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DEPARTMENT OF TRANSPORT
METEOROLOGICAL BRANCH
TORONTO WEATHER OFFICE
BOX 159
TORONTO A M F, ONTARIO

AN EXPLANATION FOR THE OBSERVED ICE GROWTH AT PORT HARRISON

BY

D.M. LEAHEY

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CANADA - DEPARTMENT OF TRANSPORT - METEOROLOGICAL BRANCH
315 Bloor Street, West,
Toronto 5, Ontario.

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ABSTRACT

An analysis of ice measurements at Port Harrison has led to the following conclusions.

(1) At the mouths of rivers emptying into bodies of freezing salt water, ice growth will be increased by losses of heat from the fresh to the salt water.

(2) A study of ice thickness at Port Harrison can determine the time at which fresh water ceased to flow past the measuring site. This knowledge should be useful in estimating the winter discharge of the Innuksuak River.

EXPLICATION DE LA CROISSANCE DE LA GLACE OBSERVÉE À PORT HARRISON

par

D. M. Leahey

RÉSUMÉ

Une analyse des mesures des glaces à Port Harrison a permis de tirer les conclusions suivantes:

(1) A l'embouchure des rivières qui se déversent dans des étendues d'eau salée au point de congélation, la croissance de la glace est accrue par la transmission de chaleur de l'eau douce à l'eau salée.

(2) Une étude de l'épaisseur de la glace à Port Harrison peut permettre de déterminer le moment où l'eau douce a cessé de couler au poste de mesure. Cette connaissance devrait aider à l'évaluation du débit hivernal de la rivière Innuksuak.

AN EXPLANATION FOR THE OBSERVED ICE GROWTH AT PORT HARRISON

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D. M. Leahey

(Manuscript Received February 17, 1967)

1. Introduction

Both Port Harrison (58° 27'N, 78° 09'W) and Baker Lake (64° 18'N, 96° 00'W) are beside bodies of fresh water. Reported ice thicknesses at the former station have been consistantly 20 to 25 cm greater than those at the latter.

This difference is surprising as Port Harrison is warmer and has more snow cover than Baker Lake.

A modified version of Stefan's growth equation is used in this paper to explain the anomaly. Additional ice-growth at Port Harrison is shown to result from the presence of salt water which intrudes up the mouth of Innuksuak River from Hudson Bay. An analysis of the ice measurements also shows that the winter discharge of the Innuksuak River ceases before the onset of the melt season.

2. Stefan's Equation, Modified to take into Account Variable Snow Cover

The question of ice growth due to heat transfer was treated theoretically by Stefan (1891). The heat conducted by a unit area of ice from the water in a period of time dt is:

$$k \frac{d\phi}{dZ}$$

where $\frac{d\phi}{dZ}$ is the vertical temperature gradient at the ice-water interface. k is the coefficient of thermal conductivity of ice. $\frac{d\phi}{dZ}$ may be approximated by $\frac{\beta}{l}$ where l is the thickness of the ice (cm) and β is the difference in temperature (°C) between the lower and upper surfaces of the ice.

This heat loss results in the formation of an additional layer of ice. In this manner:

$$\frac{k\beta}{l} = \varsigma L dl$$

where L is the heat of fusion and ζ is the density of the ice. If we assume that k, L and ζ are independent of time and ice thickness.

$$k \int_0^t \beta \, dt = \zeta L \int_0^l l \, dl$$

$$\text{or } k \sum \beta = \frac{\zeta L l^2}{2}$$

Here, the integral has been approximated by the summation over the growing period of the daily mean values of β. This is Stefan's equation. As blankets of snow usually cover the ice one cannot assume that the temperature of the ice surface equals the air temperature. Stefan's equation can, however, be modified to take into account variable snow cover. Assuming that there is not heat storage in the snow:-

$$K_s \frac{(\theta_i - \theta_s)}{\delta} = - Q \qquad (1)$$

where K_s = the thermal conductivity of the snow (cal/day. cm.$^\circ$ C).

θ_i = temperature of the ice surface $^\circ$ C

θ_s = temperature of the snow surface $^\circ$ C

δ = depth of the snow (cm)

Q = heat absorbed at the snow surface (cal/cm.^{2}. day.)

