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# DRY AND WET SPELLS AT WINNIPEG

BY

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W. Simpson
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## ABSTRACT

A first order Markov chain probability model is shown to conform closely to the long term average distribution of wet and dry spells at Winnipeg during the months of April to October, inclusive. Consequently, other parameters, such as, (a) probability of changeable weather, from wet to dry, and vice-versa, (b) number of wet days per month, and (c) average lengths of wet and dry spells, were calculated using the model. These values are shown to be in fair agreement with actual average values over the long term period. However, short term deviations from the model may be large.

## PÉRIODES SÈCHES ET PÉRIODES HUMIDES À WINNIPEG

par

W. Simpson
et
C. D. Henry

## RÉSUMÉ

Les auteurs démontrent que le modèle de probabilité à chaîne de Markov de premier ordre se conforme étroitement à la distribution moyenne à long terme des périodes sèches et des périodes humides à Winnipeg durant les mois d'avril à octobre inclusivement. Par conséquent, d'autres paramètres, comme (a) la probabilité de temps variable, de l'humide au sec et inversement, (b) le numbre de jours humides par mois, et (c) les durées moyennes des périodes humides et des périodes sèches, ont été calculés au moyen de ce modèle. Les auteurs démontrent que ces valeurs concordent bien avec les valeurs moyennes réelles pendant la période étendue. Toutefois, les écarts à court terme par rapport au modèle peuvent être considérables.

# DRY AND WET SPELLS AT WINNIPEG

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## 1. Introduction

Studies have been made of various statistical models which might be used to describe the duration and frequency of dry and wet spells. In this paper, the first order Markov chain model is shown to apply reasonably well to April through October dry and wet spells at Winnipeg. Data on dry and wet spells are presented.

## 2. Dry and Wet Spells

A day was considered to be wet if one tenth of an inch or more of precipitation was recorded during that day. Otherwise, it was a "dry day". This definition was chosen to avoid considering a dry spell as ended at the fall of only a very small amount of precipitation. It seems unlikely that rainfall of less than one tenth of an inch would have much effect on low soil moisture, forest fire hazards, blowing dust, or other conditions associated with dry spells. This definition has the possibly bad feature of classifying most periods of showery weather (which would generally be considered as wet spells) as several short dry periods separated by one-day and two-day wet spells. No "wet spells" longer than five days were found in the period of record used.

The definition used in this paper will probably give results which agree with the general notion of a dry spell. Several very long dry spells were found. Table 1 gives the number of dry spells of various lengths that were found in the period of record studies. Table 2 is a list of the longest dry spells found with the dates they started and ended and the amount of precipitation during that period. The periods used are listed below under "data". The years 1961, 1962 and 1963 were also used for June, July and August.

## 3. First Order Markov Chain

Let P(w/w) represent the probability that a day is wet given that the previous day was wet and P(d/d) represent the probability that a day is dry given that the previous day was dry.

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The first order Markov chain is developed from the assumption that $P(d/d)$ and $P(w/w)$ are independent of the previous weather.

The relative frequencies of dry spells of one day, two days, three days, ... can be readily seen to be proportional to 1, $P(d/d)$, $[P(d/d)]^2$ ... and the relative frequencies of wet spells to 1, $P(w/w)$, $[P(w/w)]^2$ ....

The absolute probability of a wet day can be shown to be

$$\frac{P(w/d)}{P(w/d) + P(d/w)} = P(w)$$

where $P(w/d)$ represents the probability of a wet day following a dry day and $P(d/w)$ represents the probability of a dry day following a wet day.

$$\text{Thus, } P(w/d) = 1 = P(d/d)$$

$$\text{and } P(d/w) = 1 = P(w/w).$$

The mean number of wet days in a period of n days in $n\, P(w)$ and the standard deviation of this quantity, for large values of n, is

$$\sqrt{n\, P(w) \quad [1 - P(w)] \quad \frac{P(w/w) + P(d/d)}{P(w/d) + P(d/w)}}$$

These and other results derivable from the Markov chain are given in a paper by Gabriel and Neumann (1).

