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Calculation of Geostrophic Currents

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INTRODUCTION

Procedures for the calculation of currents by the dynamical method as used by various investigators and organizations have been studied with a view of evaluating their convenience, accuracy, and speed, and with an additional aim of making an improvement in the standard routines particularly for use in the Canadian Atlantic region.

A modification to accepted practice is offered as having some merits. It is based on the partial derivatives of specific volume anomaly and the differences in temperature and in salinity at corresponding isobaric levels for two oceanographic stations.

THEORY

The basis of the dynamical method of calculating ocean currents is the assumed equality of the geostrophic acceleration of the water mass and the component of the gravitational acceleration along any sloping isobaric surface. Thus:

$$2\,\omega \sin\phi \cdot v = ig \qquad (1)$$

where $\omega \sin\phi$ is the vertical component of angular velocity of the earth's rotation in latitude ϕ, v the horizontal velocity of the water mass relative to the reference level of assumed zero velocity, i the slope of the isobaric surface, and g the gravitational acceleration. This equation was derived by Mohn (1887) and independently by Sandstrom and Helland-Hansen (1905) and may be found in standard works such as "The Oceans" and Proudman's "Dynamical Oceanography".

The slope (i) of a given isobaric surface may be expressed in several ways which are related to the various procedures followed in the calculation of ocean currents. In terms of dynamic height anomalies:

$$i = \frac{10}{g} \cdot \frac{\Delta D_A - \Delta D_B}{L_{AB}} \tag{2}$$

where ΔD_A and ΔD_B are the dynamic height anomalies of the particular isobaric surface at the two stations A and B, a distance L_{AB} apart.

METHODS IN USE

A. In a routine widely used and most satisfactorily organized in "Processing Oceanographic Data" (LaFond 1951), the dynamic height anomalies are found for each standard depth by the formula:

$$\Delta D = \int_{P}^{P_0} \delta \, dp \quad \doteq \int_{z}^{z_0} \delta \, dz \tag{3}$$

The integration, performed numerically by summation of the products of specific volume anomaly δ and the pressure increments dp, is carried through from a selected reference depth of assumed zero velocity. Alternatively the summation may be made with respect to the surface. Depth in metres (z) conveniently replace pressure in decibars (p) with only slight error.

Summations are carried out in turn for each hydrographic station of the section. Slopes of the isobaric surfaces in the interval between stations are then found by taking differences and dividing by the distance between stations as in equation (2). If there are a number of hydrographic stations forming a grid, charts of dynamic topography are frequently constructed. From these, currents and directions may be readily evaluated. This procedure has been widely used by the International Ice Patrol and is described by Smith (1926).

B. If the difference in dynamic height of a given isobaric surface at two stations A and B is expressed in the form:

$$\Delta D_A - \Delta D_B = \int_{z}^{z_0} (\delta_A - \delta_B)\, dz \qquad (4)$$

a procedure for current calculations is suggested. Here the data for each individual station are processed to obtain specific volume anomaly for each standard isobaric surface. Thereafter differences in anomaly between pairs of stations are obtained followed by the integration operation, which is easily performed as the differences are small. Fewer errors will occur than in calculations of dynamic height anomalies with their greater number of significant figures. This consideration is of greatest importance where a single pair of stations only is involved, since there is no compensating advantage in finding the dynamic height anomalies at each station once for all.

This is essentially the routine used by Sandstrom (1919) in his dynamical study of Canadian Atlantic waters. Proudman (1953, pp. 64-65) includes an illustration of dynamical calculations from Helland-Hansen and Nansen (1925) who followed this procedure.

C. Werenskiold (1935) has developed a graphical procedure for calculating ocean currents. The slope of an isobaric surface as in equation (2) may be written in differential form:

$$i = \frac{10}{g} \cdot \frac{dD}{dx} \doteq \frac{d}{dx}\int_{z}^{z_0} \delta \, dz \qquad = \int_{z}^{z_0} \left(\frac{\partial \delta}{\partial x}\right)_z dz \qquad (5)$$

