important to include the effects of release of latent heat into baroclinic models. In areas of development  $\omega$  can be increased by as much as a factor of 2 or 3 by the addition of a latent heat term. Thus, it is probable that the divergence, vertical advection and tilting terms are even more important than indicated by this investigation, at least in some areas and at some levels.

## 7. Conclusions

The Central Analysis Office baroclinic model is based on a simplified form of the vorticity equation. Three case studies indicate that the sum of the terms neglected, though unimportant in most regions, can sometimes be larger in magnitude than the sum of the terms retained. This happens in the neighbourhood of developing short wave disturbances, and is particularly apparent at 1000 mb. In these areas, a less simplified version of the vorticity equation, in which the divergent advection and full divergence terms are retained, would appear to give low level vorticity tendencies much closer to reality. However, in practice there may be numerical problems to be overcome before the extra terms could be computed with sufficient accuracy to make their inclusion worthwhile.

APPROVED,



J. R. H. Noble,  
Director,  
Meteorological Branch.

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APPENDIX 1

The reason for replacing  $Q$  by  $f_0$  in the divergence term of equation (2.5) was first given by Wiin-Nielson (8). Given any horizontal vector,  $\underline{A}_H$ , the integral of  $\nabla_H \cdot \underline{A}_H$  over any area,  $\mathcal{S}$ , of a pressure surface may be transformed into a line integral, thus:

$$\mathcal{S} \iint \nabla_H \cdot \underline{A}_H dS = \oint_{\mathcal{S}} A_n dl; \quad (9.1)$$

where  $dS$  is a (scalar) areal element of  $\mathcal{S}$ ,  $dl$  is an element of the boundary line enclosing  $\mathcal{S}$ , and  $A_n$  is the component of  $\underline{A}_H$  on the boundary which is perpendicular to  $dl$  and pointing outwards from  $\mathcal{S}$ . If  $\mathcal{S}$  is a closed area,  $\mathcal{S}^*$ , in which case it would be a complete pressure surface surrounding the Earth, then:

$$\oint_{\mathcal{S}^*} A_n dl = 0; \quad (9.2)$$

and so by (9.1):

$$\oint_{\mathcal{S}^*} \nabla_H \cdot \underline{A}_H dS = 0. \quad (9.3)$$

Writing:

$$\underline{q} = Q \underline{V}_H + \omega \frac{\partial \underline{V}_H \times \underline{R}}{\partial p}, \quad (9.4)$$

the complete vorticity equation (2.1) becomes:

$$\frac{\partial Q}{\partial t} + \nabla_H \cdot \underline{q} = 0 \quad (9.5)$$

Examining the integral of  $\frac{\partial Q}{\partial t}$  over a closed pressure surface  $\mathcal{S}^*$  surrounding the Earth, using equation (9.5):

$$\begin{aligned} \oint_{\mathcal{S}^*} \frac{\partial Q}{\partial t} dS &= - \oint_{\mathcal{S}^*} \nabla_H \cdot \underline{q} dS \\ &= 0 \end{aligned} \quad (9.6)$$

by (9.3).

Replacing (9.5) with (2.5), it can be seen that the same property is preserved:

$$\begin{aligned}
 \oint_{S^*} \frac{\partial Q}{\partial t} dS &= - \oint_{S^*} \left\{ \frac{v^r}{-H} \cdot \nabla_H Q - f_0 \frac{\partial \omega}{\partial p} \right\} dS & (9.7) \\
 &= - \oint_{S^*} \nabla_H \cdot \left( Q \frac{v^r}{-H} + f_0 \frac{v^d}{-H} \right) dS \\
 &= 0 & \text{by (9.3).}
 \end{aligned}$$

Note that the replacement of  $Q$  by  $f_0$  is essential for this condition to be satisfied by (2.5). Some controversy surrounds the usefulness of this substitution. It seems that it is advantageous for long period forecasts and disadvantageous for short period forecasts.

If instead of (2.5) a simplification of the vorticity equation is used in which the divergent advection term is retained, it is no longer advantageous to substitute  $f_0$  for  $Q$ , even for long range forecasts;

$$\frac{\partial Q}{\partial t} + \frac{v^r}{-H} \cdot \nabla_H Q = Q \frac{\partial \omega}{\partial p} \quad (9.8)$$

In this case:

$$\begin{aligned}
 \oint_{S^*} \frac{\partial Q}{\partial t} dS &= - \oint_{S^*} \left( \frac{v^r}{-H} \cdot \nabla_H Q - Q \frac{\partial \omega}{\partial p} \right) dS & (9.9) \\
 &= - \oint_{S^*} \nabla_H \cdot \left( Q \frac{v^r}{-H} \right) dS \\
 &= 0 & \text{by (9.3).}
 \end{aligned}$$

TABLE 1

Values of the Individual Terms in the Vorticity Equation as Computed at Point A for the Case of 12Z, February 24th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\omega$                                 | -0.9    | -8.1   | -8.3   | -4.8   |
| $\frac{\partial \omega}{\partial p}$     | 600     | 250    | -68    | -166   |
| $Q$                                      | 5980    | 5460   | 4290   | 3250   |
| $f$                                      | 2990    | 2990   | 2990   | 2990   |
| vertical advection                       | 2       | 46     | 56     | 3      |
| divergent advection                      | 39      | 24     | -4     | 4      |
| rotational advection                     | -92     | -19    | 21     | -127   |
| rotational twisting                      | 0       | -24    | -5     | 5      |
| divergent twisting                       | -1      | 1      | 1      | 0      |
| total twisting                           | -1      | -23    | -4     | 5      |
| $(Q - f_0)$ -divergence                  | 137     | 44     | -4     | 8      |
| $Q$ -divergence                          | 360     | 136    | -29    | -54    |
| $f_0$ - divergence                       | 223     | 92     | -25    | -62    |
| sum missing terms                        | 178     | 90     | 43     | 18     |
| sum usual terms                          | 130     | 73     | -4     | -189   |
| complete $\frac{\partial Q}{\partial t}$ | 308     | 163    | 39     | -170   |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ ,  $Q$ ,  $f$ , 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 2

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point B for the Case of 12Z, February 24th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 1.5     | 4.7    | -0.2   | -6.1   |
| Q  | -330    | 40     | 308    | 61     |
| f  | 5570    | 5330   | 4370   | 3580   |
|  | 2950    | 2950   | 2950   | 2950   |
| vertical advection                       | -1      | -19    | -1     | 8      |
| divergent advection                      | -106    | -41    | 1      | -11    |
| rotational advection                     | 139     | 9      | -42    | 93     |
| rotational twisting                      | 7       | -60    | -54    | -37    |
| divergent twisting                       | -3      | 3      | 4      | 2      |
| total twisting                           | 4       | -57    | -50    | -35    |
| (Q - f <sub>0</sub> )-divergence         | -61     | 7      | 21     | -1     |
| Q-divergence                             | -182    | 22     | 135    | 22     |
| f <sub>0</sub> -divergence               | -121    | 15     | 114    | 23     |
| sum missing terms                        | -164    | -110   | -30    | -39    |
| sum usual terms                          | 18      | 24     | 72     | 115    |
| complete $\frac{\partial Q}{\partial t}$ | -147    | -86    | 42     | 77     |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 3

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point C for the Case of 12Z, February 24th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\omega}{\partial p}$              | 0       | -4.7   | -4.0   | -3.3   |
| $\frac{\partial \omega}{\partial p}$     | 400     | 130    | -40    | -83    |
| Q  | 5050    | 6060   | 4730   | 4990   |
| f  | 3560    | 3560   | 3560   | 3560   |
| vertical advection                       | 0       | 5      | 18     | 3      |
| divergent advection                      | 97      | 12     | -5     | -16    |
| rotational advection                     | 54      | 36     | 102    | 235    |
| rotational twisting                      | 0       | 6      | -8     | -3     |
| divergent twisting                       | 0       | 3      | 2      | 0      |
| total twisting                           | 0       | 8      | -6     | -3     |
| (Q - f <sub>0</sub> )-divergence         | 54      | 31     | -4     | -11    |
| Q-divergence                             | 201     | 81     | -19    | -42    |
| f <sub>0</sub> -divergence               | 147     | 50     | -15    | -31    |
| sum missing terms                        | 150     | 56     | 3      | -27    |
| sum usual terms                          | 201     | 85     | 87     | 204    |
| complete $\frac{\partial Q}{\partial t}$ | 352     | 141    | 91     | 178    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>).

TABLE 4

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point D for the Case of 12Z, February 24th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 0       | -8.6   | -5.2   | -4.6   |
| Q  | 780     | 170    | -143   | -114   |
| f  | 1530    | 4540   | 4390   | 2900   |
|  | 3720    | 3720   | 3720   | 3720   |
| vertical advection                       | 0       | -82    | 20     | 25     |
| divergent advection                      | 21      | 8      | -3     | -4     |
| rotational advection                     | -37     | 59     | -6     | 147    |
| rotational twisting                      | 0       | -8     | -2     | 1      |
| divergent twisting                       | 0       | 1      | 0      | 0      |
| total twisting                           | 0       | -7     | 2      | 1      |
| (Q - f <sub>0</sub> )-divergence         | -169    | 15     | -10    | 9      |
| Q-divergence                             | 118     | 79     | -63    | -33    |
| f <sub>0</sub> -divergence               | 287     | 64     | -53    | -24    |
| sum missing terms                        | -148    | -67    | 5      | 31     |
| sum usual terms                          | 250     | 123    | -59    | 105    |
| complete $\frac{\partial Q}{\partial t}$ | 102     | 56     | -55    | 136    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup>; hr<sup>-2</sup>)

TABLE 5

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point E for the Case of 00Z, March 18th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 1.1     | 10.1   | 9.0    | 2.3    |
| Q  | -770    | -266   | 184    | 238    |
| f  | 6240    | 6230   | 5750   | 5850   |
|  | 3450    | 3450   | 3450   | 3450   |
| vertical advection                       | 0       | -17    | -15    | 7      |
| divergent advection                      | -215    | -80    | -7     | -2     |
| rotational advection                     | 122     | -4     | -20    | 48     |
| rotational twisting                      | -1      | 13     | 0      | 14     |
| divergent twisting                       | 0       | -9     | -10    | -3     |
| total twisting                           | 0       | 4      | -10    | 10     |
| (Q - f <sub>0</sub> )-divergence         | -195    | -67    | 38     | 51     |
| Q-divergence                             | -479    | -165   | 106    | 139    |
| f <sub>0</sub> -divergence               | -284    | -98    | 68     | 88     |
| sum missing terms                        | -410    | -159   | 6      | 66     |
| sum usual terms                          | -162    | -102   | 48     | 136    |
| complete $\frac{\partial Q}{\partial t}$ | -572    | -261   | 55     | 202    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 6

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point F for the Case of 00Z, March 18th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\omega}{\partial p}$              | -0.7    | -9.5   | -13.1  | -9.9   |
| Q  | 680     | 412    | 67     | -264   |
| f  | 6820    | 7090   | 6250   | 5140   |
|  | 3680    | 3680   | 3680   | 3680   |
| vertical advection                       | -2      | 18     | 73     | 18     |
| divergent advection                      | 100     | 44     | 2      | -15    |
| rotational advection                     | -21     | 46     | 66     | 142    |
| rotational twisting                      | 5       | 21     | 28     | 5      |
| divergent twisting                       | -2      | -5     | -5     | 0      |
| total twisting                           | 2       | 17     | 23     | 5      |
| (Q - f <sub>o</sub> )-divergence         | 211     | 140    | 17     | -38    |
| Q-divergence                             | 462     | 292    | 42     | -136   |
| f <sub>o</sub> -divergence               | 251     | 152    | 25     | -98    |
| summissing terms                         | 311     | 219    | 115    | -30    |
| sumusual terms                           | 229     | 199    | 91     | 44     |
| complete $\frac{\partial Q}{\partial t}$ | 541     | 418    | 206    | 15     |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 7