The average length of a dry spell in days is $\frac{1}{P(w/d)}$. This may be shown by taking the weighted average of all possible lengths of dry spells using as weighting factors the relative frequencies of the various spells. That is, average length of dry spell.

$$= \frac{\sum\limits_{n=1}^{\infty} n\, [P(d/d)]^{n-1}}{\sum\limits_{n=1}^{\infty} [P(d/d)]^{n-1}} = \frac{1}{1 - P(d/d)} = \frac{1}{P(w/d)}.$$

Similarly, the average length of a wet spell is $\dfrac{1}{P(d/w)}$.

As a measure of the changeability of the weather, one can calculate $P(c)$, the probability of a change in the weather (from wet to dry or from dry to wet) between one day and the next.

From this definition, it follows that

$$P(c) = P(w)\, P(d/w) + P(w/d) \left[1 - P(w)\right]$$

$$= \frac{2\, P(w/d)\, P(d/w)}{P(w/d) + P(d/w)}\ .$$

## 4.   Data

The data used were taken from summaries of Winnipeg weather observations. The periods used were:

for April and May, 1884-1964 except 1913,

for June, July and August, 1884-1896 and 1931-1960,

for September and October, 1884-1963 except 1912, 1913.

Originally, the months of June, July and August were studied for 1931 - 1960. A good fit to the Markov chain was found. A similar study was then made for 1884-1896 to detect any possible changes in the climate. No significant differences in lengths of dry and wet spells were found between the early period and the recent period of record.

For each of these periods and for the period 1931-1960, the probabilities $P(d/d)$ and $P(w/w)$ were estimated from actual relative frequencies. These probabilities are given in Table 3.

The goodness of fit of the Markov chain model was tested using chi-square tests (Anderson and Goodman, 1957) (2). The null hypothesis tested was that the weather on a certain day is independent of the weather two days previously, given the weather on the intermediate day. The results of these tests are given in Table 4.

The 90 per cent confidence limit for chi-square is 2.71 (for one degree of freedom). Since all values obtained are lower than this, one can conclude that there is no significant departure from a first order Markov chain.

Another test of the fit of the Markov chain was a comparison of actual and predicted relative frequencies of dry spells of different durations. The standard deviations of actual from predicted values were calculated and are given in Table 5. The scale used is a relative one in the sense that a constant multiplier is arbitrary. This was chosen so that the predicted frequency of one-day dry spells was unity and the total frequency of all actual dry spells was equal to the total frequency of all predicted dry spells. The standard deviations (of actual from predicted) are quite large compared with the predictions and indicate that the Markov chain predictions are useful only as estimates of long-term averages.

The standard deviations are shown on graphs 1, 2, 3 and 4 along with actual frequencies and Markov chain predictions. The Markov chain prediction is shown by a solid line, actual frequencies by X's and one standard deviation from the predicted line by broken lines.

The actual frequencies were obtained by counting the dry spells in each month. In cases where the dry spell overlapped two months, it was generally included with the count for the month in which it ended. An exception was that dry spells continuing from October to November were considered as being in October. Many of the long "October" dry spells represented on graph 4 occurred largely in November.

Once the fit of the Markov chain model has been established and P(d/d) and P(w/w) have been estimated, other quantities describing the distribution of dry and wet days can be calculated. The following quantities were calculated and are given in Table 6.

(a) P(c) - the absolute probability of the weather changing from one type of day (wet or dry) to the other type.

(b) Average number of wet days per month together with the standard deviation in this quantity.

(c)    Average length of dry spell.

(d)    Average length of wet spell.

The actual values of (b), (c) and (d) were also obtained as a further check on the fit of the Markov chain. The quantities computed from the Markov chain are seen to be in fair agreement with the values computed directly.

It must be kept in mind that these values are averages over many years of records and short term deviations, from these, values are large.

It is noted that, of the months from April to October, June has the largest number of wet days and also has the most change-able weather (highest value of P(c)).

5.    Conclusion

The distribution of dry and wet days in the months of April through October at Winnipeg follows closely the first order Markov chain model. This model describes well the long term average distribution. Over a short time period, deviations from the predictions of the model may be very great.

APPROVED,

J. R. H. Noble,
Director,
Meteorological Branch.

6.    References

(1)    Gabriel, K.R. and J. Neumann, 1962: A Markov Chain Model for Daily Rainfall Occurrence at Tel Aviv. Quart. J. R. Met. Soc., 88, pp 90-95.