If the term $\left(\partial\delta/\partial x\right)_z$ is written in its equivalent form (see any standard calculus text) then $\left(\frac{\partial \delta}{\partial x}\right)_z = -\left(\frac{\partial z}{\partial x}\right)_\delta \left(\frac{\partial \delta}{\partial z}\right)_x$ and equation (6) becomes:

$$i = \int_{z_0}^{z} \left(\frac{\partial z}{\partial x}\right)_\delta \left(\frac{\partial \delta}{\partial z}\right)_x dz \qquad = \int_{\delta_0}^{\delta} \left(\frac{\partial z}{\partial x}\right)_\delta d\delta \qquad (6)$$

This latter expression gives the slope of the isobaric surface i, as the summation of the products of the slope if isosteric surfaces and the appropriate specific volume anomaly increments, made through a vertical column. The summation may be carried out along any vertical line of a cross-sectional plot of the isosteres. A transparent sheet ruled with slanting lines and a scale of slopes is placed over the section graph and the slopes read off directly. Suitable allowance must of course be made for the scale distortion of the graph of hydrographic section.

D. Klein (1956) has recently introduced a graphical method for determining dynamic depth anomalies (and hence calculating currents). He uses a specially ruled plotting paper having on the X axis salinity (and alternatively depth), and on the Y axis the specific volume anomaly. Temperature is given by isothermal lines which cross the plotting paper on a bias. As the partial derivative of specific volume with respect to salinity (with temperature constant) is practically constant the isothermal lines are straight.

Klein's procedure includes the making of three plots, the first, a plot of depth (X axis) and temperature (slanting lines). This permits later interpolation for standard depths. The second plot is of salinity (X axis) and temperature (slanting lines) with each depth appropriately marked on the graph. The Y coordinate gives the corresponding specific volume anomaly (while an auxiliary scale reads σ_t). A third plot is then constructed from this, giving depth (X axis) and specific volume anomaly (Y axis).

The area under this last plot represents the dynamic height anomaly. Klein performs this area summation numerically using an adding machine routine. It may be here noted that the writer found that a planimeter gave a rapid and accurate summation.

By superimposing a second plot of depth vs. specific volume for a neighbouring hydrographic station on the same plotting sheet and summing the area between these curves with a planimeter, the writer measured the dynamic depth differences for the two stations.

This method of Klein was found to give a convenient graphical representation of water conditions on which stability characteristics and other variables might be vividly noted. Klein made additional provision with auxiliary scales to include the maximum information regarding each oceanographic station.

A revision of the plotting sheets to include a wider range of salinities and perhaps a reduction in scale might make this a valuable aid in the description of water conditions and computing ocean currents in the Canadian Atlantic region.

It should be mentioned that Klein's plot makes no provision for inclusion for the small correction terms for pressure in the specific volume anomaly. This, he points out, is no serious objection to the method in most instances, considering the small size of these corrections and the assumptions involved in the dynamical method. He refers to a method of introducing these corrections where desired.

It is the writer's opinion that the operations on the graphs, especially transferring points from one to the other, is a potential source of inaccuracy, and a source of eye strain that together constitute a serious objection to the method. In all fairness, however, an extended trial of the method would be necessary to properly evaluate it.

NEW METHOD

The writer has examined the possibility of setting up a calculating routine in which the difference of specific volume anomaly for two stations, is expressed as the sum of two factors due to temperature and salinity. One may write as a first order approximation:

$$\delta_A - \delta_B = \left(\frac{\partial \delta}{\partial T}\right)_S (T_A - T_B) + \left(\frac{\partial \delta}{\partial S}\right)_T (S_A - S_B) \qquad (7)$$

for each isobaric surface.

In the standard notation of "The Oceans" (Sverdrup et al. 1942), the specific volume anomaly δ is the sum of a part Δ independent of pressure, and pressure dependent terms δ_{TP} and δ_{SP}. Thus:

$$\delta = \Delta + \delta_{TP} + \delta_{SP}$$

and equation (7) becomes:

$$\delta_A - \delta_B = \left\{\left(\frac{\partial \Delta}{\partial T}\right)_S + \left(\frac{\partial \delta_{TP}}{\partial T}\right)_S\right\} (T_A - T_B) + \left\{\left(\frac{\partial \Delta}{\partial S}\right)_T + \left(\frac{\partial \delta_{SP}}{\partial S}\right)_T\right\} (S_A - S_B)$$

These four coefficients may be found from four brief tables (or alternatively graphs) which need be no more subdivided than into unit degrees of temperature and one part per thousand salinity. The pressure dependent terms are effectively small correction terms which may be neglected except at greater depths.