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point G for the Case of 00Z, March 18th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 3.9     | -8.1   | -11.8  | -10.6  |
| Q  | 5330    | 5330   | 5020   | 4610   |
| f  | 3480    | 3480   | 3480   | 3480   |
| vertical advection                       | 0       | 8      | 24     | 19     |
| divergent advection                      | 75      | 24     | -3     | -8     |
| rotational advection                     | -190    | 6      | 49     | 136    |
| rotational twisting                      | -2      | 29     | 38     | 21     |
| divergent twisting                       | 1       | -3     | -3     | 1      |
| total twisting                           | -1      | 26     | 36     | 22     |
| (Q - f <sub>0</sub> )-divergence         | 154     | 85     | 15     | -20    |
| Q-divergence                             | 503     | 279    | 58     | -101   |
| f <sub>0</sub> -divergence               | 349     | 194    | 43     | -81    |
| sum missing terms                        | 228     | 144    | 72     | 13     |
| sum usual terms                          | 159     | 200    | 91     | 54     |
| complete $\frac{\partial Q}{\partial t}$ | 387     | 343    | 163    | 67     |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 8

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point H for the Case of 00Z, March 18th, 1965.

| level                                       | 1000 mb | 850 mb | 700 mb | 500 mb |
|---|---------|--------|--------|--------|
| $\omega$                                    | -1.5    | -9.1   | -12.4  | -11.3  |
| $\frac{\partial \omega}{\partial p}$        | 580     | 361    | 100    | -266   |
| Q   | 1770    | 2910   | 3910   | 3520   |
| f   | 3470    | 3470   | 3470   | 3470   |
| vertical advection                          | -12     | -65    | -36    | 58     |
| divergent advection                         | -91     | -47    | -7     | 10     |
| rotational advection                        | -137    | 132    | 168    | 215    |
| rotational twisting                         | -3      | -15    | -6     | 28     |
| divergent twisting                          | -7      | 2      | 3      | 1      |
| total twisting                              | -10     | -13    | -3     | 29     |
| (Q - f)-divergence                          | -111    | -28    | 2      | 5      |
| Q-divergence                                | 103     | 105    | 39     | -94    |
| f <sub>o</sub> -divergence                  | 214     | 133    | 37     | -99    |
| summissing terms                            | -225    | -153   | -45    | 102    |
| sumusual terms                              | 77      | 266    | 205    | 116    |
| complete $\frac{\partial \phi}{\partial t}$ | -148    | 113    | 160    | 218    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>).

TABLE 9

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point I for the Case of 00Z, January 8th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 6.4     | 6.4    | 9.4    | 11.3   |
| Q  | 0       | -101   | -156   | 189    |
| f  | 1740    | 3180   | 4770   | 7460   |
|  | 3270    | 3270   | 3270   | 3270   |
| vertical advection                       | 59      | 64     | 111    | 102    |
| divergent advection                      | 11      | -8     | -16    | -7     |
| rotational advection                     | -208    | -106   | -117   | -319   |
| rotational twisting                      | 26      | 24     | 13     | 20     |
| divergent twisting                       | 4       | 3      | 3      | -2     |
| total twisting                           | 30      | 27     | 16     | 18     |
| (Q - f <sub>0</sub> )-divergence         | 0       | 5      | -17    | 71     |
| Q-divergence                             | 0       | -32    | -74    | 141    |
| f <sub>0</sub> -divergence               | 0       | -37    | -57    | 70     |
| sum missing terms                        | 100     | 88     | 94     | 184    |
| sum usual terms                          | -208    | -144   | -174   | -249   |
| complete $\frac{\partial Q}{\partial t}$ | -108    | -55    | -81    | -66    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 10

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point-J for the Case of 00Z, January 8th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\omega$                                 |         |        |        |        |
| $\frac{\partial \omega}{\partial p}$     | -0.1    | -2.6   | -5.3   | -7.4   |
| Q  | 6560    | 6210   | 5870   | 5900   |
| f  | 3870    | 3870   | 3870   | 3870   |
| vertical advection                       | 0       | 6      | 7      | 4      |
| divergent advection                      | 9       | 3      | -4     | -3     |
| rotational advection                     | 16      | -15    | 25     | 299    |
| rotational twisting                      | -2      | 0      | -8     | -8     |
| divergent twisting                       | 0       | 1      | 1      | 0      |
| total twisting                           | -2      | 1      | -8     | -8     |
| (Q - f <sub>0</sub> )-divergence         | 47      | 44     | 32     | -17    |
| Q-divergence                             | 108     | 109    | 88     | -45    |
| f <sub>0</sub> -divergence               | 61      | 65     | 56     | -28    |
| sum missing terms                        | 54      | 54     | 27     | -23    |
| sum usual terms                          | 76      | 50     | 80     | 271    |
| complete $\frac{\partial Q}{\partial t}$ | 130     | 104    | 108    | 249    |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 11

Values of the Individual Terms in the Vorticity Equation as  
Computed at Point K for the case of 00Z, January 8th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | -0.3    | -1.3   | -5.6   | -9.9   |
| Q  | 391     | 178    | -14    | -109   |
| f  | 5770    | 5200   | 4220   | 3360   |
|  | 3950    | 3950   | 3950   | 3950   |
| vertical advection                       | 1       | 26     | 31     | 5      |
| divergent advection                      | 30      | 11     | -3     | -8     |
| rotational advection                     | -23     | 22     | 73     | 99     |
| rotational twisting                      | 7       | 5      | 11     | 11     |
| divergent twisting                       | -1      | -1     | -2     | -1     |
| total twisting                           | 6       | 4      | 9      | 10     |
| (Q - f <sub>0</sub> )-divergence         | 81      | 27     | -1     | 4      |
| Q-divergence                             | 226     | 93     | -6     | -37    |
| f <sub>0</sub> -divergence               | 145     | 66     | -5     | -41    |
| sum missing terms                        | 117     | 68     | 36     | 11     |
| sum usual terms                          | 122     | 88     | 67     | 58     |
| complete $\frac{\partial Q}{\partial t}$ | 239     | 156    | 103    | 70     |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

TABLE 12

Values of the Individual Terms in the Vorticity Equation as Computed at Point L for the Case of 00Z, January 8th, 1965.

| level                                    | 1000 mb | 850 mb | 700 mb | 500 mb |
|--|---------|--------|--------|--------|
| $\frac{\partial \omega}{\partial p}$     | 22.8    | 16.8   | 10.8   | 4.9    |
| Q  | 406     | 400    | 353    | 245    |
| f  | 6900    | 7280   | 7430   | 7280   |
|  | 4620    | 4620   | 4620   | 4620   |
| vertical advection                       | 68      | 30     | 3      | 7      |
| divergent advection                      | 38      | 10     | -4     | 2      |
| rotational advection                     | 20      | -30    | -54    | -148   |
| rotational twisting                      | -16     | -20    | -13    | -7     |
| divergent twisting                       | -3      | -2     | -3     | -4     |
| total twisting                           | -19     | -21    | -16    | -11    |
| (Q - f <sub>0</sub> )-divergence         | 130     | 143    | 131    | 88     |
| Q-divergence                             | 280     | 291    | 262    | 178    |
| f <sub>0</sub> -divergence               | 150     | 148    | 131    | 90     |
| sum missing terms                        | 217     | 161    | 115    | 87     |
| sum usual terms                          | 171     | 118    | 76     | -57    |
| complete $\frac{\partial Q}{\partial t}$ | 388     | 279    | 191    | 29     |

(Units:  $\omega$ , mb hr<sup>-1</sup>;  $\frac{\partial \omega}{\partial p}$ , Q, f, 10<sup>-4</sup> hr<sup>-1</sup>; rest 10<sup>-4</sup> hr<sup>-2</sup>)