(2)    Anderson, T. W. and L. A. Goodman, 1957: Ann. Math. Stat., 28, pp 89-110.

## TABLE 1

## OCCURRENCE OF DRY SPELLS AT WINNIPEG

| No. of Dry Spells in: | April May | | June, July, August | | September | | October | |
|---|---|---|---|---|---|---|---|---|
| For the Period: | 1931 -1960 | 80 Years | 1931 -1960 | 46 Years | 1931 -1960 | 78 Years | 1931 -1960 | 78 Years |
| Of Duration: 1 | 16 | 56 | 78 | 106 | 13 | 40 | 8 | 21 |
| (in days) 2 | 22 | 68 | 63 | 90 | 14 | 33 | 4 | 16 |
| 3 | 15 | 45 | 56 | 83 | 11 | 30 | 12 | 24 |
| 4 | 14 | 40 | 37 | 53 | 10 | 27 | 5 | 17 |
| 5 | 16 | 46 | 30 | 50 | 7 | 21 | 3 | 10 |
| 6 | 16 | 36 | 30 | 52 | 6 | 19 | 2 | 13 |
| 7 | 7 | 27 | 27 | 39 | 7 | 14 | 3 | 7 |
| 8 | 12 | 21 | 18 | 28 | 5 | 8 | 1 | 6 |
| 9 | 9 | 21 | 12 | 21 | 3 | 14 | 5 | 14 |
| 10 | 4 | 12 | 12 | 17 | 1 | 8 | 4 | 9 |
| 11 | 8 | 22 | 13 | 17 | 2 | 3 | 6 | 12 |
| 12 | 6 | 16 | 6 | 12 | 1 | 6 | 1 | 5 |
| 13 | 5 | 12 | 6 | 8 | 2 | 6 | 1 | 4 |
| 14 | 0 | 13 | 6 | 10 | 1 | 5 | 2 | 8 |
| 15 | 2 | 12 | 3 | 6 | 2 | 6 | 1 | 6 |
| 16 | 5 | 7 | 4 | 5 | 4 | 5 | 3 | 7 |
| 17 | 1 | 2 | 4 | 5 | 0 | 2 | 2 | 4 |
| 18 | 3 | 7 | 2 | 2 | 4 | 4 | 4 | 8 |
| 19 | 2 | 5 | 2 | 3 | 1 | 4 | 1 | 8 |
| 20 | 4 | 7 | 2 | 2 | 0 | 0 | 2 | 5 |
| 21 | 3 | 4 | 2 | 2 | 2 | 4 | 3 | 4 |
| 22 | 1 | 3 | 4 | 4 | 1 | 2 | 2 | 3 |
| 23 | 0 | 3 | 1 | 1 | 0 | 2 | 0 | 3 |
| 24 | 0 | 2 | 0 | 0 | 0 | 1 | 1 | 4 |
| 25 | 1 | 5 | 1 | 4 | 0 | 2 | 1 | 3 |
| Longer than 25 days | 26-3 | 26-5 | 28-1 | 27-2 | 26-1 | 26-2 | 26-1 | 26-5 |
| | 27-1 | 27-4 | 29-1 | 28-1 | | 28-1 | 29-1 | 27-3 |
| | | 28-4 | | 29-2 | | | 30-1 | 28-2 |
| | 31-2 | 29-4 | 31-1 | 30-1 | | 30-1 | 32-1 | 29-2 |
| | | 30-1 | | 31-1 | | 38-1 | 36-1 | 30-3 |
| | | 31-3 | | 32-1 | | | 40-1 | 31-1 |
| (Duration-No. of Occurrences) | | 32-1 | | 48-1 | | 43-1 | 41-1 | 32-2 |
| | | 33-1 | | 53-1 | | | 54-1 | 33-1 |
| | | 34-1 | | | | | 70-1 | 34-2 |
| | | 35-2 | | 54-1 | | | | 35-3 |
| | | 46-1 | | 64-1 | | | | 36-1 |
| | | | | | | | | 38-1 |
| | | 66-1 | | | | | | 40-2 |
| | | | | | | | | 41-1 |
| | | | | | | | | 43-1 |
| | | | | | | | | 45-1 |
| | | | | | | | | 52-1 |
| | | | | | | | | 54-1 |
| | | | | | | | | 60-1 |

## TABLE 2

## LONG DRY SPELLS

66 days - 1915 - February 15 to April 21.
     - 0.31 inches of precipitation in period.