Solving equation (1) for θ_i we have

$$\theta_i = \theta_s - \frac{Q\delta}{K_s} \qquad (2)$$

If we take the freezing point of fresh water to be 0° C, $\beta = - \theta$. Using this result together with the relation expressed in equation (2) we have

$$l^2 = - 2k / \zeta L \sum (\theta_s - Q\delta / K_s) \qquad (3)$$

This is the modified version of Stefan's equation applicable to fresh water ice.

3. Values of K_s and $2k/\rho L$

A formula has been presented by Abels (1892) which relates the heat conductivity of snow to its density ρ_o.

$$K_s = .0068\ \rho_o^2 \text{ cal/cm. sec. } ^\circ C.$$

Bilello (1964), has published weekly values of snow densities measured at northern stations for the period 1960-61. The average of his values for ρ_o is $.380 g/cm^3$. Substituting this value into Abel's equation and converting from seconds to days we find $K_s = 85$ cal/cm. day. $^\circ C$. With this K_s equation (3) becomes:

$$l^2 = -2k/\rho L \sum (\theta_s - \theta\ \delta/85) \qquad (4)$$

An effective value of $2k/\rho L$ for fresh water ice was found from Baker Lake data by the method of least squares. i.e. equation (4) was adjusted so that

$$\frac{\partial}{\partial\, k/\rho L} \sum' (l - l')^2 = 0 \qquad (5)$$

here l' the observed value of ice thickness

l the ice thickness predicted by equation (4)

$\sum'$ summation over the number of observations

Solving equation (5) for $2k/\rho L$ we get:

$$2k/\rho L = \left(\frac{\sum' l' (- \sum (\theta_s - Q\,\delta/85))^{\frac{1}{2}}}{\sum' \sum (\theta_s - Q\,\delta/85)} \right) \qquad (6)$$

For this equation, θ_s was assumed equal to the air temperature. Ice thickness and snow depth data were obtained from an annual circular published by the Canadian Meteorological Service entitled "Ice thickness Data from Canadian Selected Stations". Q was assumed to equal the climatic values of the net radiation as published by Vowinckel and Orvig, (1964).

A value of 10.8 was calculated for $2k/\rho L$ from 4 years data (1960-64). Equation (4) now becomes

$$l^2 = -10.8 \sum (\theta_s - Q\delta/85) \qquad (7)$$

The substantial success of this equation can be seen in Figure 1. The correlation between the observed and predicted values of l was .996. Further tests of the accuracy of equation (7) with ice growth data from Knob Lake and Norman Wells yielded correlation coefficients in excess of .97. A variation of the modified Stefan equation suitable to the growth of sea water ice has been tested with similar results (Leahey, 1966).

4. Additional Ice Growth at Port Harrison Due to the Presence of Saltwater

Figure 2 shows the observation site at Port Harrison. The Innuksuak River flows past the station in summer at a speed of about 4 knots. (Hudson Bay Pilot). It is shallow with an average width of less than 100 feet.

At Port Harrison the average snow depth over the period to be treated never exceeded 13 cm. Under this circumstance of light snow cover, the modified Stefan's equation will not be sensitive to values of Q. (Leahey, 1966). In working with Port Harrison, data Q will be assumed equal to the radiation loss at Knob Lake (50° 48'N, 60° 49' W). This was the nearest station at which estimates of Q were readily available.

Ice measurements at Port Harrison were consistently greater than at other stations with comparable climates. Certainly, as illustrated in figure 3, ice at the main measuring site grows faster than one would expect from the simple theory of equation (7).

To account for the extra ice growth, it was necessary to postulate a heat sink other than the atmosphere. As salt water freezes at a lower temperature than fresh it can provide such a sink.

Let us assume that the fresh water loses heat at a rate q_a up through the ice to the atmosphere and at a rate q_b down to the salt water. This is illustrated in figure 3. The thickness of the layer of brackish water will vary with tide and turbulence.