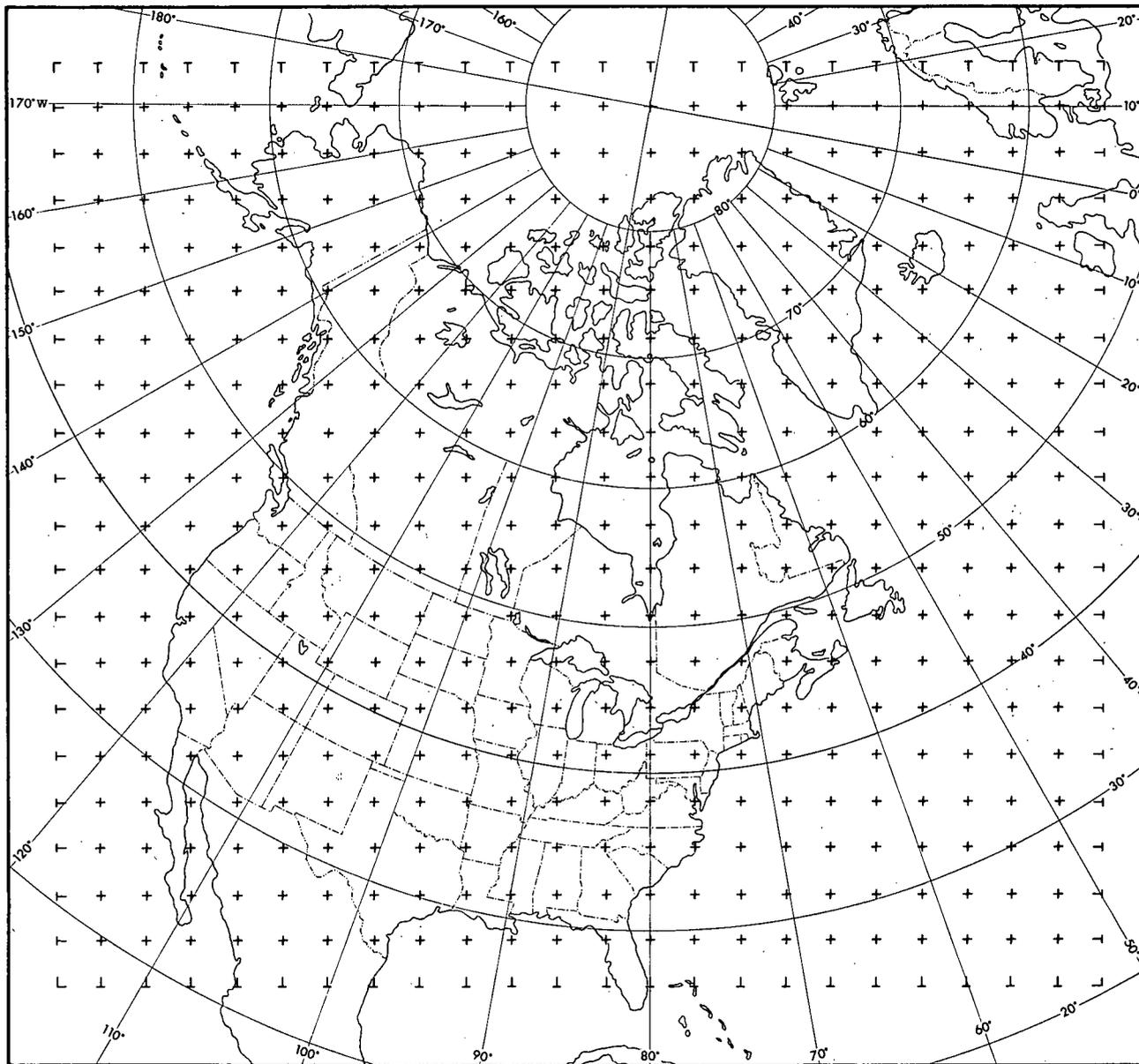


Figure 1  
The 504-Points Grid

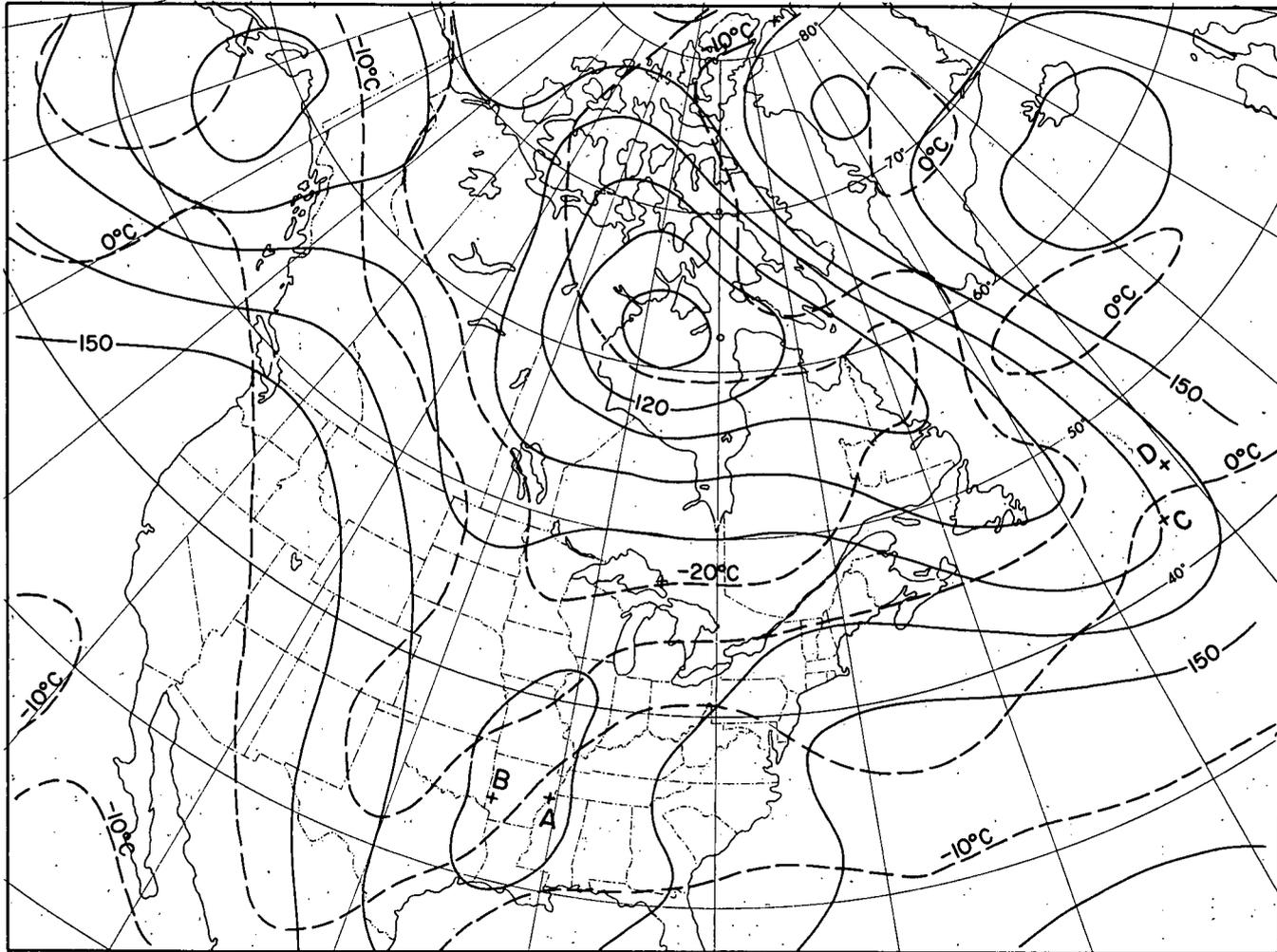


Figure 2  
850 MB Feb. 24 1965 1200Z

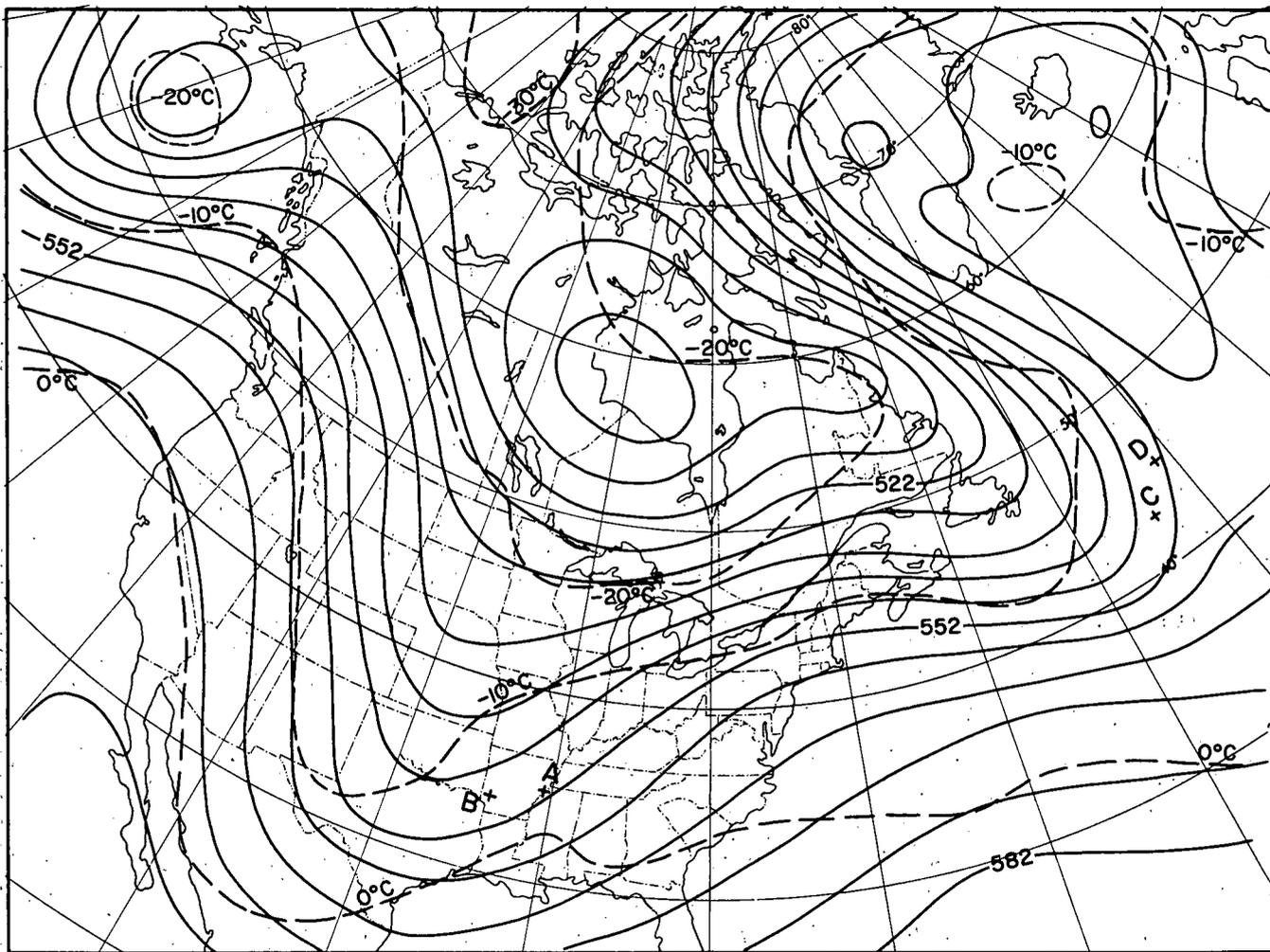


Figure 3  
500 MB Feb. 24 1965 1200Z

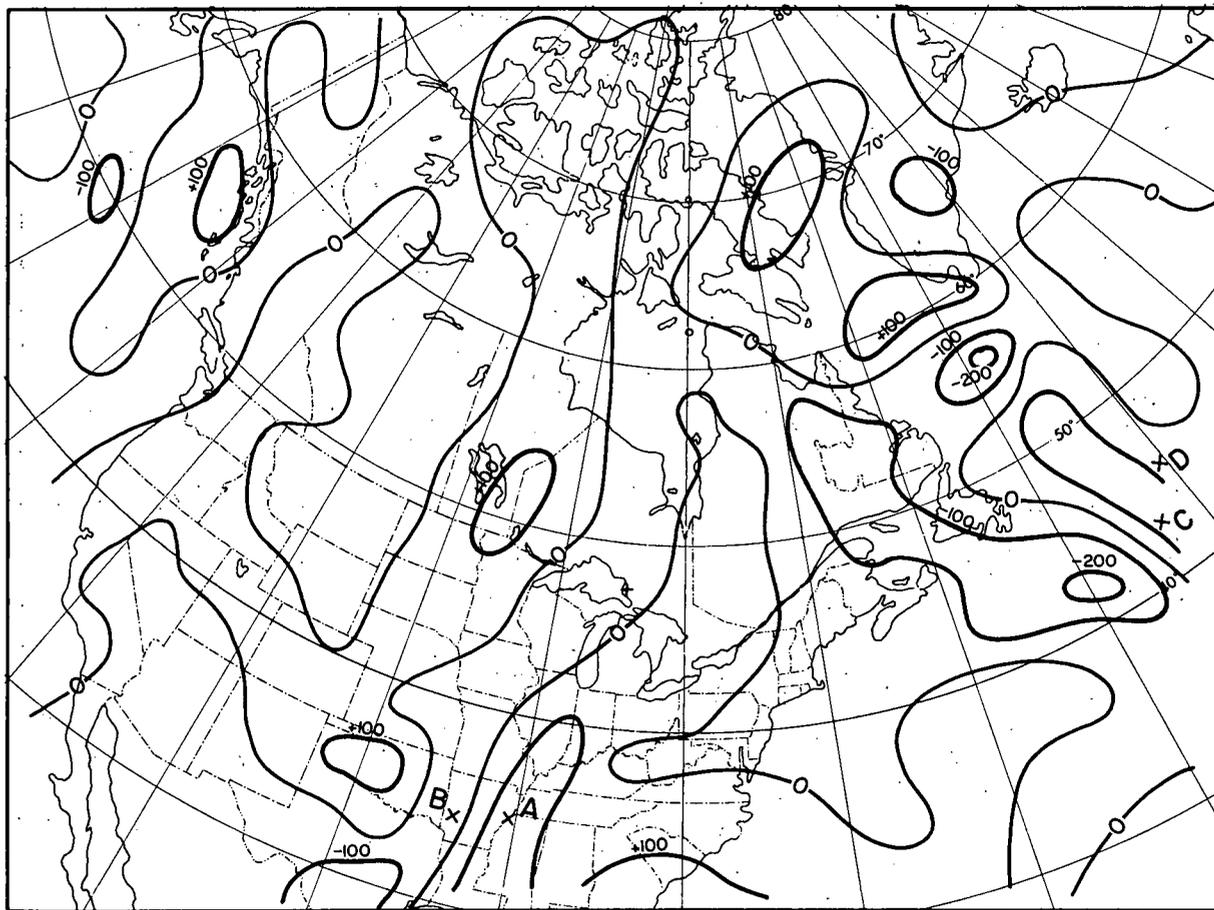


Figure 4  
Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Feb. 24 1965 1200Z

