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Technical Memoranda

EIGENVECTOR DERIVED TEMPERATURE

AND

PRECIPITATION PATTERNS IN THE MARITIMES

by

E.W. BRANDON and R.T. DYER

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ABSTRACT

Results of some of the research carried out at the MWO, Halifax using eigenvector techniques, are presented. The technique is briefly explained and the results of the research are used to demonstrate some of the advantages of this approach in the analysis of climatological data.

CONFIGURATIONS DE LA TEMPÉRATURE ET DES PRÉCIPITATIONS DANS LES MARITIMES DÉRIVÉES DE L'EIGENVECTEUR

par

E. W. Brandon et R. T. Dyer

RÉSUMÉ

Les auteurs présentent les résultats de certaines recherches effectuées au Bureau météorologique des Maritimes à Halifax à l'aide de techniques qui font intervenir l'eigenvecteur.

Ils expliquent brièvement la méthode et se servent des résultats de la recherche pour montrer certains de ses avantages dans l'analyse de données climatologiques.

EIGENVECTOR DERIVED TEMPERATURE AND PRECIPITATION PATTERNS IN THE MARITIMES

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E.W. Brandon and R. T. Dyer*

1. Introduction

There exists an extensive background of literature relating eigen-vector techniques to meteorological problems. Mateer (1) employed eigenvector techniques to examine the information content of Umkehr observations. Stidd (2) has produced a remarkably lucid account of his work on precipitation patterns in Nevada using eigenvector methods.

There are several advantages associated with the use of eigen-vectors in analyzing patterns of data such as monthly mean temperatures, precipitation, etc. It is usually possible to relate the set of significant eigenvectors to physical processes. Corresponding derived fields yield information on the geographical distribution of those processes. Physical insights into the predominant climatic controls of the region under study are thus gained by an approach which has the advantage of complete objectivity. The use of this approach has the added advantage, as a rule, of significantly economizing in data storage requirements.

The climate of the Maritimes is the product of an assortment of interacting influences. In an attempt to clarify the relative importance of the various climatic controls, an investigation of precipitation and temperature patterns, employing eigenvector methods, was conducted during the summer of 1974. The results of this work are presented here.

2. Data Input

The problem of data acceptance arose at the outset. Ideally only data from those locations whose records extend over a reasonably long period should be accepted for a project of this kind. Furthermore, in order to avoid contamination from short term trends, the records from each station should involve the same time period. In this study a com-

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promise procedure was utilized in order to ensure a sufficiently dense network. It was decided, arbitrarily, that data from all stations whose records extended back over a period longer than 10 years would be accepted. There were 82 such temperature records and 93 such precipitation records.

3.   Background Theory

Let P_{ij} denote the value of the variable p at the station j for the month i, where j goes from 1 to N, (N = Number of stations), and i goes from 1 to 12. Let P denote the matrix of P_{ij}, that is:

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdots P_{1N} \\ P_{21} & P_{22} & \cdots P_{2N} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ P_{12,1} & P_{12,2} & \cdots P_{12,N} \end{pmatrix}$$

If P' denotes the transposed matrix of P_{ij}, then:

$$P' = \begin{pmatrix} P_{11} & P_{21} & \cdots P_{12,1} \\ P_{12} & P_{22} & \cdots P_{12,2} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ P_{1N} & P_{2N} & \cdots P_{12,N} \end{pmatrix}$$

The product, PP', is a 12 x 12 (square) symmetric matrix.

The following theorem is proven in most textbooks on the subject: The necessary and sufficient condition that the eigenvalues of a matrix S should be real, and that the eigenvectors of S should be mutually orthogonal, is that S should be symmetric.

Kutzbach (3) introduces eigenvectors from a very illuminating viewpoint. Regarding each column of P as a vector, one seeks a vector

V which bears the closest resemblance to the ensemble of column vectors of P. Resemblance in this case is defined in the following manner:

Let L and V be two normalized (unit) vectors, that is to say,

$$L'L = 1 = V'V,$$

then the degree of resemblance between L and V is defined as the quantity $(V'L)^2$. The maximum value which $(V'L)^2$ can attain is 1, when L and V are identical. To obtain a solution in the more general case in question, it is necessary to maximize the quantity:

$$(V'P)^2/N$$

where V and each of the column vectors of P are normalized vectors.

It turns out that the solutions to this maximization problem, which for P are 12 in number, are the eigenvectors of the matrix PP' in their normalized form. Thus a set of 12 normalized eigenvectors, e_1, e_2, $\ldots e_{12}$, is obtained, each eigenvector having 12 components. Corresponding to these eigenvectors are the 12 eigenvalues, L_1, L_2, $\ldots L_{12}$, where

$$(PP')e_f = L_f e_f,$$

and

$$e_f = \begin{pmatrix} e_{1f} \\ e_{2f} \\ \cdot \\ \cdot \\ e_{12,\,f} \end{pmatrix}, \quad f = 1, 2, \ldots 12$$

Arranging the eigenvalues in descending order of magnitude results in an arrangement of the corresponding eigenvectors in descending order of importance in explaining the variance in the data. In fact, the fraction of the total variance explained by the first k eigenvectors is:

$$(L_1 + L_2 + \ldots + L_k)/(L_1 + L_2 + \ldots + L_{12})$$

The **matrix** E_k of the first k eigenvectors may be formed:

$$E_k = \begin{pmatrix} e_{11} & e_{12} & \cdots & e_{1k} \\ e_{21} & e_{22} & \cdots & e_{2k} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ e_{12,1} & e_{12,2} & & e_{12,k} \end{pmatrix}$$

where, from the orthogonality relationship

$$E_k E_k' = I$$

A matrix M_k may be constructed, where

$$E_k' P = M_k, \quad \text{or}$$

$$P = E_k \cdot M_k$$

M_k is a matrix of N columns and k rows. If k were equal to N, the equation

$$P = E_N M_N$$

would return the input data exactly. If, however, only the first k eigen-vectors are retained, the values returned by the reconstruction process, $P = E_k M_k$, would not be exact, but may be within the range of accuracy considered desirable.