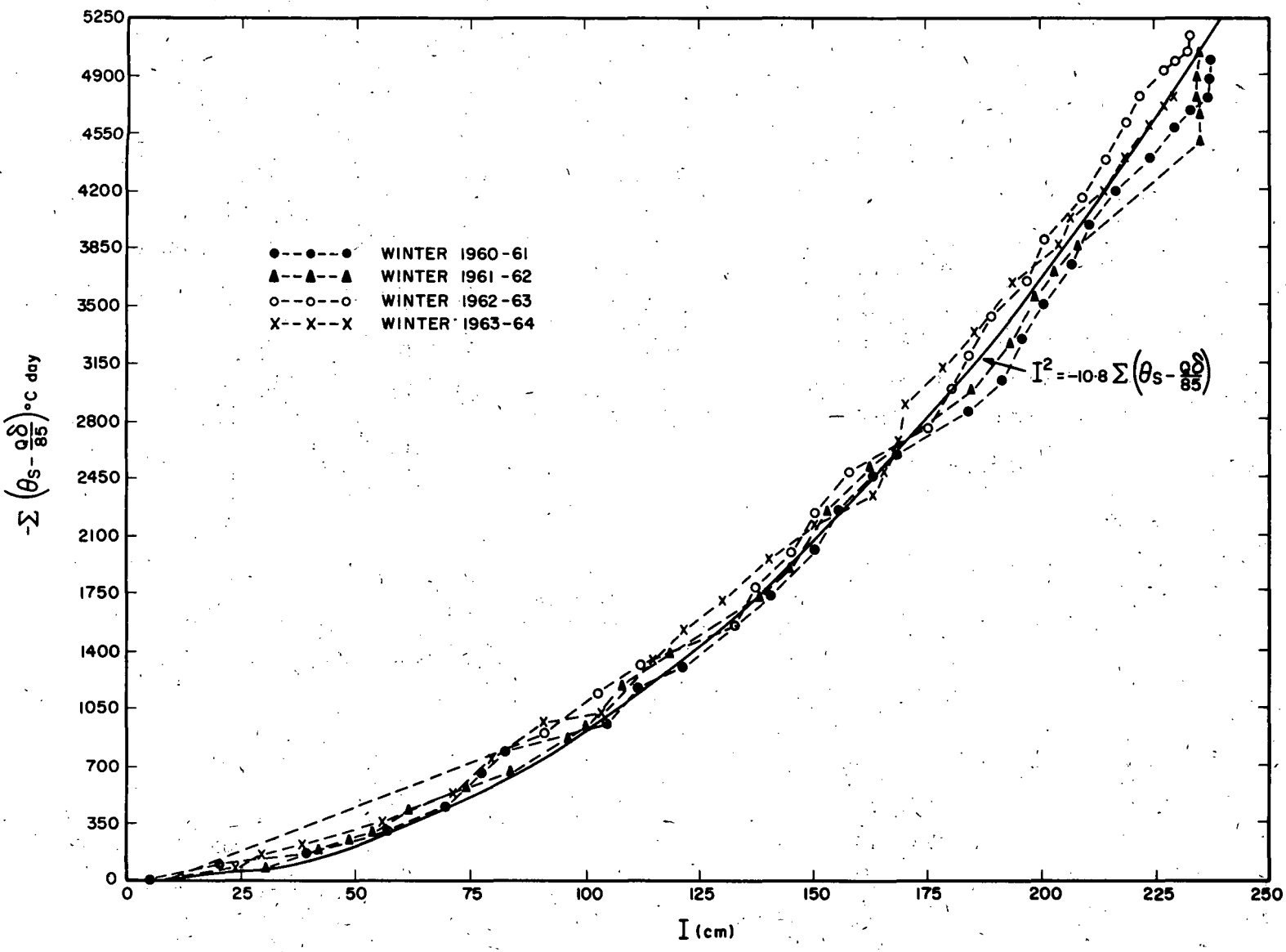

Figure 1
Plot of Observed Ice Thickness I, against $-\left(\theta_S - \frac{QS}{85}\right)$ for Baker Lake