54 days - 1900 - April 9 to June 1.
     - 0.21 inches of precipitation in period.

53 days - 1917 - April 18 to June 9.
     - 0.08 inches of precipitation in period.

64 days - 1961 - May 8 to July 10.
     - 0.21 inches of precipitation in period.

52 days - 1961 - October 12 to December 2.
     - 0.18 inches of precipitation in period.
       (All of August, 1961 was also dry   ( 0.16 inches) ).

54 days - 1948 - August 22 to October 14.
     - 0.11 inches of precipitation in period.

70 days - 1939 - October 7 to December 15.
     - 0.26 inches of precipitation in period.

60 days - 1901 - October 9 to December 7.
     - 0.27 inches of precipitation in period.

## TABLE 3

### PROBABILITIES P(d/d) AND P(w/w) DERIVED
### FROM ACTUAL RELATIVE FREQUENCIES

| Month | P(d/d) | | P(w/w) | |
|---|---|---|---|---|
| | Full Period of Record | 1931-1960 | Full Period of Record | 1931-1960 |
| April | .894 | .906 | .242 | .299 |
| May | .862 | .859 | .308 | .272 |
| June | .783 | .779 | .252 | .274 |
| July | .820 | .822 | .207 | .195 |
| August | .842 | .823 | .254 | .256 |
| September | .859 | .867 | .281 | .298 |
| October | .907 | .913 | .274 | .298 |

## TABLE 4

### CHI-SQUARE TEST OF GOODNESS OF FIT OF FIRST ORDER MARKOV CHAIN MODEL TO ACTUAL FREQUENCY DISTRIBUTION

| Months | Period of Record | $X^2$ for Intermediate Day Dry | $X^2$ for Intermediate Day Wet |
|---|---|---|---|
| April, May | 30 yrs. | 2.4 | 2.2 |
| June, July, August | 30 yrs. | 0.02 | 0.05 |
| September | 78 yrs. | 0.6 | 0.04 |
| October | 78 yrs. | 1.5 | 0.07 |

## TABLE 5

### STANDARD DEVIATIONS OF ACTUAL FREQUENCIES OF DRY SPELLS FROM PREDICTED FREQUENCIES

| Months | for dry spells of 1-15 days | for dry spells of 16 or more days |
|---|---|---|
| April, May | .083 | .030 |
| June, July August | .034 | .009 |
| September | .060 | .028 |
| October | .163 | .055 |

## TABLE 6

### VALUES OF VARIOUS PARAMETERS DERIVED FROM MARKOV CHAIN MODEL

| Month | April | May | June | July | August | September | October |
|---|---|---|---|---|---|---|---|
| P(c) | .185 | .230 | .337 | .293 | .261 | .236 | .165 |
| Mean No. of Wet Days Per Month (Actual No.) | 3.7 | 5.1 | 6.6 | 5.8 | 5.6 | 4.8 | 3.5 |
| (No. Calculated from Markov Chain) | 3.7 | 5.15 | 6.75 | 5.7 | 5.4 | 4.9 | 3.5 |
| Standard Deviation of Wet Days/Month (Actual Value) | 1.9 | 2.8 | 2.5 | 2.0 | 2.55 | 2.6 | 2.0 |
| (Value Calculated from Markov Chain) | 2.1 | 2.5 | 2.4 | 2.2 | 2.3 | 2.3 | 2.1 |
| Mean Length of Dry Spell (Actual No. of Days) | 10.5 | 8.4 | 4.9 | 5.2 | 7.5 | 7.3 | 12.8 |
| (No. Calculated from Markov Chain) | 9.4 | 7.3 | 4.6 | 5.6 | 6.3 | 7.1 | 10.7 |
| Mean Length of Wet Spell (Actual No. of Days) | 1.34 | 1.43 | 1.32* | 1.32* | 1.32* | 1.40 | 1.42 |
| (No. Calculated from Markov Chain) | 1.32 | 1.44 | 1.34 | 1.26 | 1.34 | 1.39 | 1.38 |
| No. of Years of Records Used | 80 | 80 | 43 | 43 | 43 | 78 | 78 |