The values of the multipliers in the M-matrix may be plotted against the station positions in the input data to compose a set of charts which, in conjunction with the tabulated values of the k eigenvectors, may be used to reconstruct the data within the permissible range of accuracy prescribed by using a curtailed set of eigenvectors and M-values.

4. Results

Although the first eigenvectors resulting from the analysis of Maritime precipitation and temperature data explained over 95% of the variance in the data, it was found that greater overall accuracy could be attained if the first 3 eigenvectors of precipitation and the first two eigenvectors of temperature were used.

The first 3 eigenvectors of Maritime precipitation are plotted against time in Figs. 1, 2 and 3, while in Figs. 4 and 5 the first two eigenvectors of temperature are similarly depicted. These eigenvectors are tabulated in tables 1 and 2. The M-values are analyzed in Figs. 6 7, 8, 9 and 10.

In order to clarify the reconstruction process, the following example may prove helpful.

The first three M-values of precipitation for Moncton, N.B., taken from Figs. 1, 2 and 3 are:

$$11.50, \quad .70, \quad -.29$$

The eigenvector values for the month of August, taken from table 2, eighth row, are:

$$.277, \quad .170, \quad .739$$

Reconstructing the data:

$$11.50 \text{ x } .277 + .70 \text{ x } .170 - .29 \text{ x } .739 = 3.09$$

The value for the mean precipitation for August, listed in the input data, was 2.93 inches. The percentage error is then 5.1%.

5.   Conclusions and Discussion

The eigenvectors are orthogonal, and so uncorrelated. This implies that the M-value patterns associated with the eigenvectors are unrelated. For instance Figs. 6, 7 and 8 depict the M-values associated with the first, second and third eigenvectors, respectively, of precipitation. These patterns may be interpreted in terms of influences which are entirely separate and an attempt has been made to do this.

The first eigenvector of Maritimes temperatures is related to the seasonal variation of solar heating. The pattern of the associated M-values (Fig. 9) is similar to available annual mean temperature charts for these regions, which is not surprising since over 95% of the variance in the data is explained by the first eigenvector. The second eigenvector of Maritimes temperatures seems to be representative of the moderating influence of the adjacent water bodies on temperatures in these regions. The patterns of the corresponding M-values (Fig. 10) seem to confirm this impression. The chart shows large positive values off the southern coast of Nova Scotia and large negative values in northwestern New

Brunswick. Positive values will contribute a net warming effect in the winter months and will contribute towards lowering temperatures in the summer months, since the signs of the second eigenvector values are negative for the warmer months, and vice versa. The moderating effect of the Bay of Fundy shows up well in Fig. 10 in the shape of a ridge of higher values.

East coast lows tend to intensify during the late fall and throughout the winter. Precipitation amounts from these systems are much greater during the late fall when air temperatures are high enough to permit the air to retain considerable moisture. During the warmer months low pressure systems are usually ill defined or in any case show little tendency to intensify. The first eigenvector of precipitation reflects the annual frequency and intensity of synoptic scale storms affecting the Maritimes. The pattern of the associated M-values (Fig. 6) resembles very closely the available charts of mean annual precipitation in these regions.

The second eigenvector of precipitation for the Maritimes may be interpreted as representing seasonal variation of storm tracks affecting these regions modified by the effect of the proximity of large bodies of water upon organized systems. This interpretation is in line with the associated M-value distribution shown in Fig. 7. These M-values when multiplied by the appropriate eigenvector values indicate a large positive contribution to precipitation amounts in inland New Brunswick during the warmer months with a negligible or negative contribution in coastal sections and elsewhere in the Maritimes. This picture is as expected to conform to the interpretation given. Storm tracks are further north in the warmer months while during these months the effect of cooler bodies of water upon organized systems is a negative one. The presence of large bodies of open water tends to enhance the precipitation from organized weather systems on a synoptic scale during the winter months. Recalling that the eigenvector values are negative during these months and that storm tracks are further south, it can only be concluded that the interpretation given holds at these times of the year as well. The Gulf becomes partially or completely frozen over during the latter part of the winter season. The net contribution to precipitation amounts indicated by the M-values and associated eigenvector values is in line with this fact.

The third eigenvector of precipitation may be interpreted as representing the effect of topography and the proximity of large bodies of open water upon air mass precipitation. The associated M-values (Fig. 8) almost duplicate in their distribution patterns the topography of the

Maritimes and correspond to a maximum amount of precipitation over higher ground in the warmer months with the opposite effect in the colder months due to the fact that these areas are farthest removed from the effects of comparatively warm bodies in the wintertime. The M-values when multiplied by the appropriate eigenvector values show the effect of the wintertime freeze-up of the Gulf on air mass precipitation in the colder seasons. Large positive M-values along the north coast of Cape Breton act in a negative way on precipitation calculations from February through May. The air to water temperature contrast remains very strong in this area throughout the spring months.

In the cases under examination, twelve charts would be required to display the data for monthly mean temperatures in the Maritimes, whereas the same information is now contained in the two M-value charts and one table of eigenvector values. A considerable saving in requirements for data storage has been effected by the use of these methods. In addition some further insight has been gained into some of the climatic controls and their geographical distribution.