For each consecutive pair of hydrographic stations and at each standard depth, differences of temperature and salinity are found and multiplied by the appropriate coefficient. addition of these products give the specific volume difference between stations at each standard depth. Numerical integration of the specific volume differences with respect to depth yields the dynamic height difference at each standard depth. From this the inclination of the isobaric surfaces is next found by division by the distance interval between stations.

A comparison of the times required to carry out current computations by this routine and that of LaFond (1951) referred to in this paper as method A, showed a saving in time by the method just described of approximately 40% for a line of five hydrographic stations. Where only two hydrographic stations were involved, and hence no repeated use of the dynamic height anomalies of a given station, the saving in time was found to be almost 60% by this proposed routine.

Though the saving in time is an advantage, it is perhaps of less importance than the reduction in the risk of numerical errors. This arises with the use of difference terms in temperature and salinity at the outset, involving fewer significant figures. In the standard method five and six figure dynamic height anomalies must be obtained, with a final operation involving the subtraction of two nearly equal quantities.

The following list enumerates the steps involved in the computation of currents in this routine. The steps are numbered and refer to the columns in Table V. This table is drawn up in the form of a worksheet which is suggested for use if routine calculations using this method are undertaken.

1. Enter the standard depths.
2. Enter T_A, temperature at station A.
3. Enter T_B, temperature at station B.
4. Enter $T_A - T_B$, difference of columns 2 and 3.
5. Enter S_A, salinity at station A.
6. Enter S_B, salinity at station B.
7. Enter $S_A - S_B$, difference of columns 5 and 6.
8. Enter $10^5(\partial\Delta/\partial S)$ from Table I (note negative sign)
   (plus $10^5(\partial\Delta_{sp}/\partial S)$ from Table III for greater depths.
9. Enter the <u>product,</u> columns 7 and 8.
10. Enter $10^5(\partial\Delta/\partial T)$ from Table II.
    (plus $10^5(\partial\delta_{TP}/\partial T)$ from Table IV for greater depths.
11. Enter the <u>product,</u> columns 4 and 10.
12. Enter the <u>sum</u> of columns 9 and 11.
13. Enter the mean value for each depth interval from column 12.
14. Enter the product of column 13 and the corresponding depth interval.
15. Enter for each standard depth the term by term sum of column 14 entries.
16. Figures for comparison purposes from calculations by standard routine (R. Trites). Maximum difference from column 15 is 3 dynamic millimetres.

The summation in column 15 are the dynamic height differences at each standard depth. From these, currents may be computed by use of equations (1) and (2).

Where calculations in the water column were made from a depth of 300 metres or less, the effect of the pressure dependent coefficients is slight, and in general, omission of these terms involves a negligible error compared to those inherent in the assumptions in the method of dynamical calculations. This simplification has been proposed by Montgomery and Wooster (1954).

Section with Shallowing Bottom

The calculation of currents in a section bounded by a shallowing bottom is always a difficult problem involving assumptions and uncertainties. Methods have been proposed by Helland-Hansen (1934), Sverdrup et al. (1942), and Groen (1948), and all require the plotting of the isosteres for the section and their extrapolation through the sloping bottom into a fictitious water mass. If specific volume anomalies were found for a single station in deep water, the specific volume differences as found by the proposed routine might be referred to this to give specific volume anomalies for all stations. Differences in the fictitious water mass might then be used to complete the routine in the shallow intervals.

1. Four routines for the dynamic computation of currents are examined and comment is offered on their use in Canadian Atlantic waters.

2. A new routine for computation of currents is described in which specific volume differences are calculated directly from temperature and salinity differences. Some advantages in time saved and in the ease of calculation are found.

3. Tables of the partial derivatives of specific volume with temperature, salinity and depth are presented for use in this calculation routine.

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TABLE I

Decrease in $\Delta_{ST}10^5$ for 1‰ Sal.