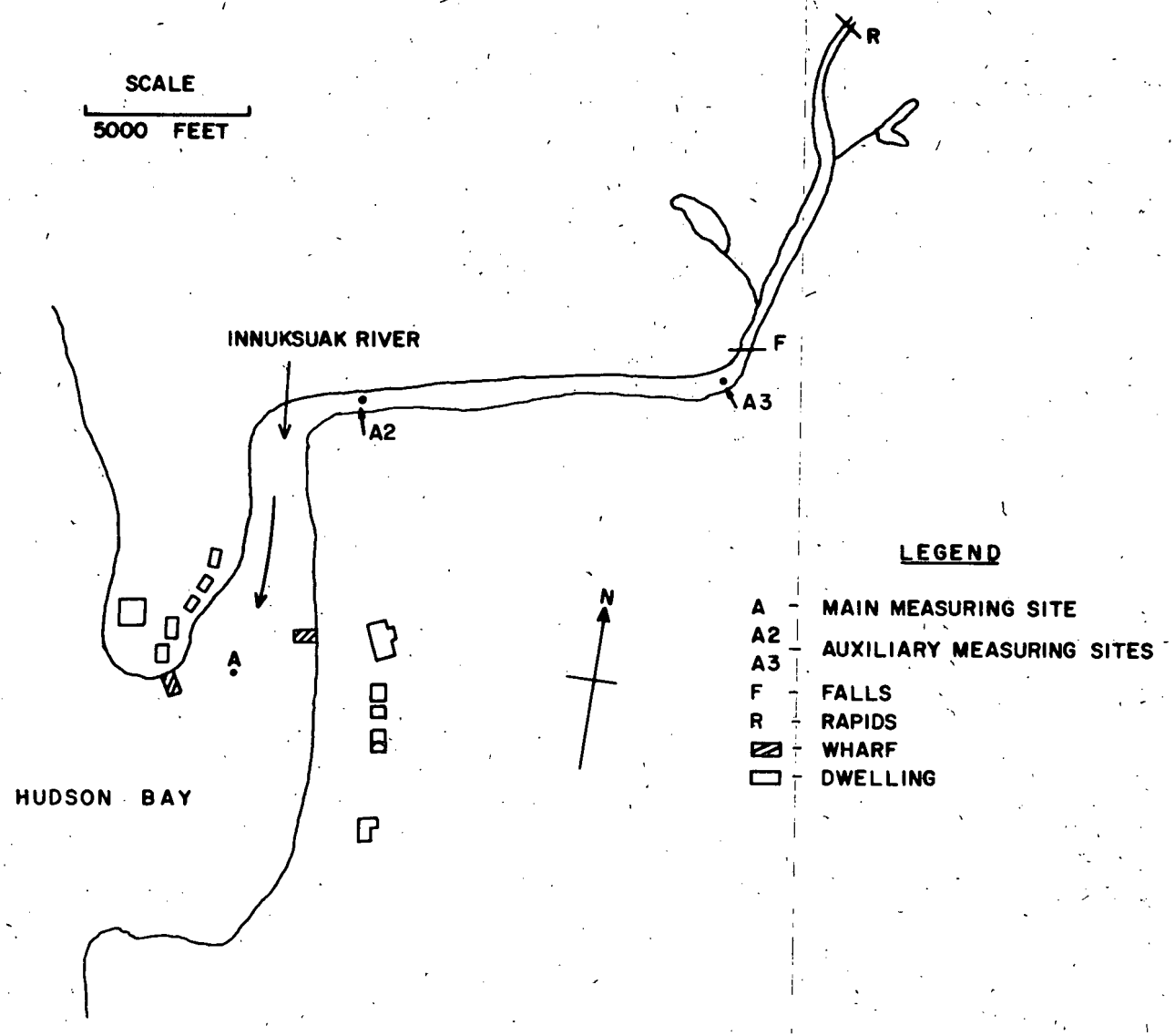

Figure 2
Map of Port Harrison (58° 27'N, 78° 09'W) and Environs

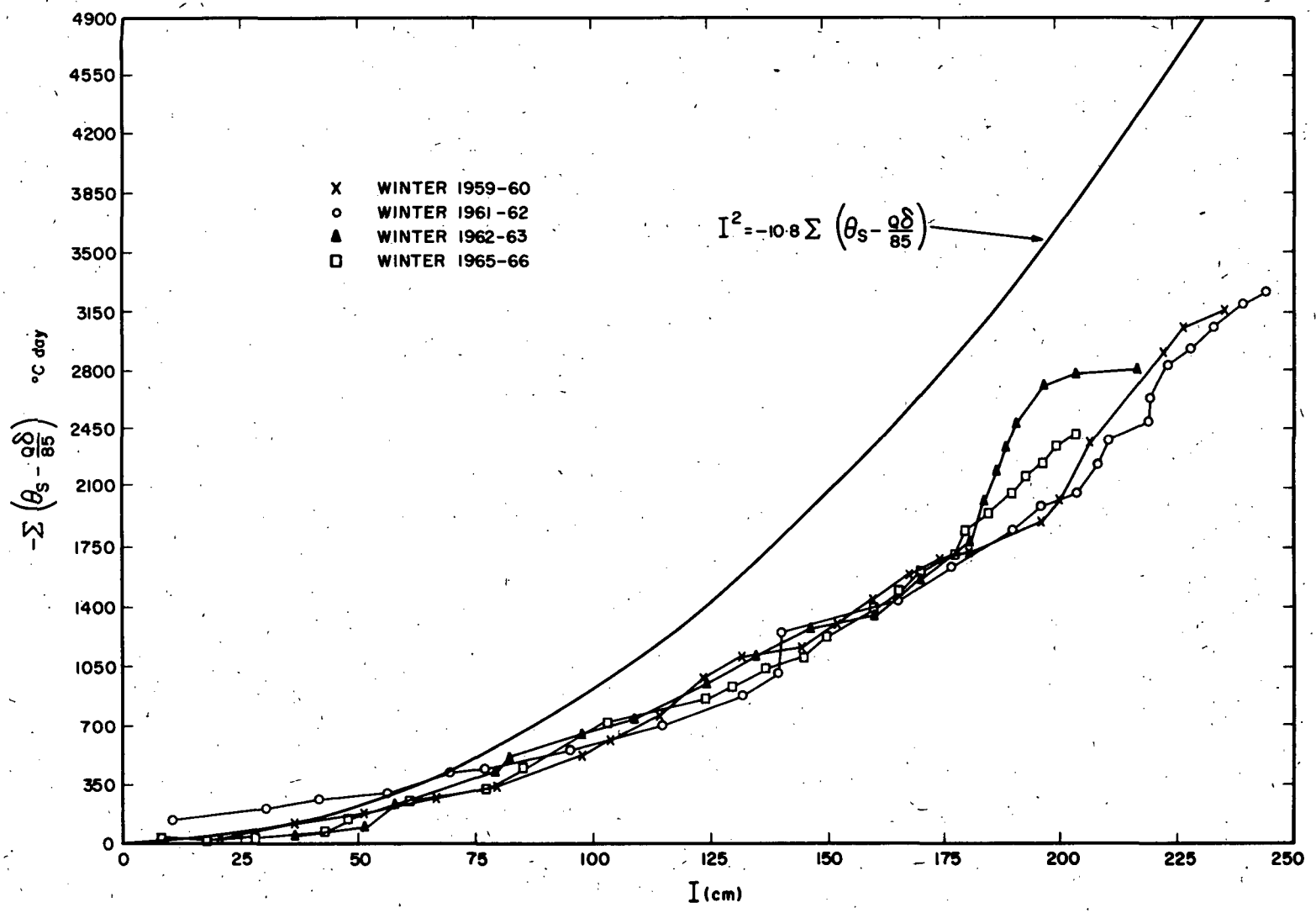

Figure 3
Plot of Observed Ice Thickness, 1, against - $(\theta_S - \frac{QS}{85})$ at Port Harrison

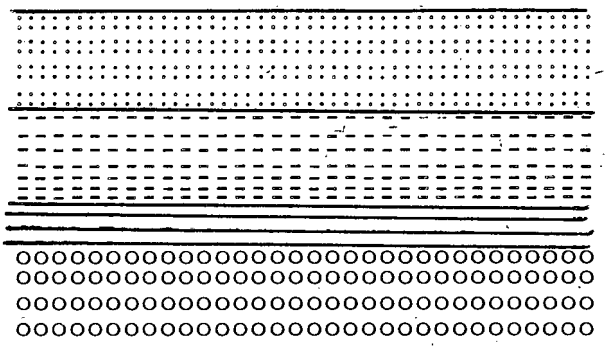

Figure 4: Showing fresh water at a mouth of a river losing heat to two sinks, through the ice to the atmosphere and to the sea water.

Now, water at 0° C loses heat in the form of latent heat.

$$\rho L \frac{dl}{dt} = q_a + q_b$$

$$= k \frac{d\phi}{dl} + k_w \frac{d\phi}{dY}$$

here, k_w heat conductivity of the brackish water

$\frac{d\phi}{dY}$ the effective temperature gradient at the fresh water - brackish water interface. The other quantities have been previously defined.

By assuming that there is a linear temperature gradient through the ice we obtain.

$$l^2 = 2\int_0^l k/\rho L \ \theta_i \, dt + 2\int_0^l \frac{k_w}{\rho L} \frac{d\phi}{dY} \, l dt$$

$$\text{or } l^2 = -10.8 \sum(\theta_s - Q\sigma/85) + 2 \sum \frac{k_w}{\rho L} \frac{d\phi}{dY} \, l \qquad (8)$$

In the above equation use has been made of the results of section 2.