\* for June, July and August combined
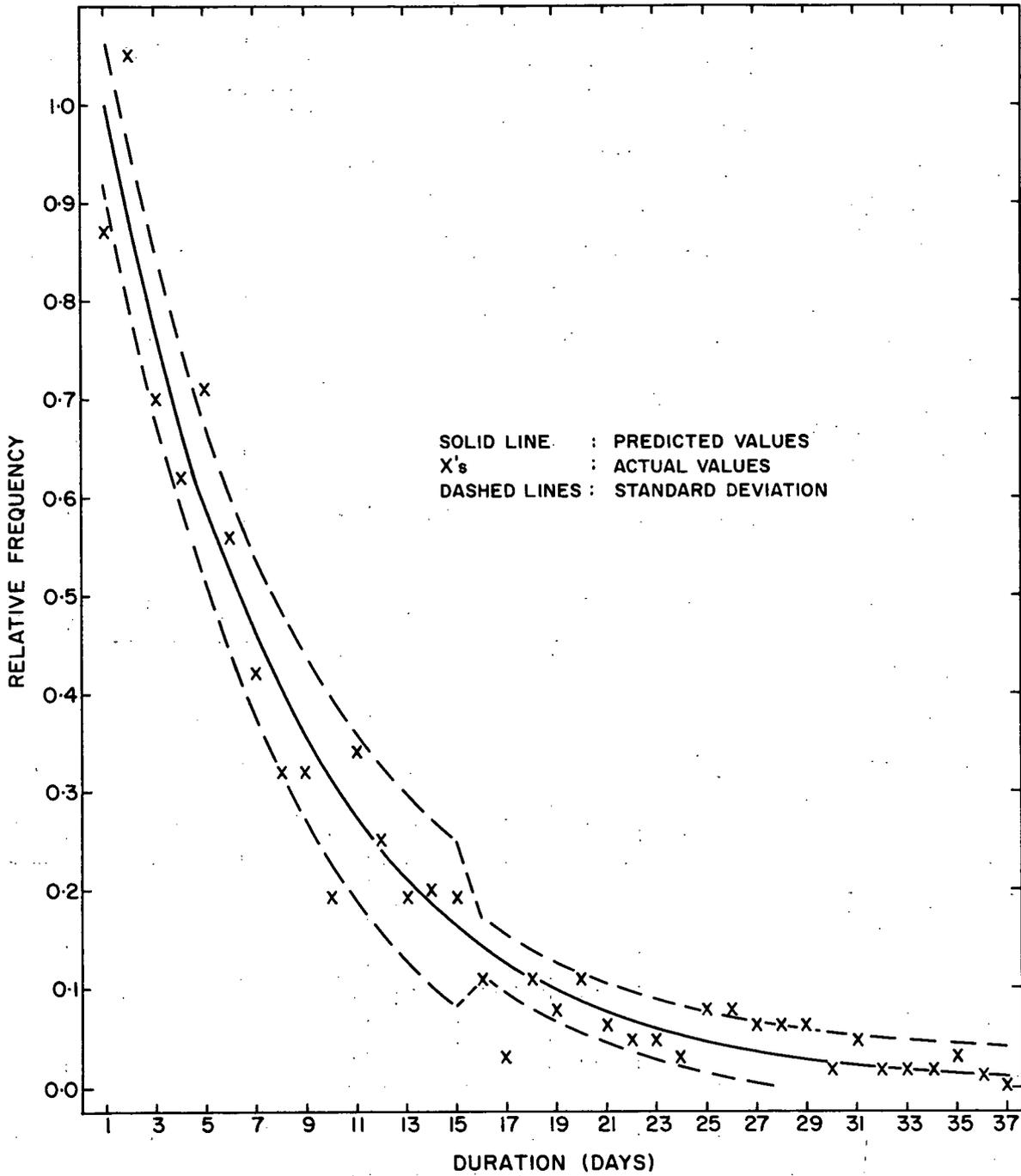
Figure 1
Comparison of Actual to Predicted Frequencies of Dry Spells for April, May.
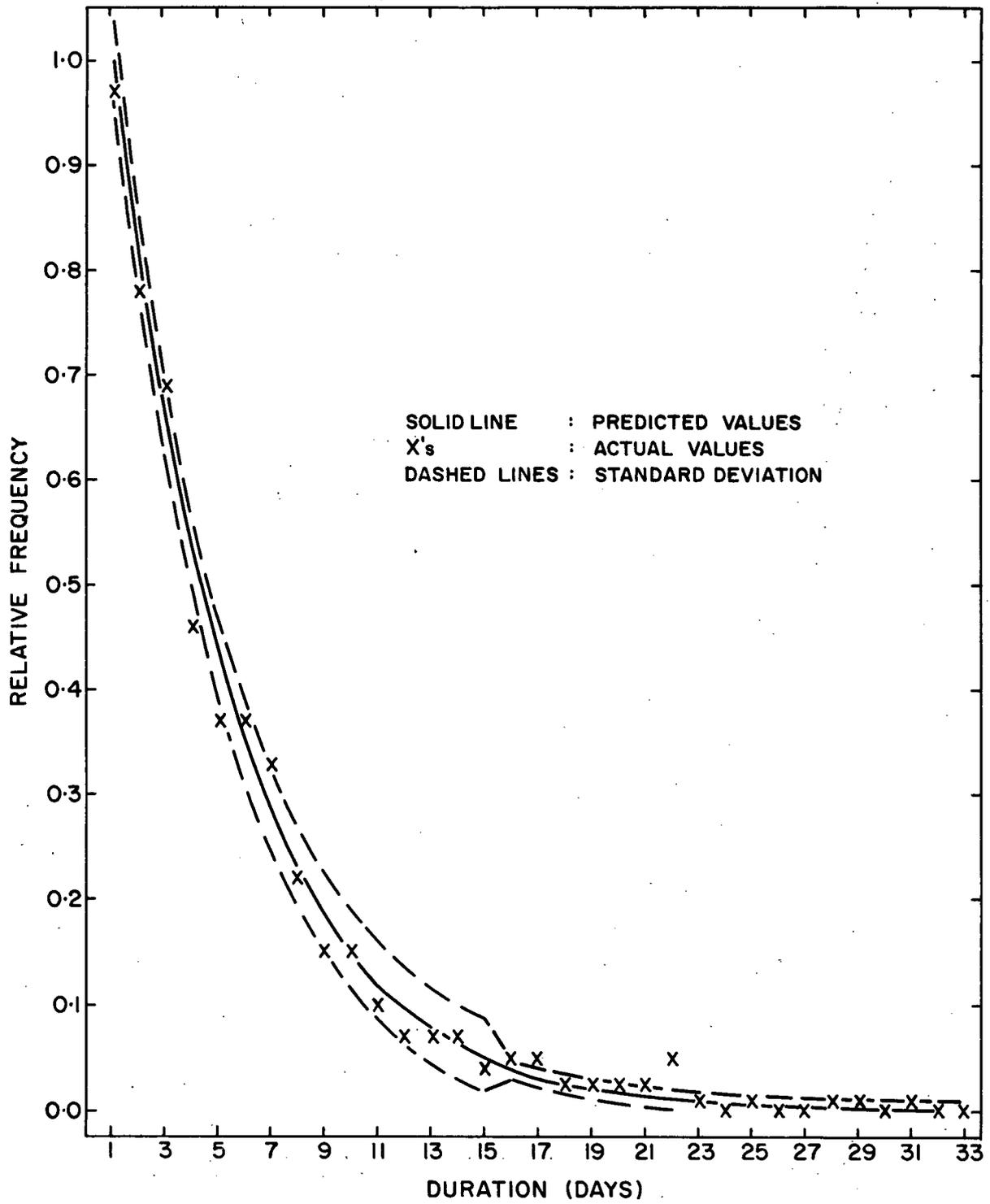
Figure 2
Comparison of Actual to Predicted Frequencies of Dry Spells for June, July, August.