T° \ S‰	28	29	30	31	32	33	34	35	36
0	76.8	76.7	76.6	76.5	76.4	76.3	76.3	76.2	76.1
1	76.5	76.4	76.4	76.3	76.2	76.1	76.0	75.9	75.8
2	76.2	76.1	76.0	76.0	75.9	75.9	75.7	75.7	75.6
3	76.0	75.9	75.8	75.7	75.7	75.6	75.5	75.4	75.4
4	75.7	75.6	75.5	75.5	75.4	75.3	75.2	75.2	75.2
5	75.5	75.4	75.3	75.2	75.1	75.1	75.0	75.0	74.9
6	75.3	75.2	75.1	75.0	74.9	74.8	74.8	74.7	74.6
7	75.0	74.9	74.8	74.8	74.7	74.6	74.6	74.5	74.4
8	74.8	74.7	74.6	74.6	74.5	74.4	74.4	74.3	74.2
9	74.6	74.5	74.4	74.3	74.3	74.2	74.2	74.1	74.0
10	74.4	74.3	74.2	74.1	74.1	74.0	74.0	73.9	73.8
11	74.2	74.1	74.0	73.9	73.9	73.8	73.8	73.7	73.7
12	74.0	73.9	73.8	73.8	73.7	73.6	73.6	73.6	73.5
13	73.8	73.7	73.7	73.6	73.6	73.5	73.4	73.4	73.3
14	73.7	73.6	73.5	73.4	73.4	73.3	73.3	73.2	73.2
15	73.5	73.4	73.4	73.3	73.2	73.2	73.1	73.1	73.0
16	73.3	73.3	73.2	73.2	73.1	73.0	73.0	72.9	72.9
17	73.2	73.1	73.1	73.0	73.0	72.9	72.8	72.8	72.8
18	73.1	73.0	73.0	72.9	72.8	72.8	72.7	72.7	72.6
19	73.0	72.9	72.8	72.8	72.7	72.6	72.6	72.6	72.5
20	72.9	72.8	72.7	72.7	72.6	72.5	72.5	72.4	72.4
21	72.8	72.7	72.6	72.6	72.5	72.4	72.4	72.3	72.3
22	72.7	72.6	72.5	72.5	72.4	72.3	72.3	72.3	72.2
23	72.6	72.5	72.4	72.4	72.3	72.3	72.2	72.2	72.1
24	72.5	72.4	72.3	72.3	72.2	72.2	72.1	72.1	72.1
25	72.4	72.3	72.3	72.2	72.1	72.1	72.1	72.0	72.0

TABLE II

Increase in $10^5\ \Delta_{ST}$ per dg. C

T° \ S‰	28	29	30	31	32	33	34	35	36
0	3.0	3.3	3.6	3.9	4.2	4.5	4.7	5.0	5.3
1	4.3	4.6	4.9	5.2	5.5	5.7	6.0	6.3	6.6
2	5.7	6.0	6.2	6.5	6.8	7.1	7.3	7.5	7.8
3	7.0	7.2	7.5	7.7	8.0	8.2	8.5	8.8	9.0
4	8.2	8.5	8.7	8.9	9.2	9.4	9.7	10.0	10.2
5	9.4	9.7	9.9	10.1	10.4	10.6	10.8	11.1	11.3
6	10.6	10.8	11.0	11.3	11.5	11.7	11.9	12.2	12.4
7	11.7	11.9	12.1	12.4	12.6	12.8	13.0	13.3	13.5
8	12.8	13.1	13.3	13.5	13.5	13.9	14.1	14.3	14.5
9	13.8	14.1	14.3	14.5	14.7	14.9	15.1	15.3	15.5
10	14.9	15.1	15.3	15.5	15.7	15.9	16.1	16.3	16.5
11	15.9	16.1	16.3	16.5	16.7	16.9	17.1	17.2	17.4
12	16.9	17.1	17.3	17.5	17.7	17.8	18.0	18.2	18.3
13	17.9	18.0	18.2	18.4	18.6	18.7	18.9	19.1	19.3
14	18.9	19.1	19.2	19.4	19.5	19.7	19.9	20.0	20.2
15	19.8	20.1	20.2	20.3	20.4	20.6	20.7	20.9	21.0
16	20.7	21.0	21.1	21.2	21.3	21.5	21.6	21.7	21.8
17	21.6	21.8	22.0	22.1	22.2	22.3	22.5	22.6	22.7
18	22.5	22.7	22.8	22.9	23.1	23.2	23.3	23.4	23.5
19	23.4	23.6	23.7	23.8	23.9	24.0	24.1	24.3	24.4
20	24.3	24.5	24.5	24.6	24.7	24.8	24.9	25.1	25.2
21	25.2	25.3	25.4	25.5	25.6	25.7	25.8	25.9	26.0
22	26.0	26.1	26.2	26.3	26.4	26.5	26.6	26.7	26.8
23	26.8	26.9	27.0	27.1	27.2	27.3	27.4	27.4	27.5
24	27.7	27.8	27.8	27.9	28.0	28.1	28.2	28.2	28.3
25	28.5	28.6	28.7	28.8	28.8	28.9	28.9	29.0	29.1