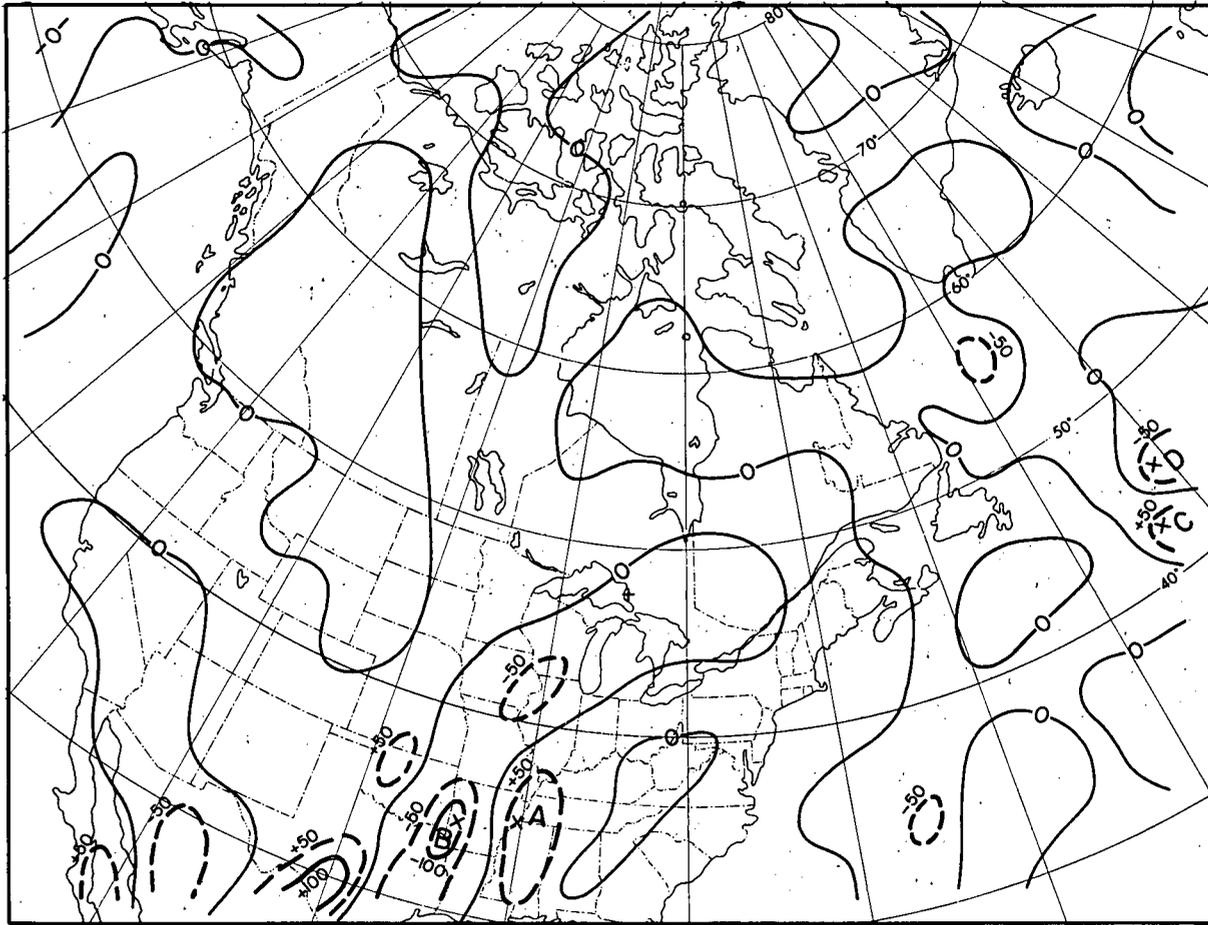


Figure 5  
Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Feb. 24 1965 1200Z

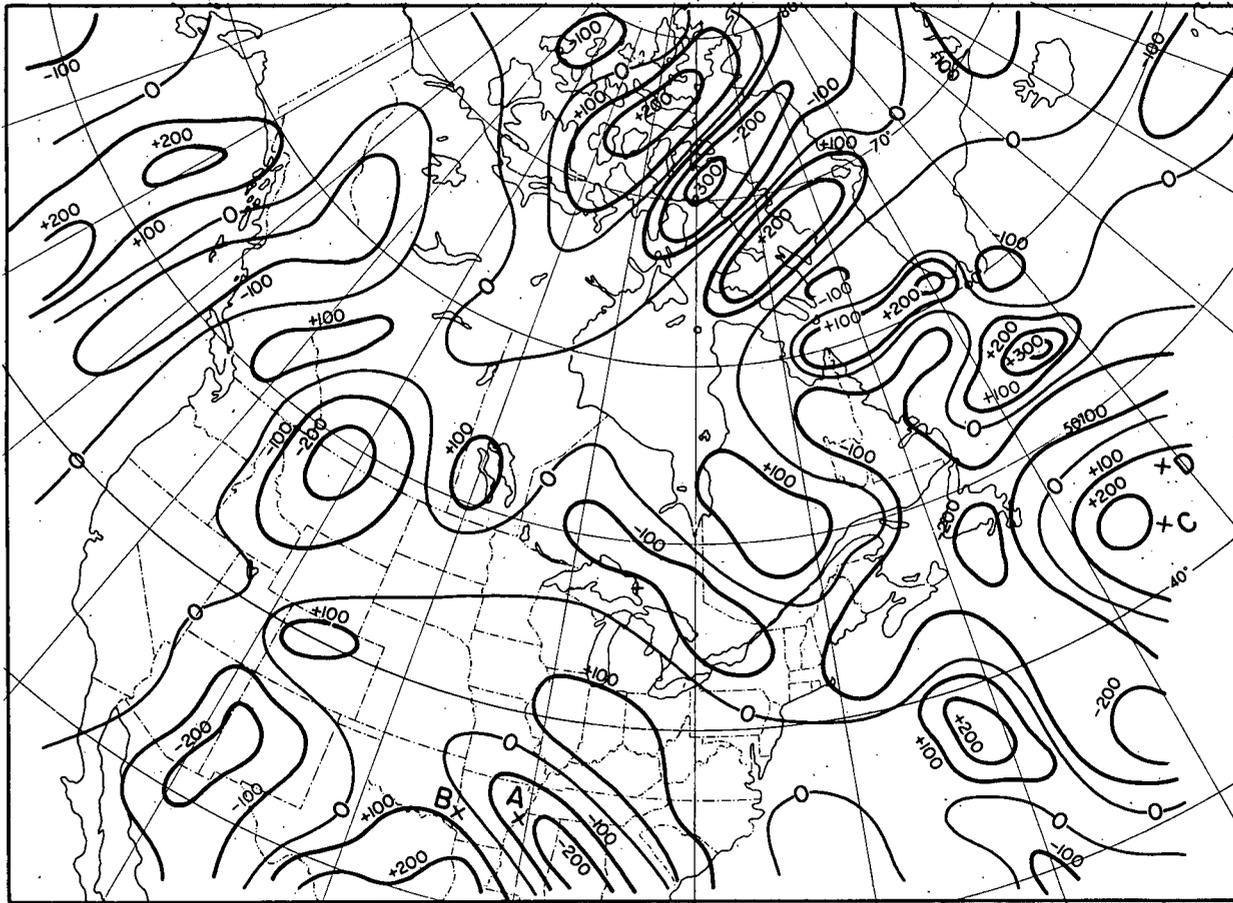


Figure 6  
Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
500 MB Feb. 24 1965 1200Z

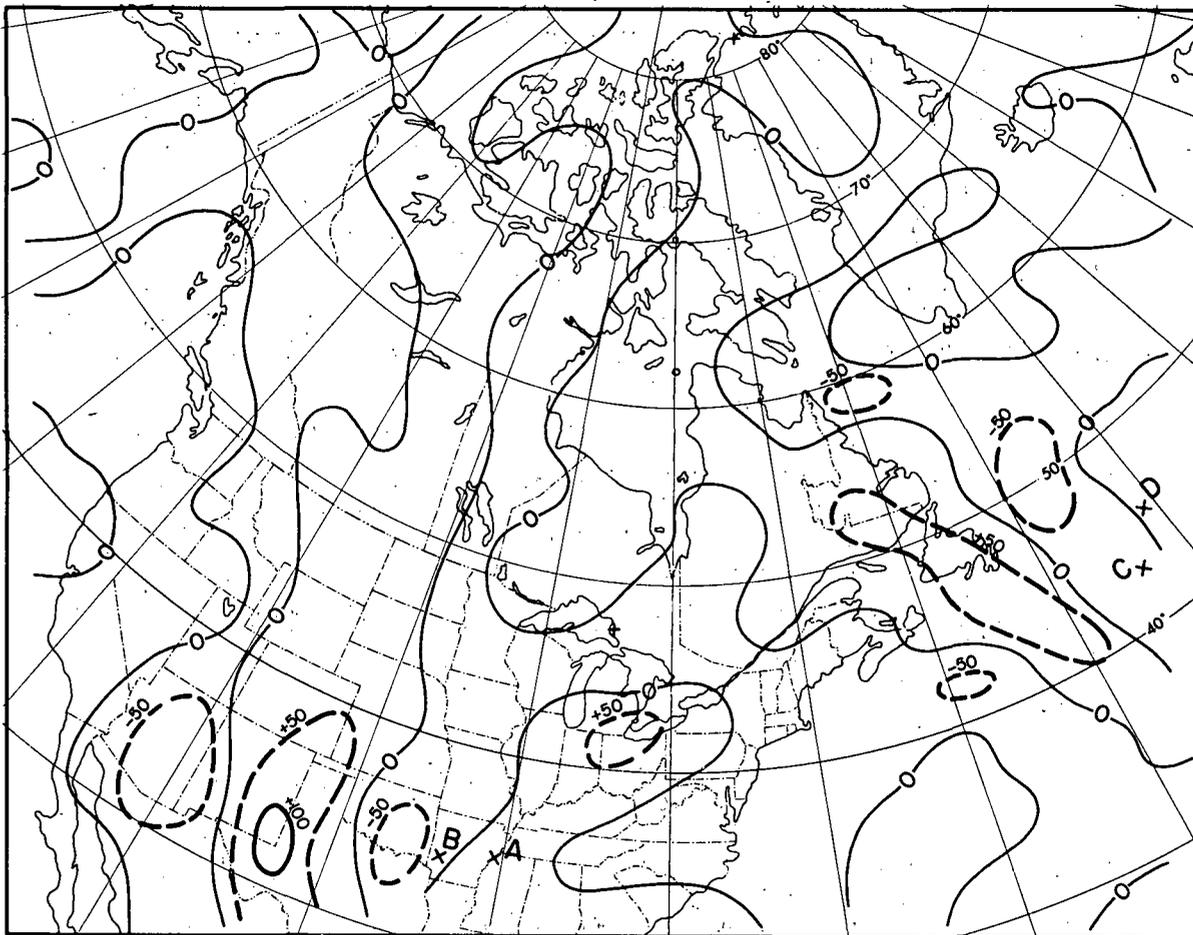


Figure 7  
Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
500 MB Feb. 24 1965 1200Z