6.   Acknowledgements

The assistance of the computer programming section at the Bedford Institute of Oceanography in providing the computer programs for the calculation of eigenvectors of a square matrix is recognized with thanks. Appreciation is expressed to Dr. A.D.J. O'Neill for reading the manuscript and for making many helpful suggestions, to Mr. A.D. Gates for his helpful comments, and to Mr. F. Amirault for providing the data.

APPROVED

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Atmospheric Environment Service

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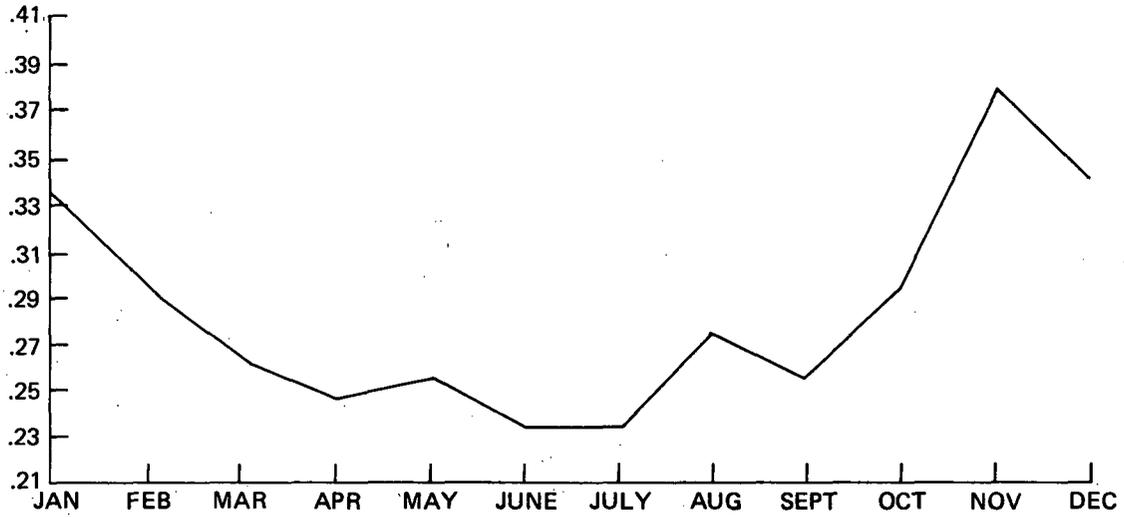
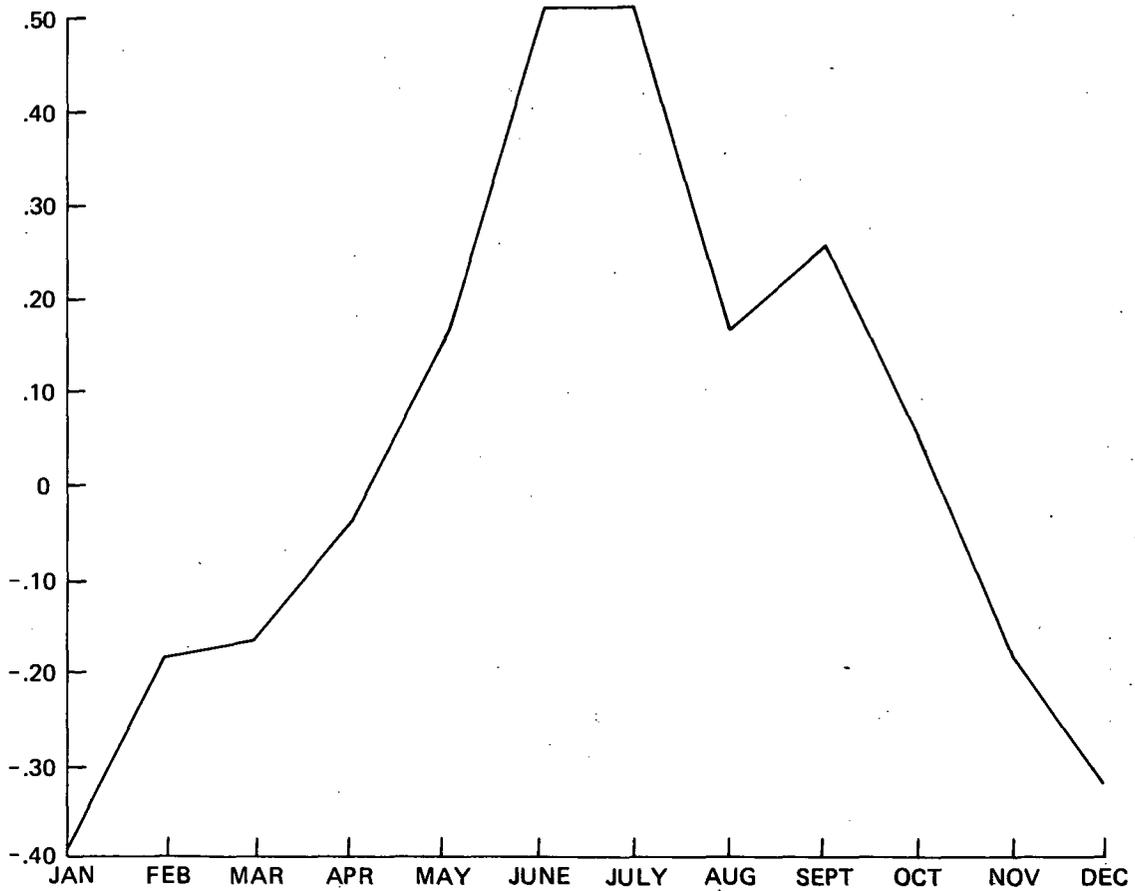Figure 1 First eigenvector of precipitation of 93 Maritimes sites.