If $\frac{2k}{\rho L}\frac{w}{}\frac{d\phi}{dY}$ is assumed constant with time, its value proper to Port Harrison can be found by having resort again to the method of least squares. We obtain:

$$a = \sum' \left(\frac{(1' \sum 1}{(10.8 \sum(\theta_s - \frac{Q\delta}{85}))^2} - \frac{\sum 1}{10.8 \sum(\theta - \frac{Q\delta}{85})} \right) \Bigg/ \sum' \left(\frac{1'}{10.8(\theta_s - \frac{Q\delta}{85})} \right)^2 \qquad (9)$$

$$a = \frac{2k}{\rho L}\frac{w}{}\frac{d\phi}{dY} \text{ (assumed constant)}$$

The quantity $\sum_1$ in equation (9) may be calculated by making the following assumptions:

(1) $1 = 1'$

(2) There is a constant rate of growth between observations.

With these two approximations an average value for a of 1.26 cm was calculated. Figure 5 shows the accuracy to which the equation

$$1^2 = -10.8 \sum(\theta_s - \frac{Q\delta}{85}) + 1.26 \sum 1$$

fits the observed data.

There is an apparent divergence of the observed from the predicted ice thickness as the ice grows beyond 150 cm.

5. Ice Growth After the Exhaustion of the Fresh Water Source

As shown in Figure 4, the "two heat sink", theory of the previous section overestimates ice growth at the end of the growing season. Maybe the observed growth can be explained if we assume that at some stage the second heat sink is removed.