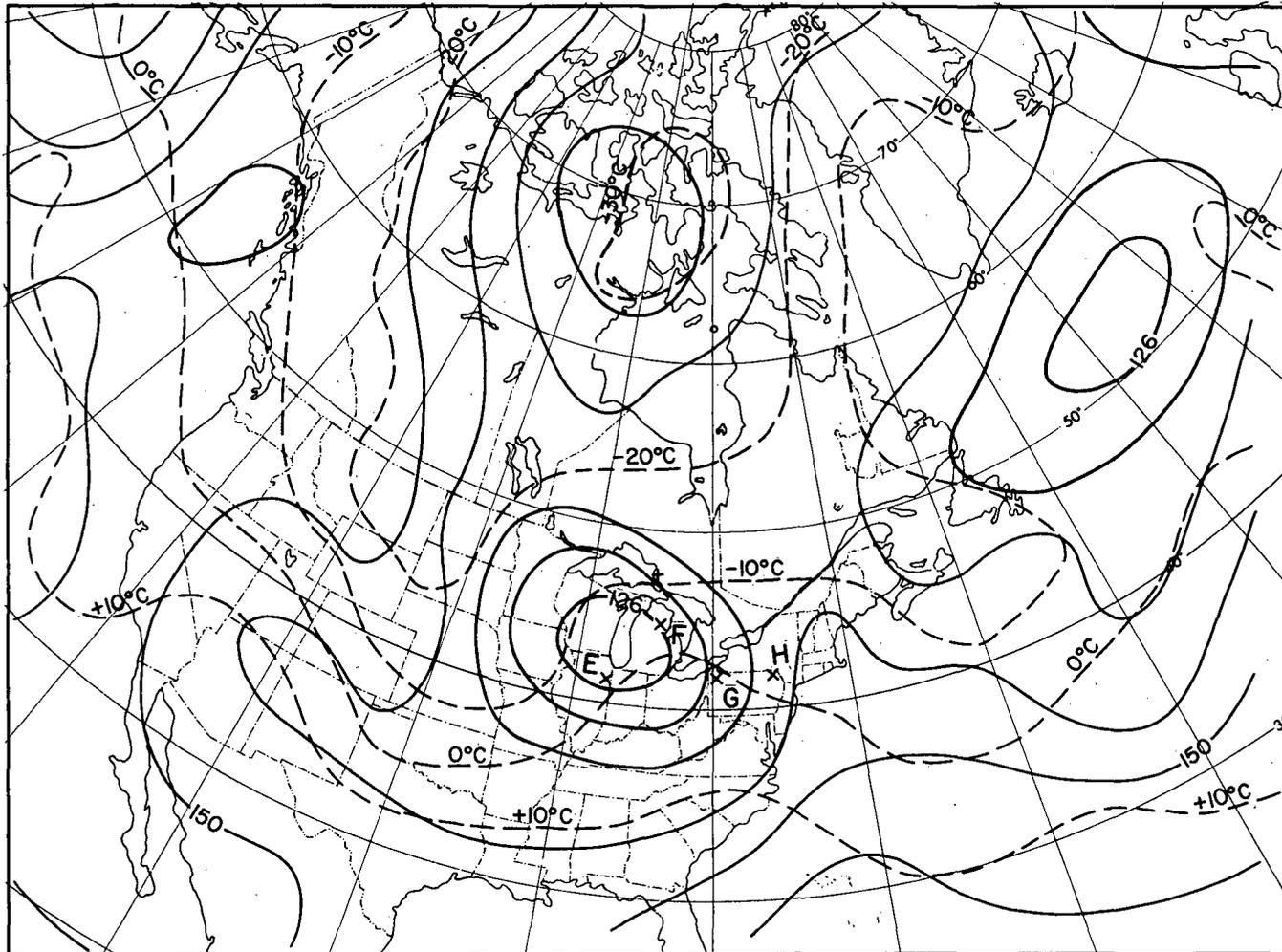


Figure 8  
850 MB Mar. 18 1965 0000Z

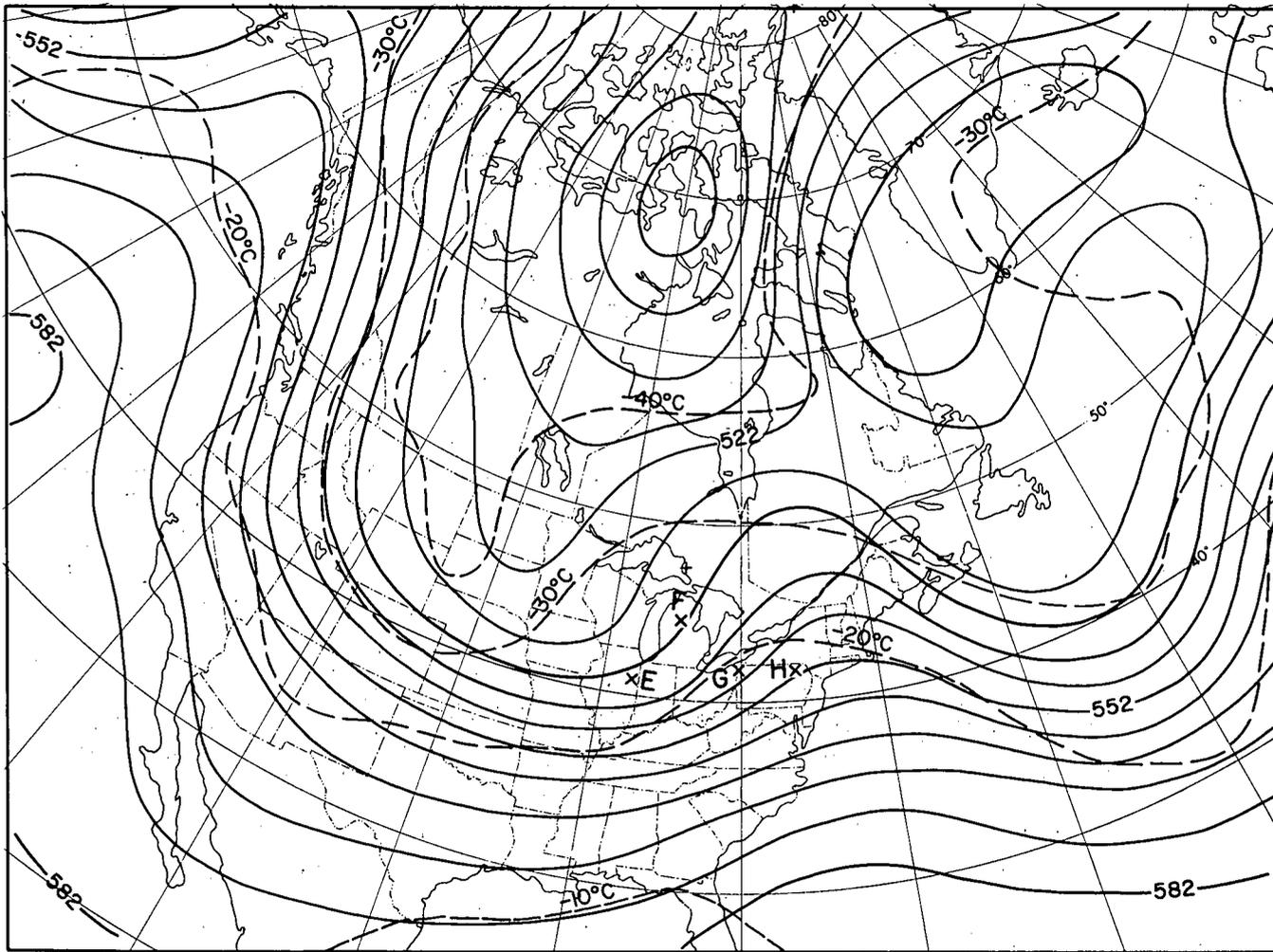


Figure 9  
500 MB Mar. 18 1965 0000Z

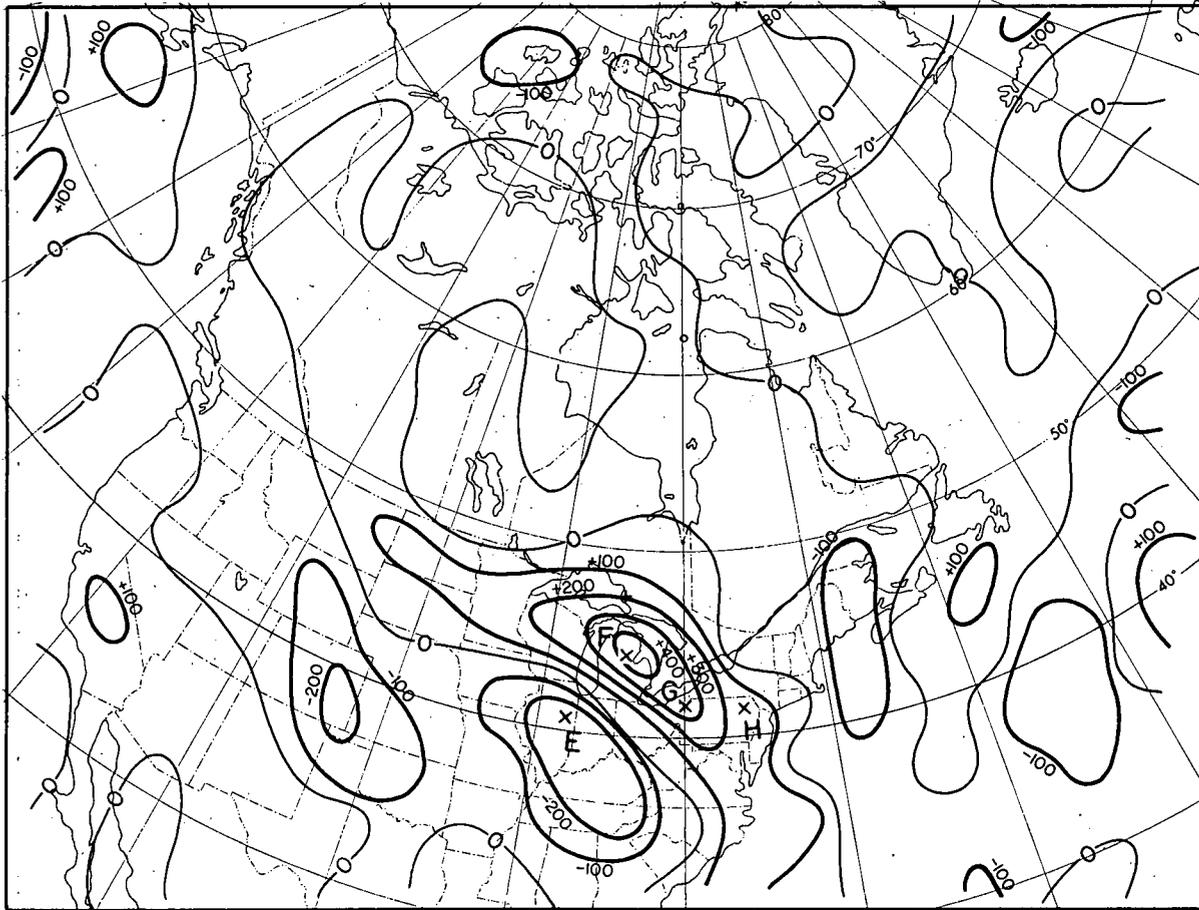


Figure 10  
Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Mar. 18 1965 0000Z