Figure 2 Second eigenvector of precipitation of 93 Maritimes sites.
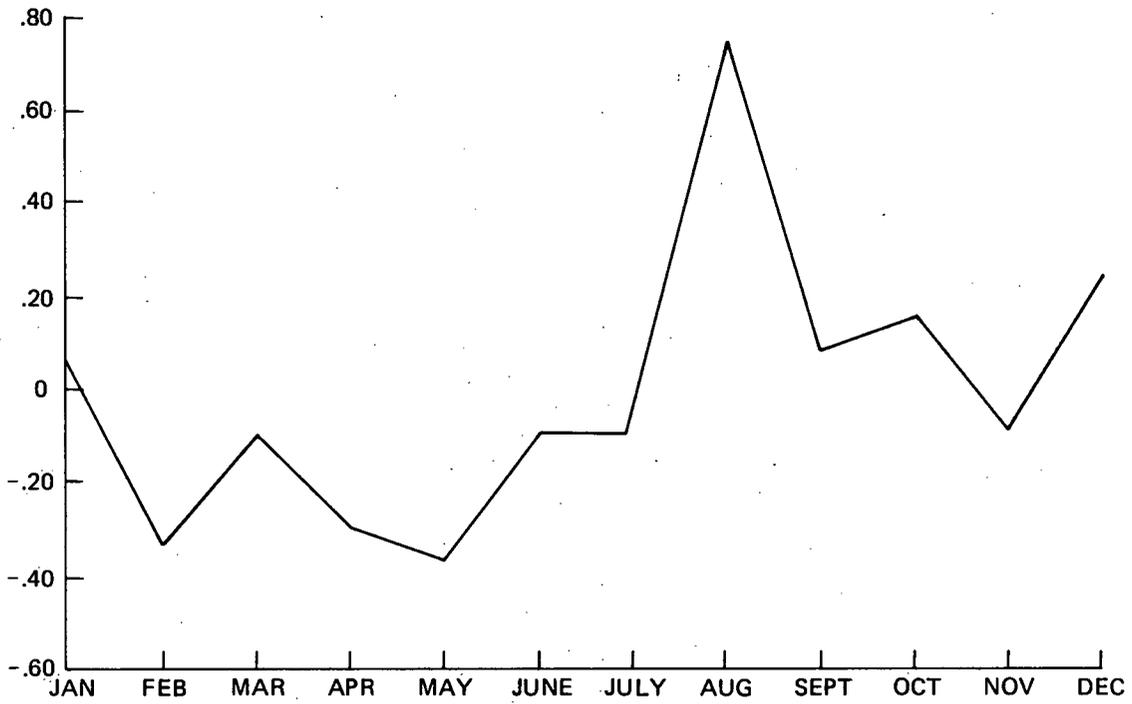
Figure 3 Third eigenvector of precipitation of 93 Maritimes sites.
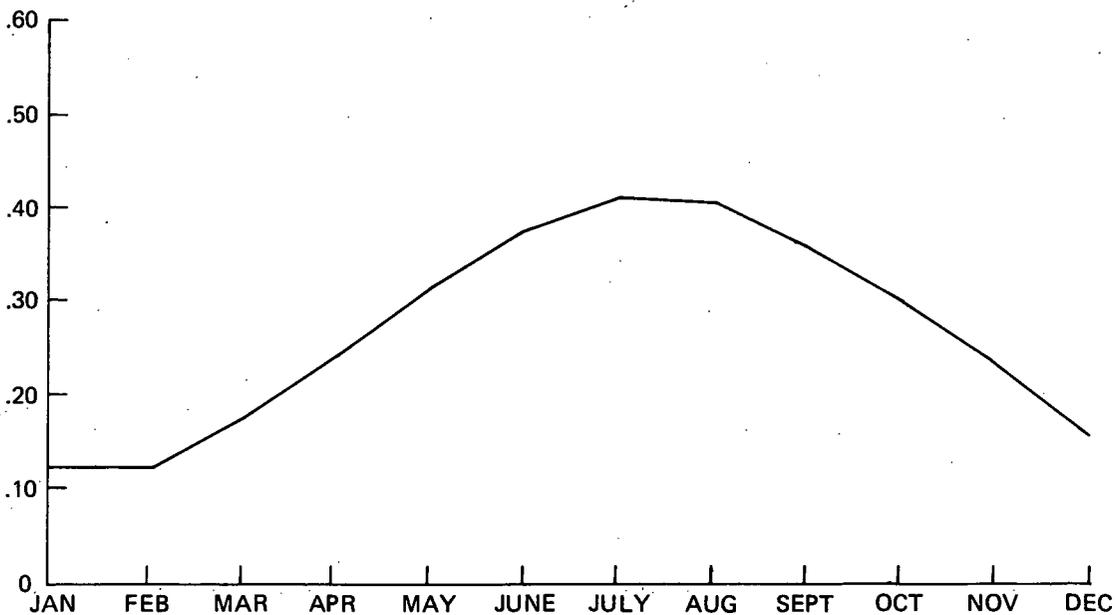
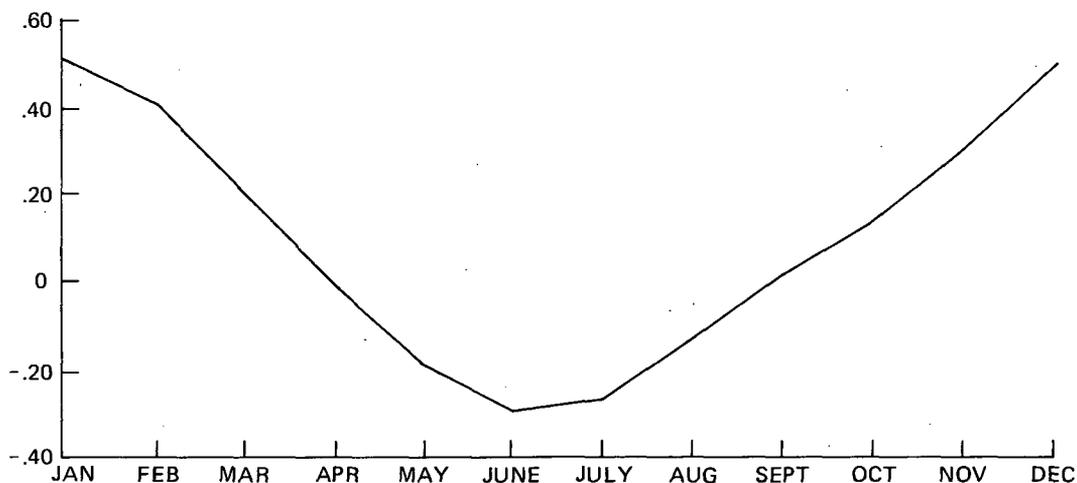Figure 4 First eigenvector of temperatures of 82 Maritimes sites.