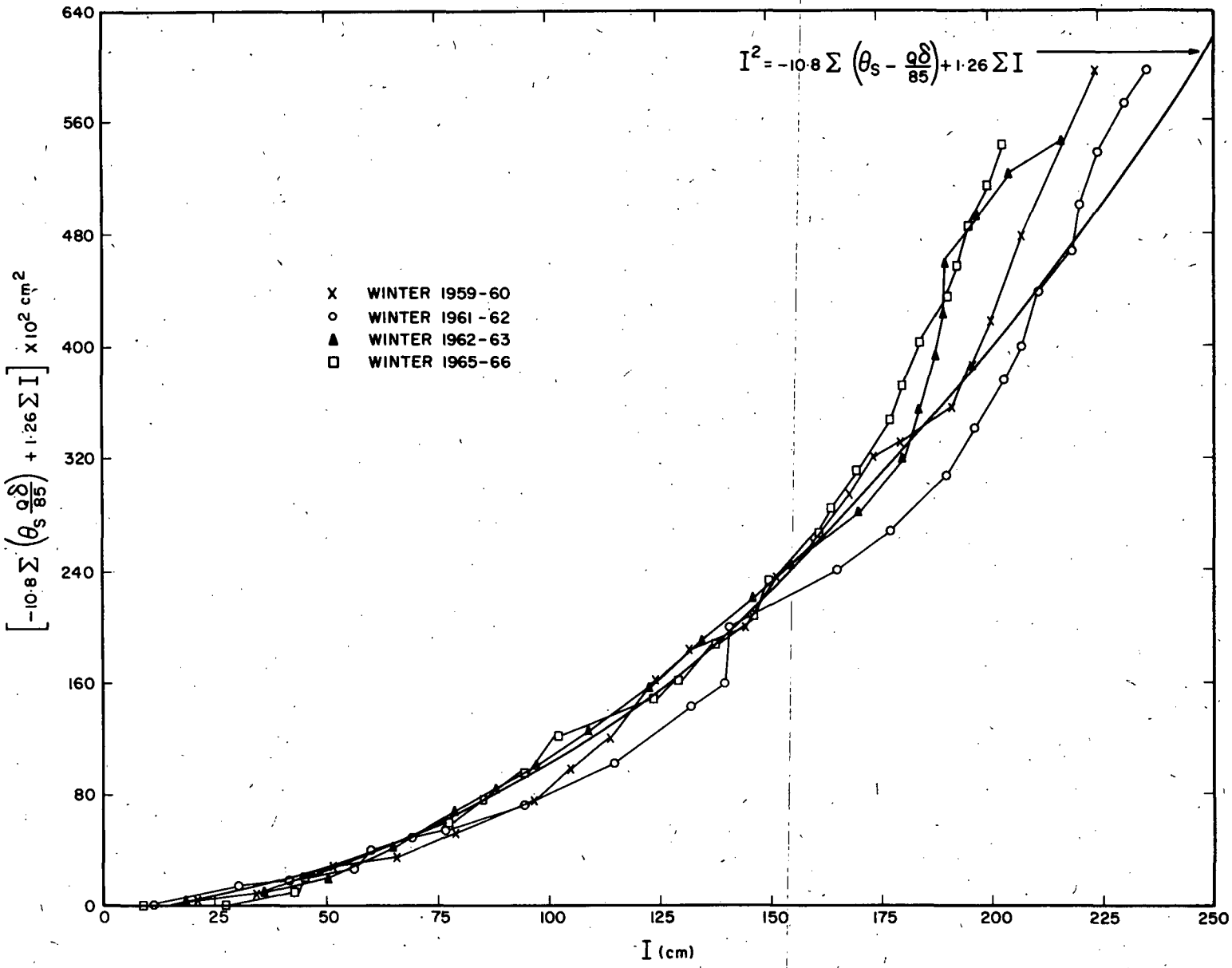

Figure 5
Plot of Observed Ice Thickness I, at Port Harrison against $-10.8 \left(\theta_S - \frac{QS}{85}\right) + 1.26$

Let us suppose that when the ice reaches a thickness l_o, the fresh water in Figure 4 is exhausted. Employing the simple theory of the previous section, it is easily shown that ice will continue to grow according to the formula

$$l^2 = l_0^2 - \sum_o B\left(\theta_s - \frac{Q\delta}{85} - C\right) \qquad (10)$$

here, $l - l_o$ = the thickness of sea ice.

$\sum_o$ = summation over the relevant period beginning with the exhaustion of the fresh water.

B = an effective constant involving the heat conductivity, density and latent heat of fusion appropriate to the entire layer of fresh water - sea water ice. It will be a function of $\frac{l - l_o}{l_o}$

C = freezing temperature of the salt water.

Lacking information about the values of B and C, the author took them to have constant values equal to those for fresh water, (i.e. 10.8 cm^2/day and 0° C respectively.)

Values of l_o were found by the method of trial and error. An l_o was assumed. Equation (10) was then employed to predict the succeeding ice growth. The value of l_o that gave the most satisfactory result was accepted. 192, 210, 169, and 163 cm were the values of l_o calculated for the winters, 1959-60, 1961-62, 1962-63, and 1965-66 respectively. One should be able to relate these to the winter discharge of the river. Such a study is, however, beyond the scope of this paper.

Equation (9) was next applied only to data collected from the beginning of the growing season to the time $l = l_o$. A new value of $\underline{a}$ equal to 1.45 cm was calculated.

The very good agreement between the subsequently predicted and observed ice growth is shown in Figure 6. The correlation coefficients were in excess of .995.

6. Additional Verifying Measurements

Ice thickness measurements were made at auxiliary stations on April 21, 1966. (See Figure 2). The findings were in agreement with theory.

At station A_2, the ice thickness was 210 cm. This is an increase of 12 cm over the value reported on April 22 at the main site. Such a difference can be expected as the layer of fresh water should persist longer at station A_2 than at the main site.

Further credence is given to the theory by the ice measurements at site A_3. Here, the ice was only 160 cm thick. This compares very favourably to the value of 159 cm which equation (7) predicts for simple fresh water growth. It appears then that salt water does not reach up river to the third measuring site.

It should be stressed that there are no actual measurements to show that salt water penetrates up river past the first bend or that fresh water ceases to flow downstream through the main measuring site.

7. Conclusion

When the only heat sink is the atmosphere, fresh water ice grows according to the equation

$$l^2 = -10.8 \sum (\theta_s - \frac{Q\delta}{85})$$

But at the mouth of the Innuksuak River, fresh water comes in contact with colder sea water from Hudson Bay. The salt water provides an additional heat sink, which precipitates faster ice growth described by the equation

$$l^2 = -10.8 \sum (\theta_s - \frac{Q\delta}{85}) + 1.45 \sum l$$

If l_o is the thickness of the ice at the time fresh water ceases to flow past the measuring site further ice growth proceeds in a manner described by the equation

$$l^2 = l_0^2 - 10.8 \sum (\theta_s - \frac{Q\delta}{85})$$

The difference $l - l_0$ is the thickness of sea ice.