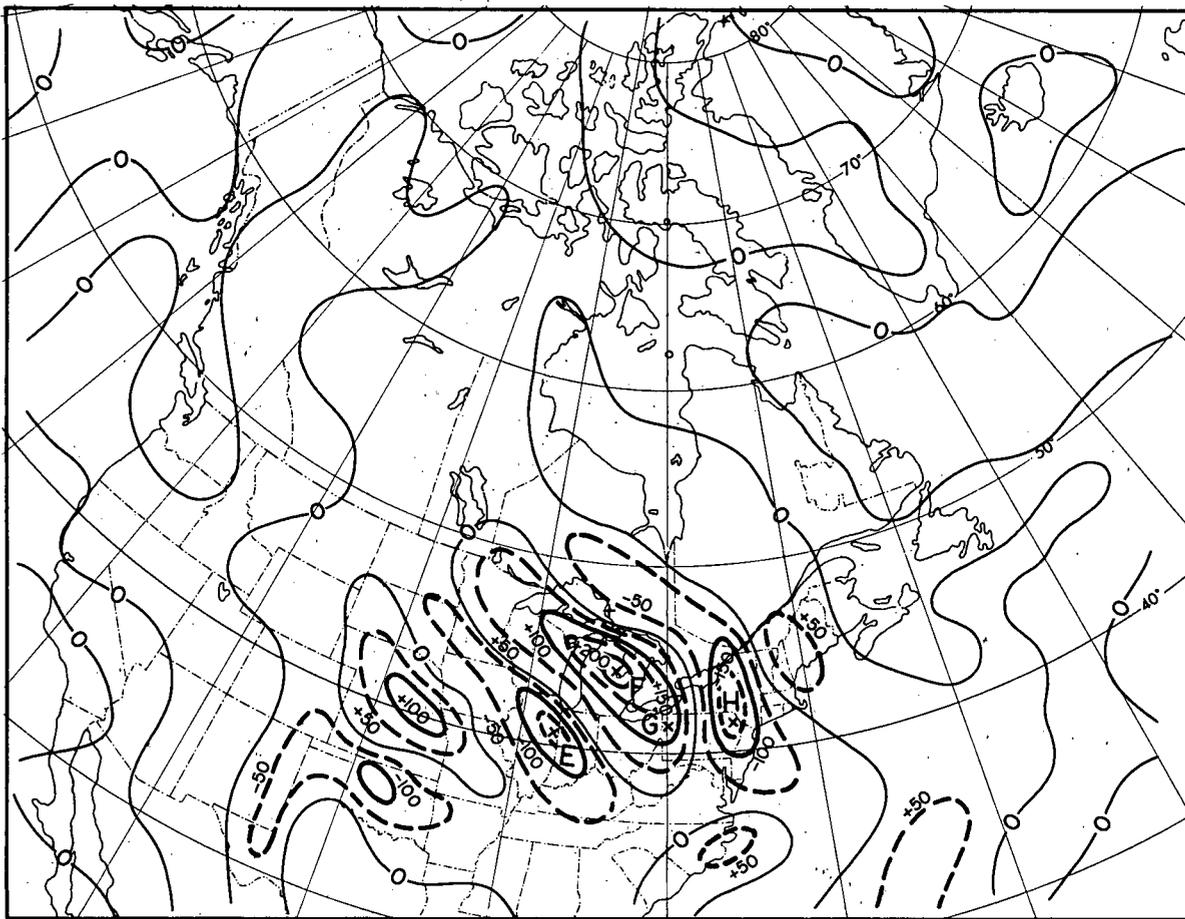


Figure 11  
Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Mar. 18 1965 0000Z

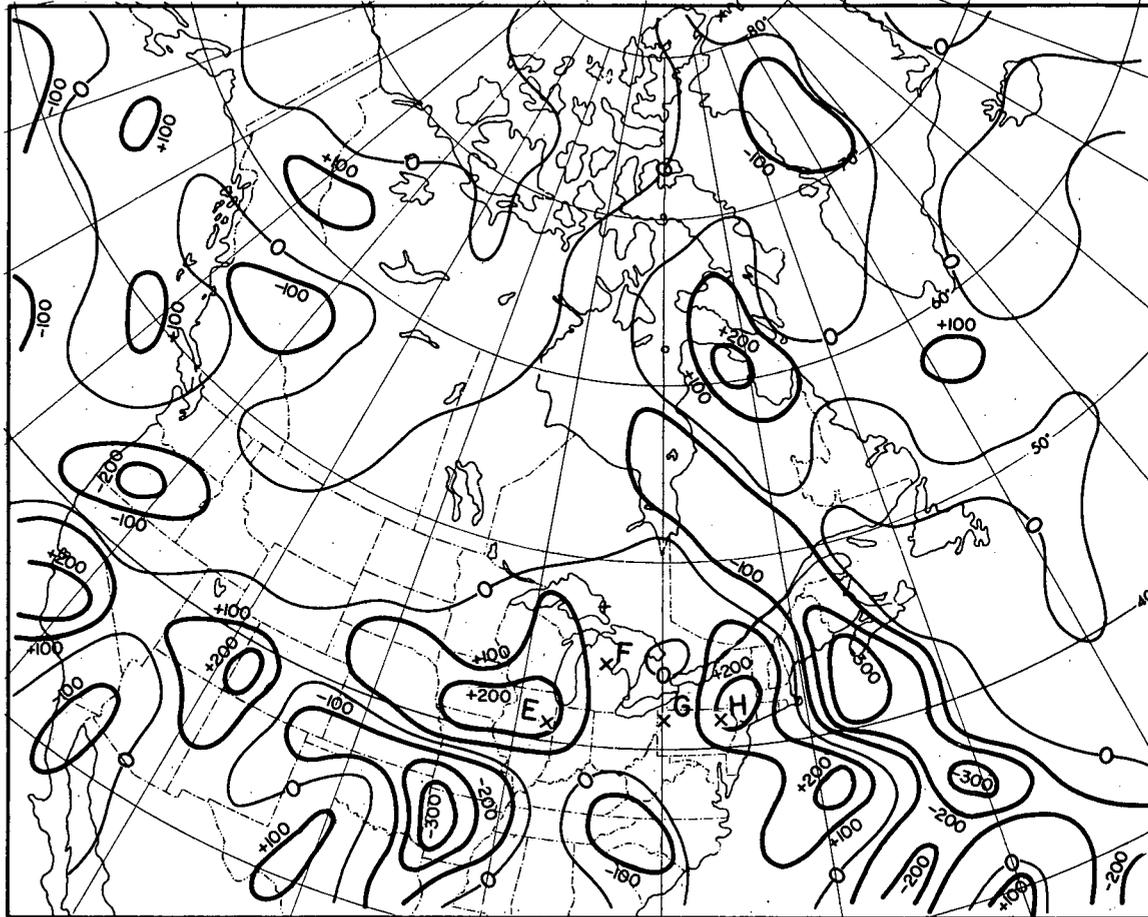


Figure 12  
 Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
 500 MB Mar. 18 1965 0000Z

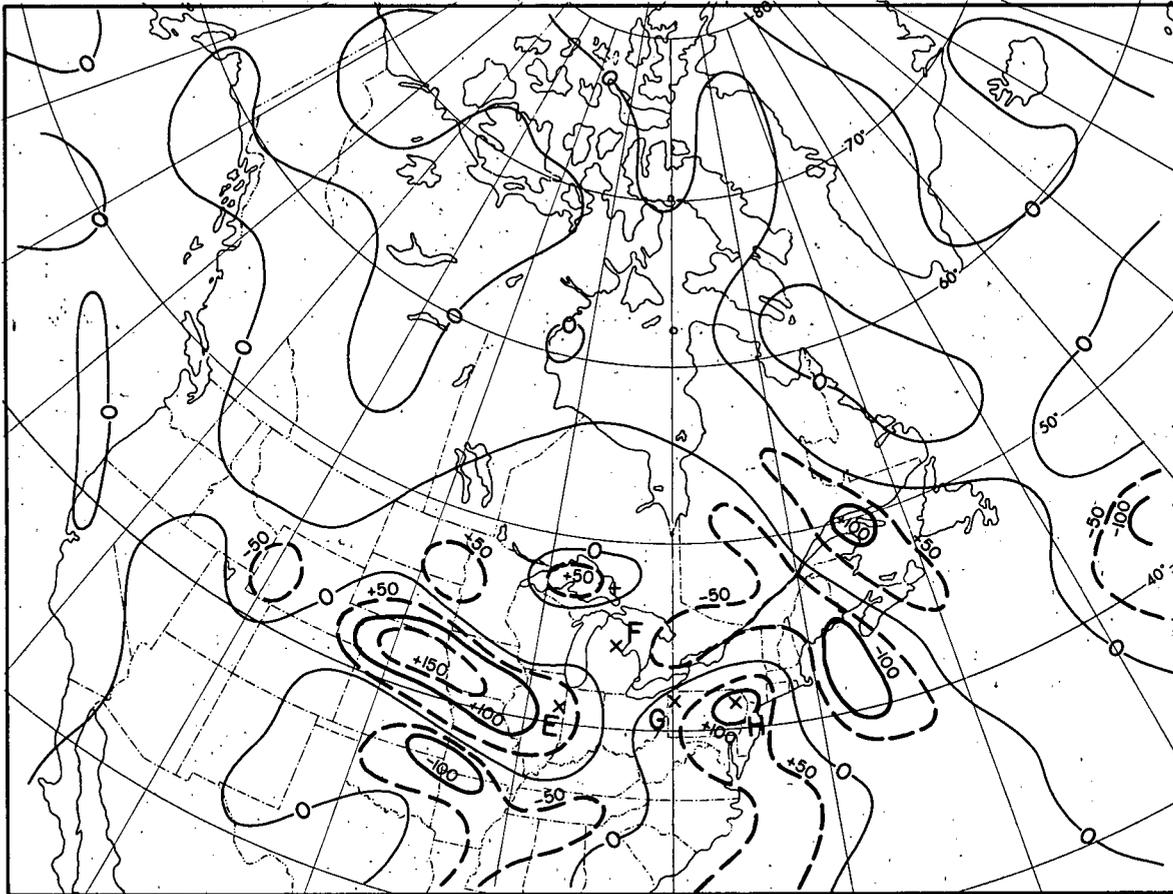


Figure 13  
 Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
 500 MB Mar. 18 1965 0000Z