Figure 5 Second eigenvector of temperatures of 82 Maritimes sites.
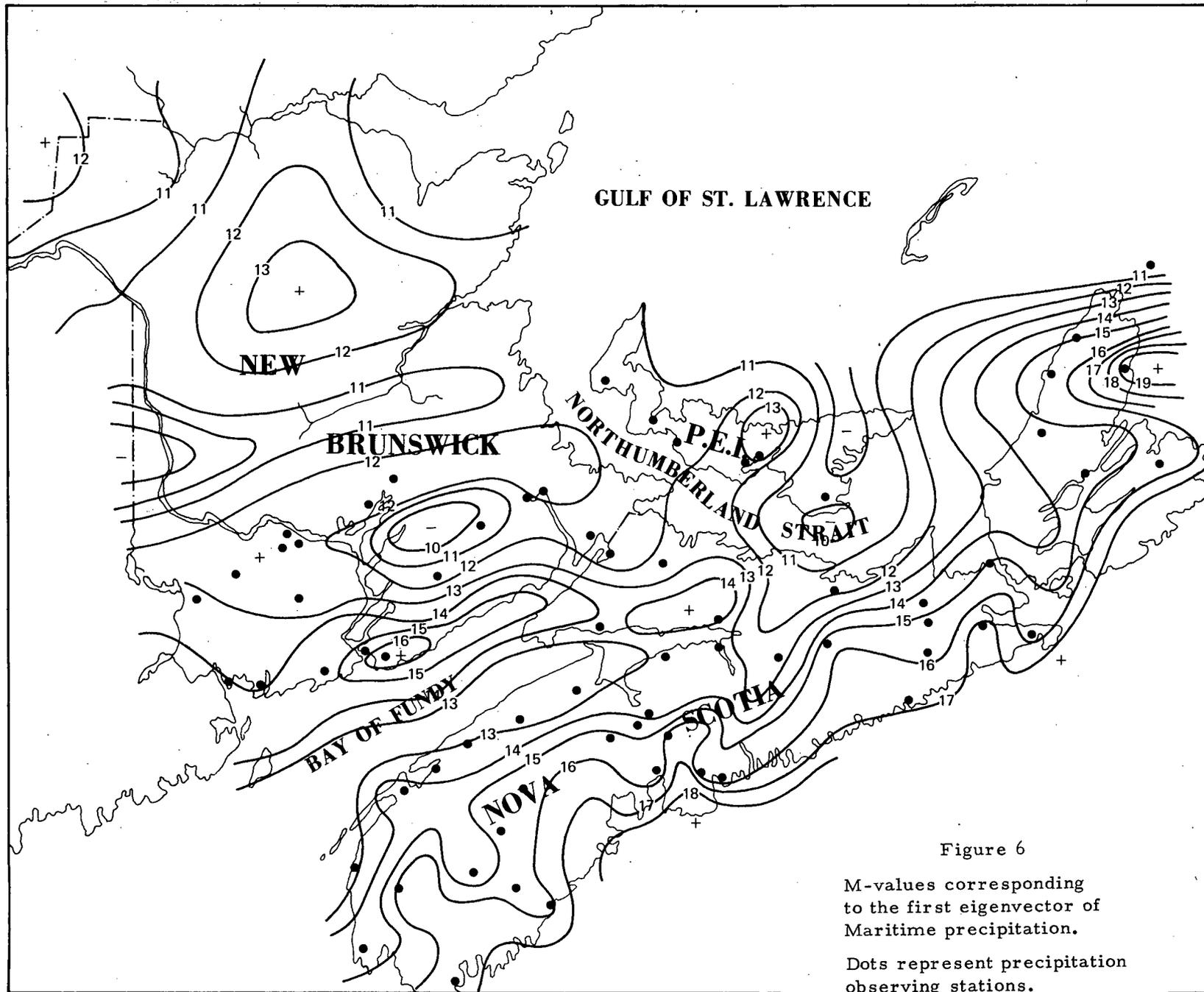
Table 1

Values of the First Two Eigenvectors of the Mean Monthly
Temperature Data of 82 Maritimes Sites.

JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
.127	.127	.178	.243	.310	.370	.413	.407	.362	.303	.241	.161
.508	.400	.196	-.017	-.189	-.287	-.273	-.137	.011	.123	.285	.486

Table 2

Values of the First Three Eigenvectors of the Mean Monthly
Precipitation Data of 93 Maritimes Sites.

JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
.335	.294	.2628	.247	.256	.238	.234	.277	.258	.294	.384	.343
-.390	-.187	-.68	-.040	.149	.517	.515	.170	.256	.065	-.174	-.316
.070	-.327	-102	-.296	-.362	-.097	-.101	.739	.074	.141	.095	.243

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Figure 6

M-values corresponding
to the first eigenvector of
Maritime precipitation.

Dots represent precipitation
observing stations.

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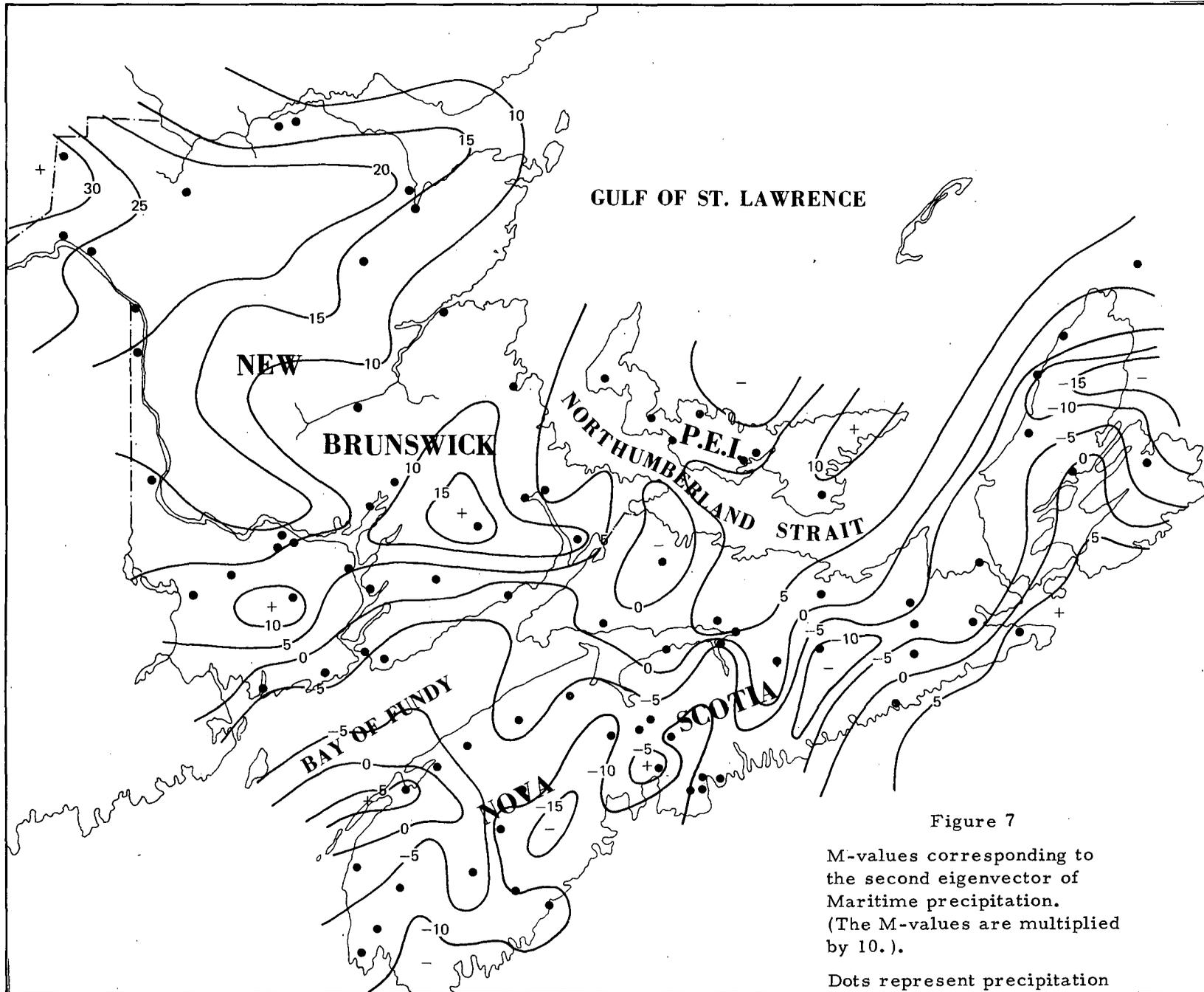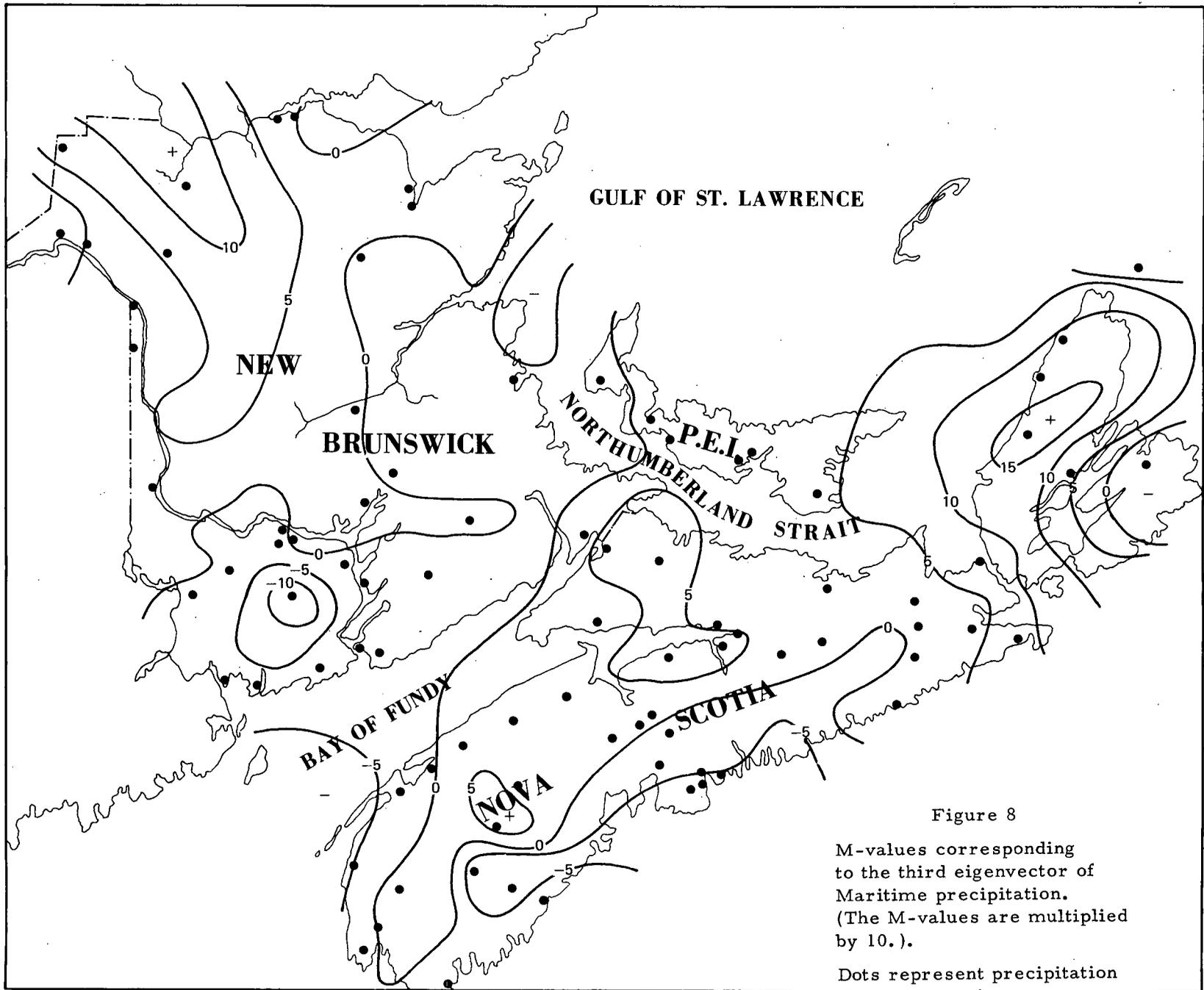BRUNSWICK