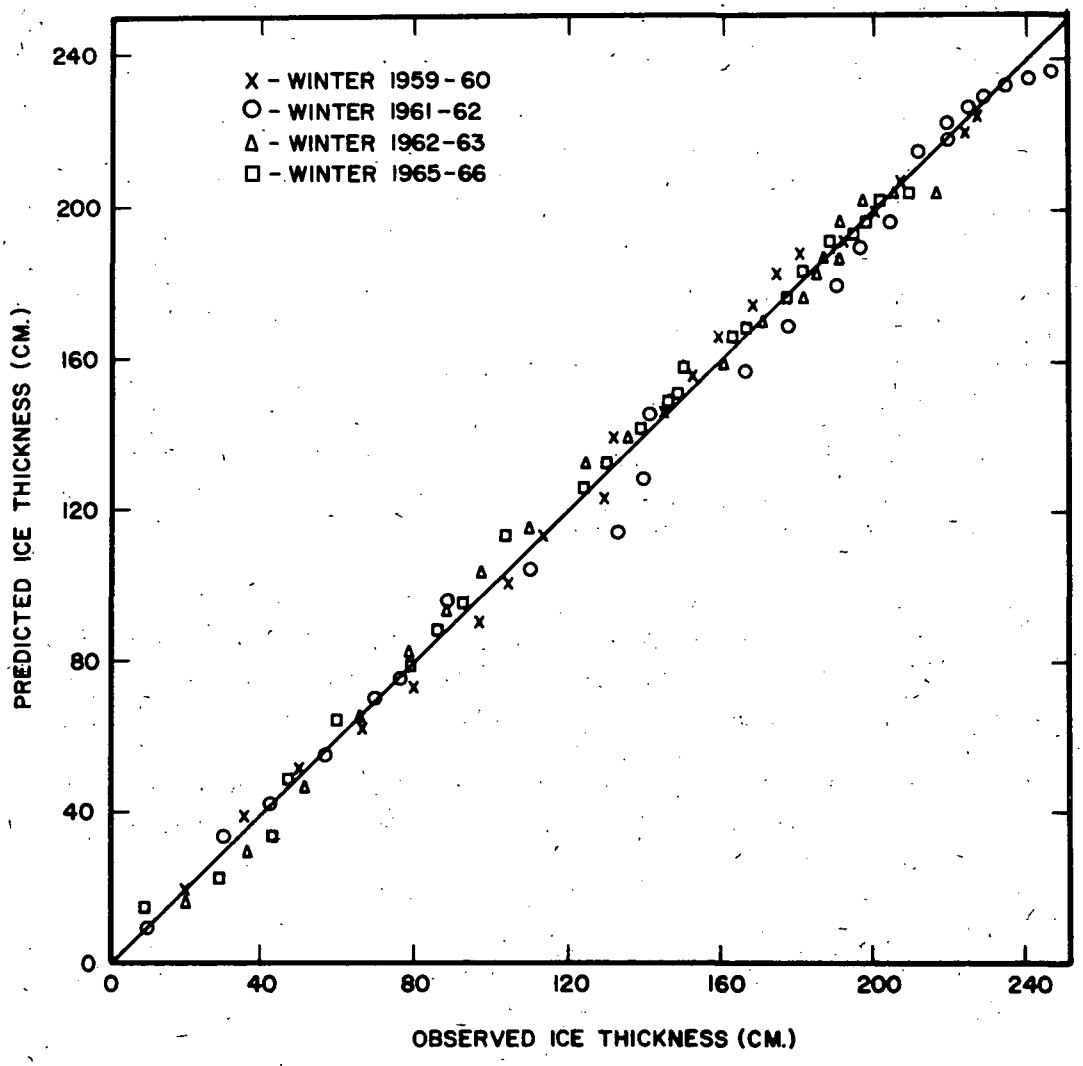

Figure 6
A Plot of Predicted Ice Thickness at Port Harrison against Observed Ice Thickness

9. Acknowledgements

I wish to thank Mr. W. E. Markham and Mr. A. P. Beaton of the Ice Central Halifax, for their suggestion and criticisms.

APPROVED,

J. R. H. Noble,
Director,
Meteorological Branch.

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