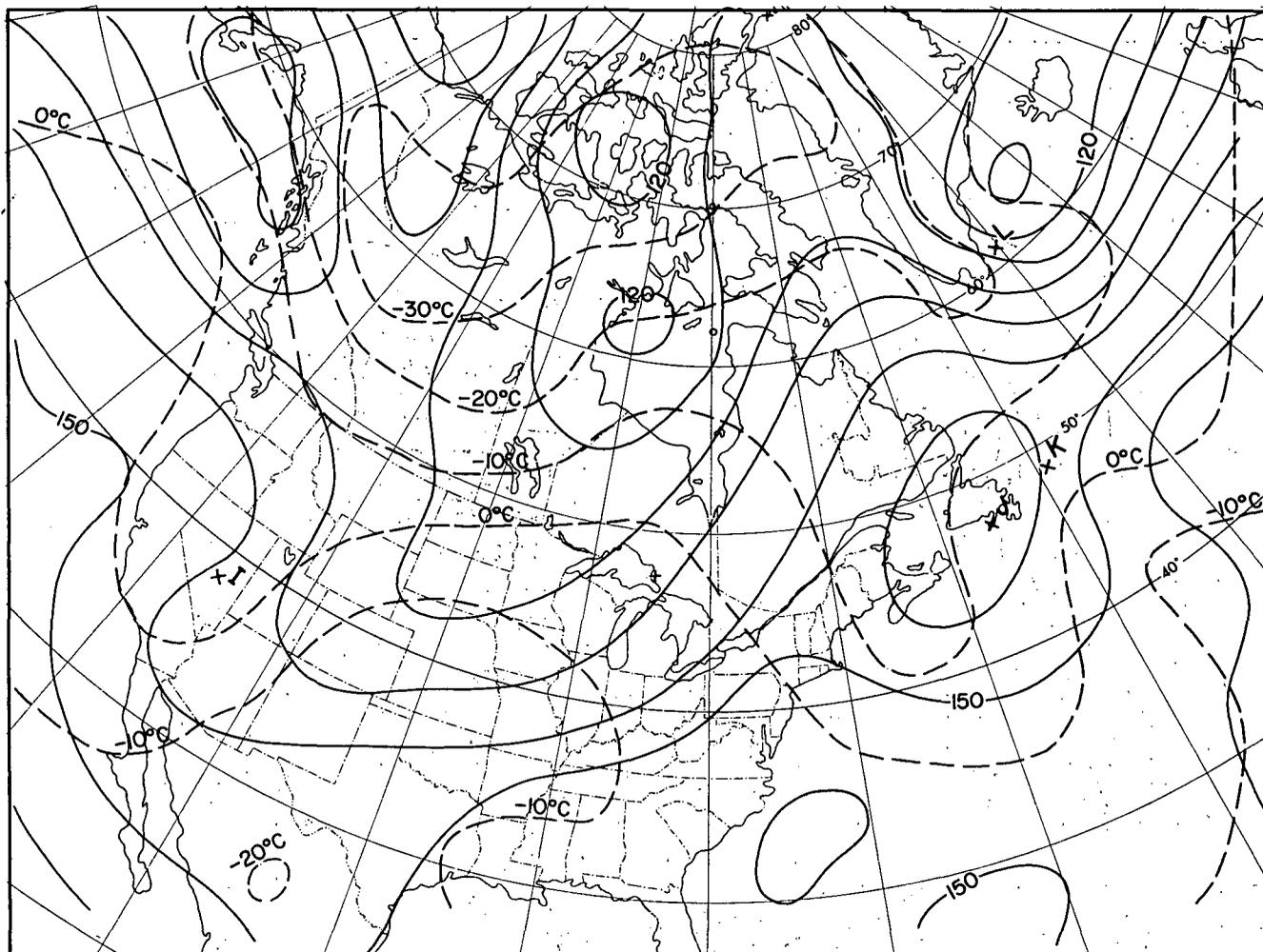


Figure 14  
850 MB Jan. 8 1965 0000Z

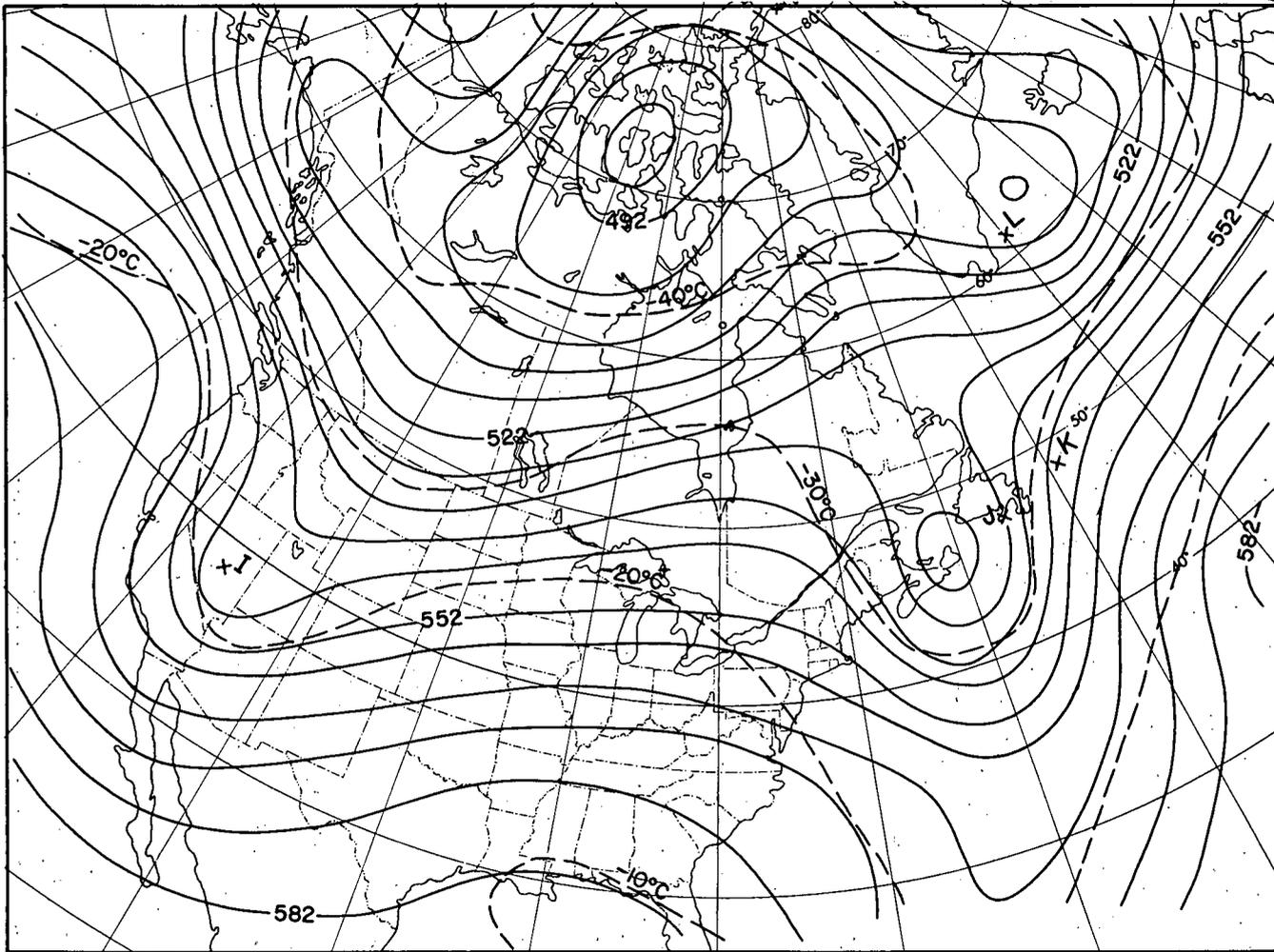


Figure 15  
500 MB Jan. 8 1965 0000Z

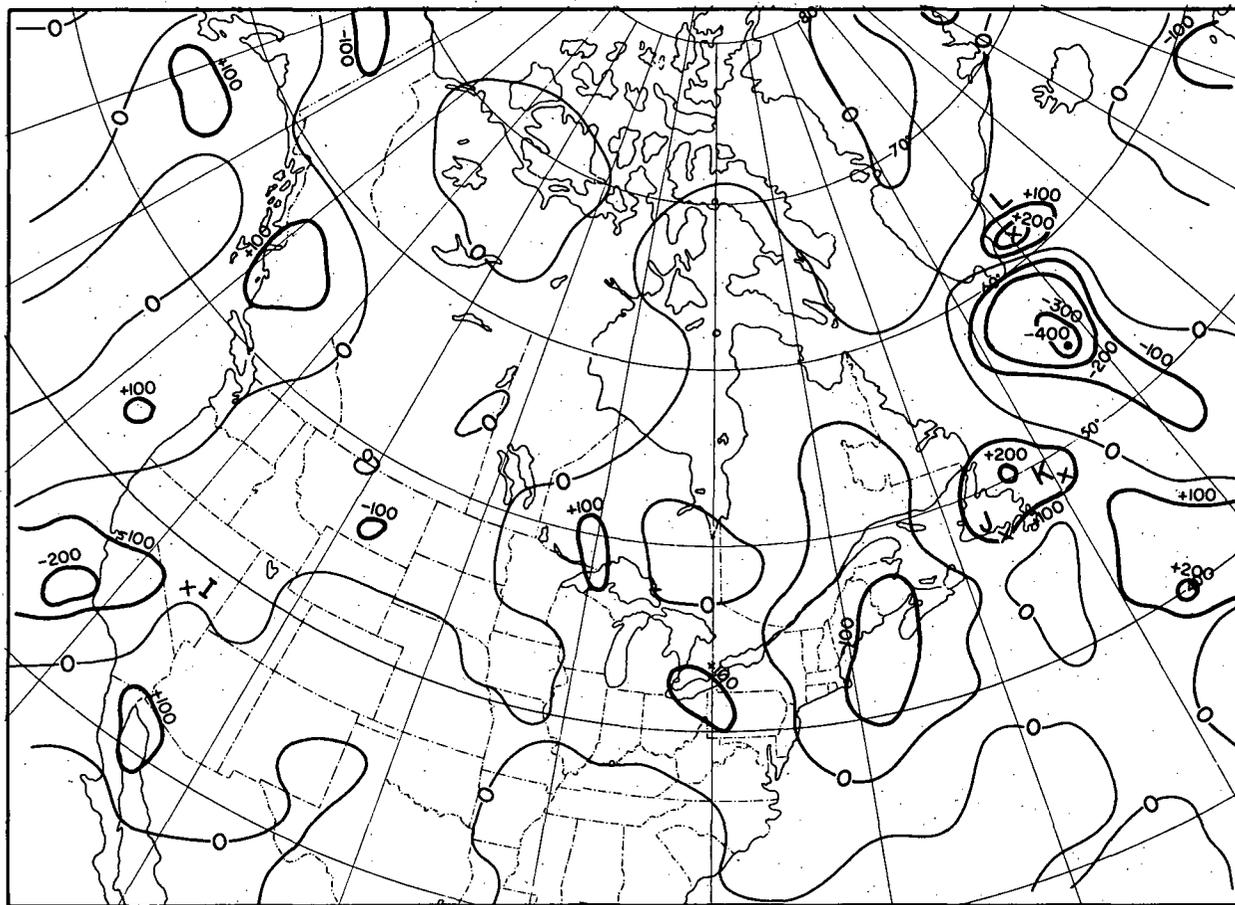


Figure 16  
Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Jan. 8 1965 0000Z