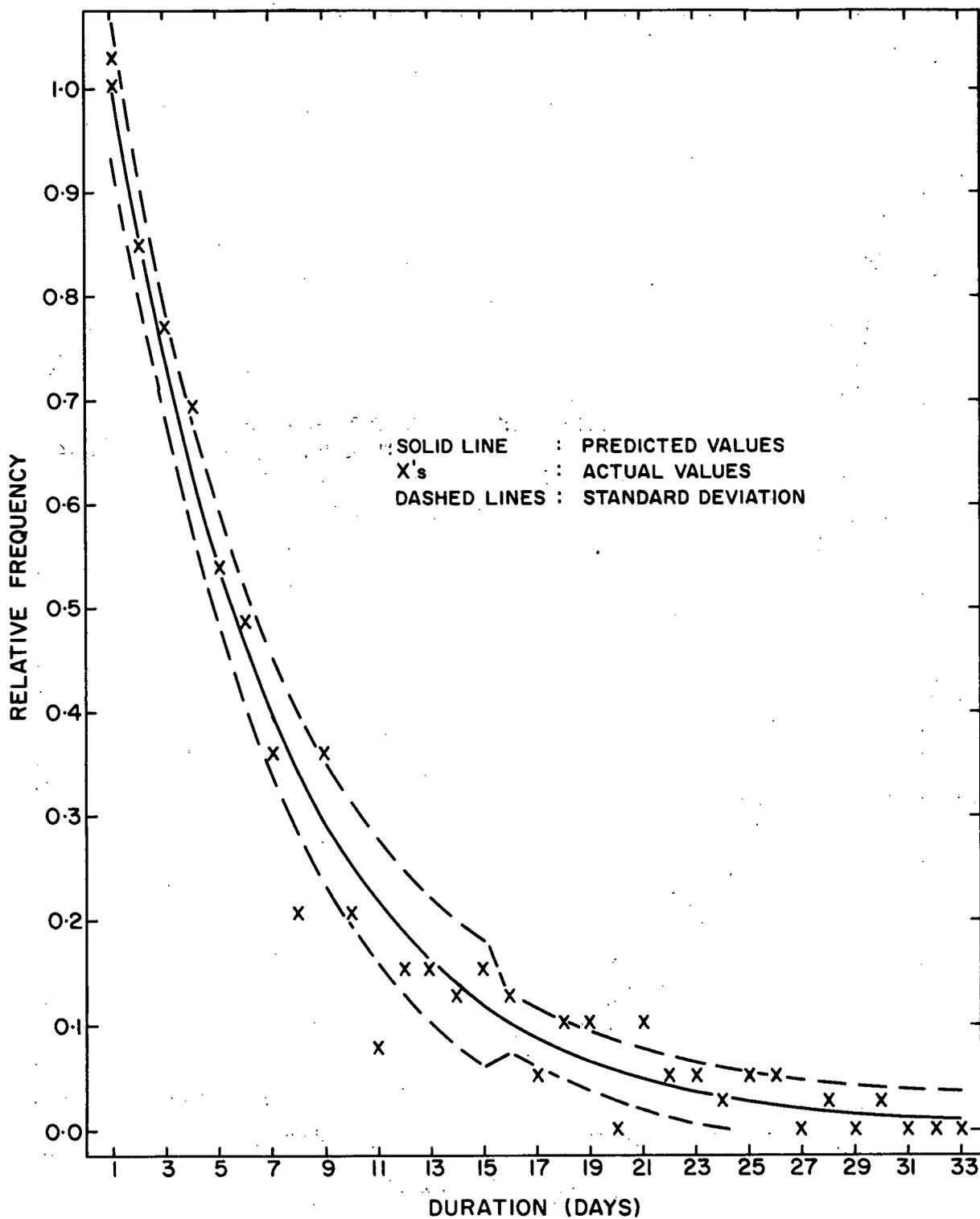
Figure 3
Comparison of Actual to Predicted Frequencies of Dry Spells for September.
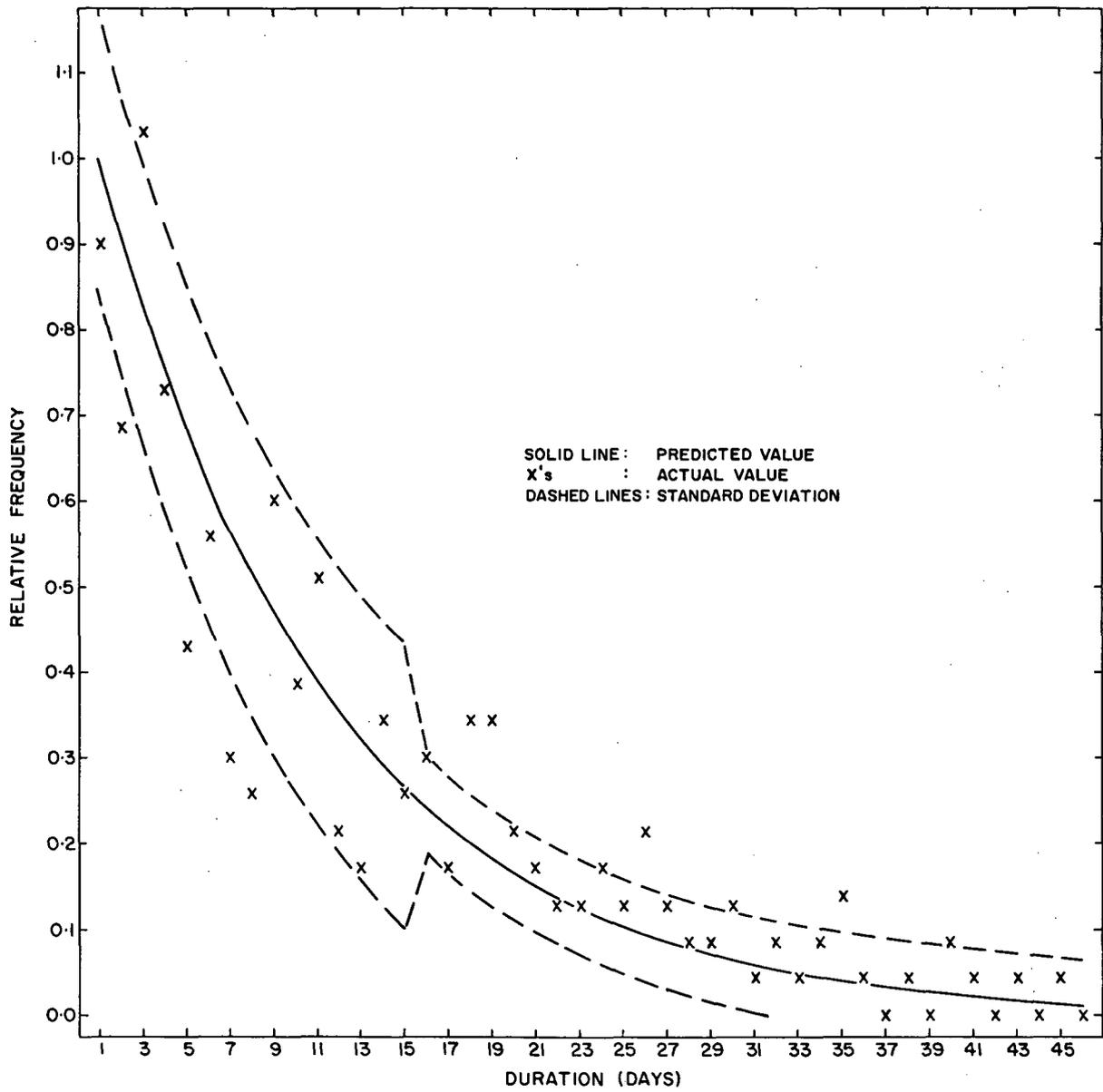
Figure 4
Comparison of Actual to Predicted Frequencies of Dry Spells for October.

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ABSTRACT: A first order Markov chain probability model is shown to conform closely to the long term average distribution of wet and dry spells at Winnipeg during the months of April to October inclusive. Consequently, other parameters, such as, (a) probability of changeable weather, from wet to dry, and vice-versa, (b) number of wet days per month, and (c) average lengths of wet and dry spells, were calculated using the model. These values are shown to be in fair agreement with actual average values over the long term period. However, short term deviations from the model may be large.