TABLE III

$$10^5\left(\frac{\partial \delta_{sp}}{\partial S}\right)$$

Metres / Sal. 20-38‰	200	400	600	800	1000	1200
	0.3	0.6	0.9	1.2	1.5	1.8

TABLE IV

$$10^5\left(\frac{\partial \delta_{TP}}{\partial T}\right)$$

Metres / T°	200	400	600	800	1000	1200
0	.6	1.2	1.7	2.2	2.7	3.2
2	.5	1.0	1.5	2.0	2.5	3.0
4	.5	.9	1.3	1.8	2.2	2.7
6	.4	.8	1.2	1.6	2.0	2.4
8	.4	.7	1.1	1.5	1.8	2.2
10	.3	.6	1.0	1.4	1.7	2.0
12	.3	.5	.9	1.2	1.5	1.8

TABLE V - DYNAMIC CALCULATIONS

Cruise: S-12	Stations; 31 = "A" 32 = "B"	Vessel: "Sackville"	Date: 7 Nov. 1952	Interval between Stations: 26 miles	Computed by: D.G.M.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Z	T_A	T_B	T_A-T_B	S_A	S_B	S_A-S_B	$10^5\frac{\partial\delta}{\partial S}$	7 x 8	$10^5\frac{\partial\delta}{\partial T}$	4 x 10	9 + 11	$10^5\overline{(\delta_A-\delta_B)}$	$(\delta_A-\delta_B)dz$	$\int(\delta_A-\delta_B)dz$	
0	10.70	9.60	+1.10	31.67	30.79	+ .88	-74.1	-65.3	15.6	17.1	-48.2			-.01368	-.0125
												-53.6	-.00536		
10	10.48	9.47	+1.01	31.87	30.86	+1.01	-74.1	-74.8	15.6	15.7	-59.1			-.00832	-.0073
												-51.1	-.00511		
20	10.49	9.21	+1.28	31.89	31.04	+ .85	-74.1	-63.0	15.6	19.9	-43.1			-.00321	-.0021
												-22.0	-.00220		
30	10.58	9.26	+1.32	32.07	31.78	+ .29	-74.1	-21.5	15.6	20.6	- .0			-.00101	+.0002
												+19.7	+.00394		
50	9.91	7.33	+2.58	31.98	32.05	- .07	-74.5	+ 5.2	13.6	35.2	+40.4			-.00495	-.0039
												+11.8	+.00295		
75	4.56	2.59	+1.97	33.08	32.63	+ .45	-75.5	-34.0	8.8	17.3	-16.7			-.00790	.0071
												-26.8	-.00670		
100	4.72	0.99	+3.73	33.81	32.90	+ .91	-75.8	-69.1	8.5 (.1)	32.1	-37.0			-.00120	-.0004
												-41.1	-.02050		
150	6.90	1.84	+5.06	34.65	33.35	+1.30	-74.4	-97.0	10.0 (3)	51.8	-45.2			+.01930	+.0206
												-17.2	-.00860		
200	6.75	5.63	+1.12	34.70	34.65	+ .05	-74.8 (+.3)	- 3.7	12.5 (4)	14.4	+10.7			+.02790	+.0292
												+11.6	+.01160		
300	5.92	4.70	+1.22	34.86	34.83	+ .03	-74.7 (+.4)	- 2.2	11.4 (.6)	14.7	+12.5			+.01630	+.0182
												+10.1	+.01010		
400	5.24	4.54	+ .70	34.91	34.90	+ .01	-75.0 (+.6)	- .7	11.1 (.8)	8.4	+ 7.7			+.00620	+.0092
												+ 6.2	+.00620		
500	4.82	4.47	+ .35	34.86	34.94	- .08	-75.0 (+.8)	+ .6	10.6 (1.0)	4.1	+ 4.7			0	0