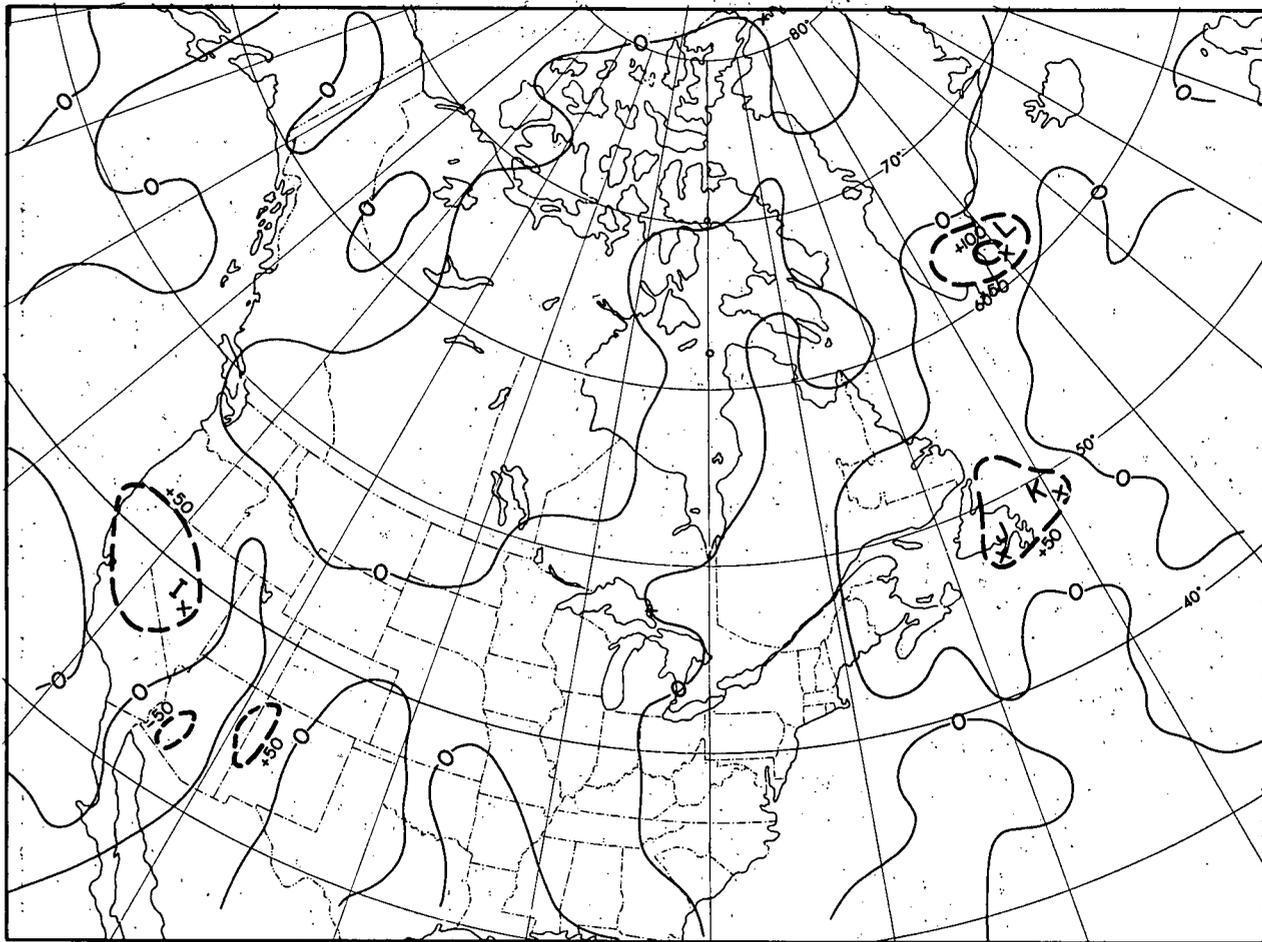


Figure 17  
Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
850 MB Jan. 8 1965 0000Z

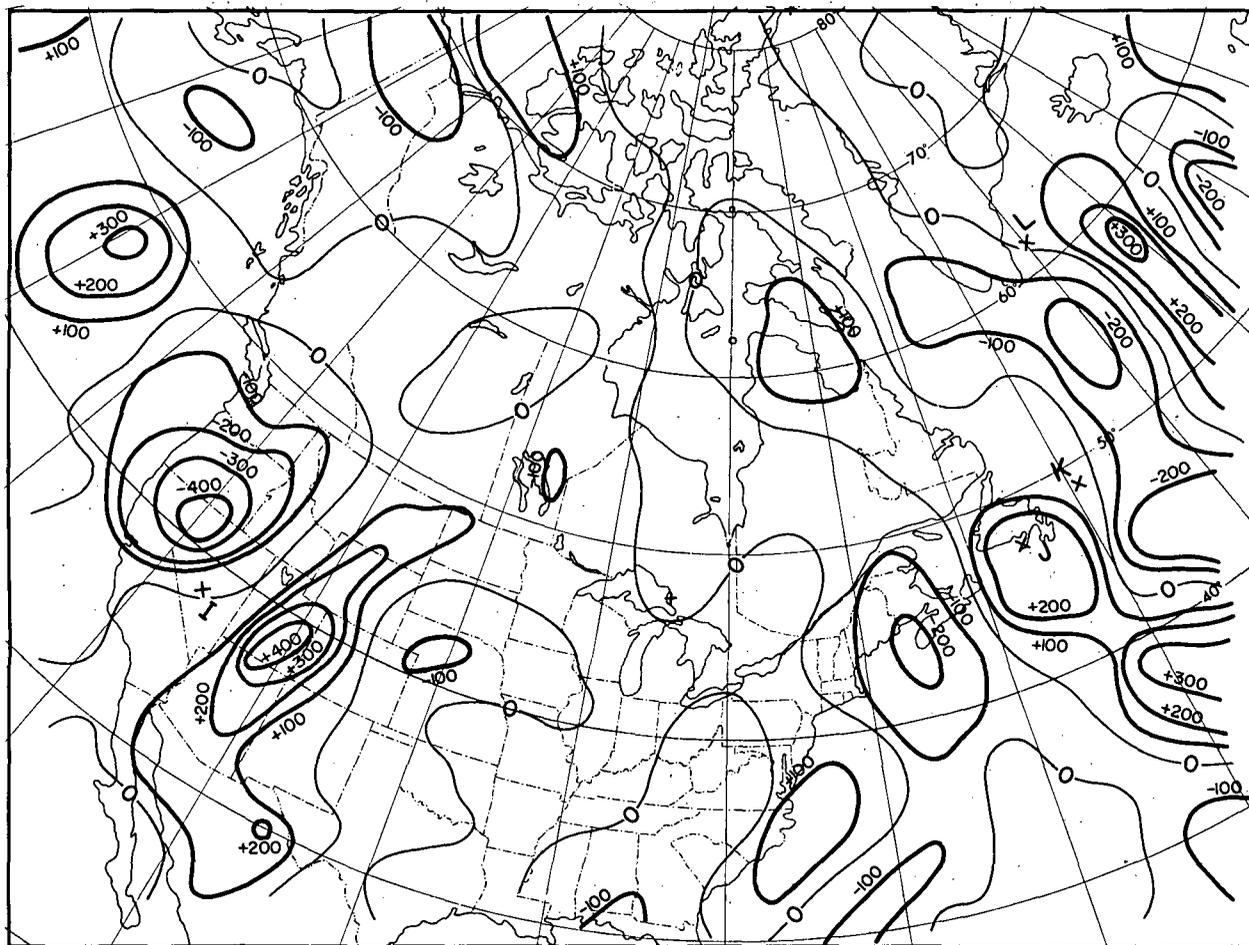


Figure 18  
Complete Vorticity Tendency (Units  $10^{-4} \text{ hr}^{-2}$ )  
500 MB Jan. 8 1965 0000Z

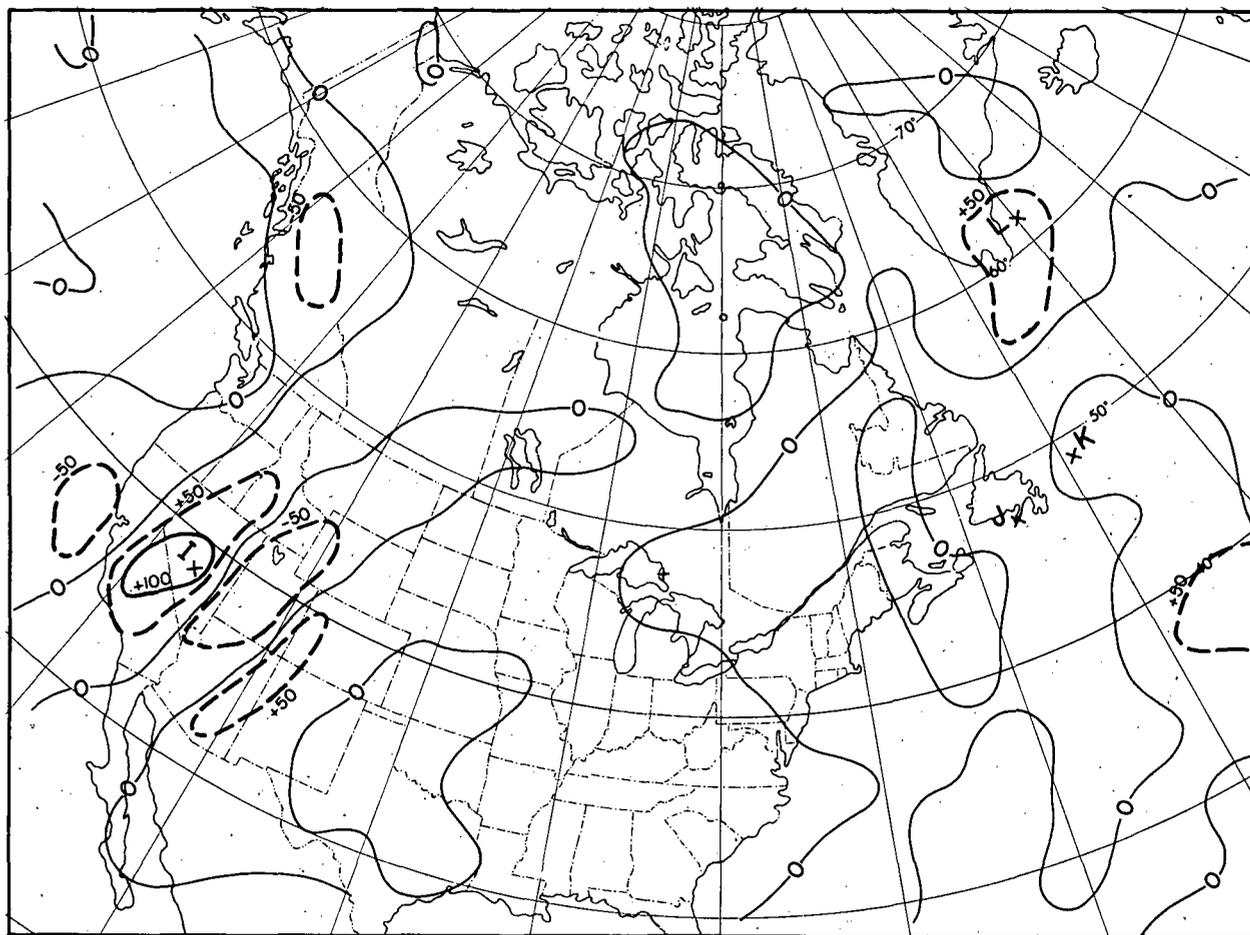


Figure 19  
Vorticity Tendency Due To Missing Terms (Units  $10^{-4} \text{ hr}^{-2}$ )  
500 MB Jan. 8 1965 0000Z

CIR-4416,  
TEC-614  
18 Apr. 1966

UDC: 551.509.313

CANADA

Department of Transport - Meteorological Branch  
315 Bloor St., W., Toronto 5, Ontario

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15 pps. 19 figs. 12 tables 8 refs.  
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ABSTRACT: For each of three synoptic situations all the terms in the vorticity equation are computed at 1000, 850, 700, and 500 mb. When the results based on diagnostic vertical motion fields are studied in some detail, they show that the terms neglected in the Central Analysis Office baroclinic model attain their maximum magnitudes in the neighbourhood of developing disturbances. In these regions, especially at 1000 mb, their sum may approach or exceed that of the terms retained.

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