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Figure 7

M-values corresponding to
the second eigenvector of
Maritime precipitation.
(The M-values are multiplied
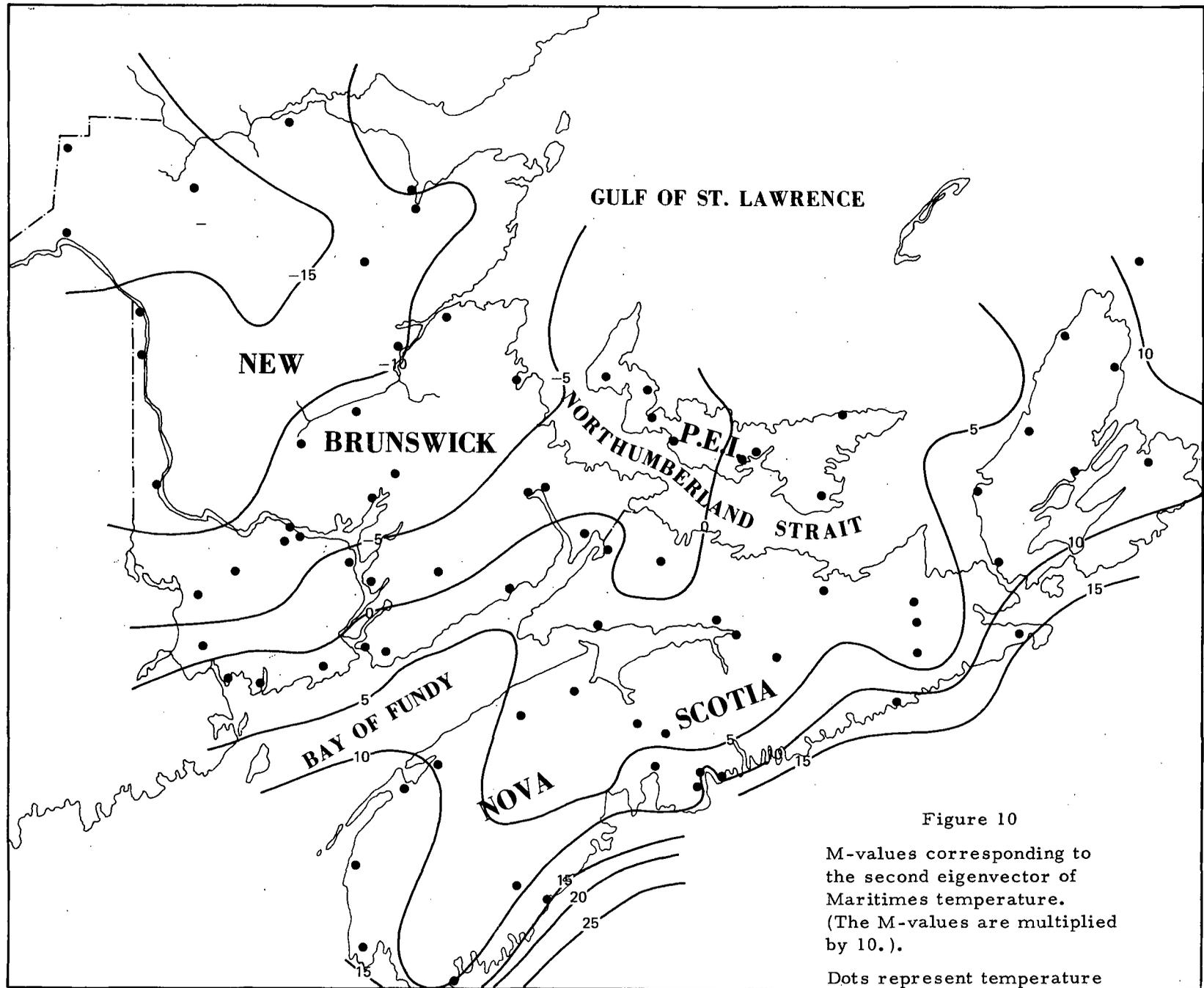by 10.).

Dots represent precipitation
observing stations.

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Figure 8

M-values corresponding
to the third eigenvector of
Maritime precipitation.
(The M-values are multiplied
by 10.).

Dots represent precipitation
observing stations.

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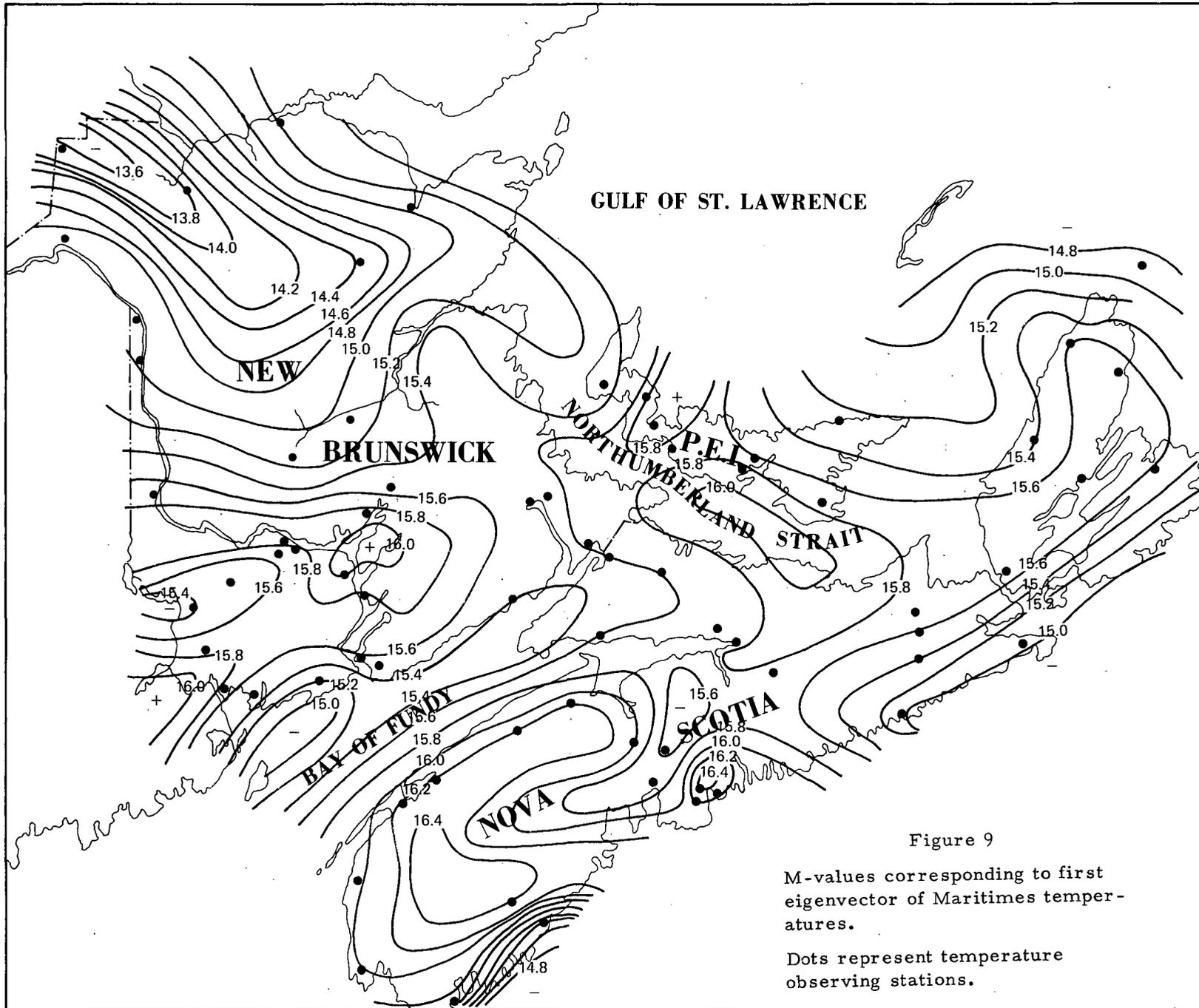
Figure 9

M-values corresponding to first eigenvector of Maritimes temperatures.

Dots represent temperature observing stations.

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Figure 10

M-values corresponding to
the second eigenvector of
Maritimes temperature.
(The M-values are multiplied
by 10.).

Dots represent temperature
observing stations.

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