THERMAL INVESTIGATIONS ON FISH MUSCLE

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Although one might not expect any degree of uniformity in the physical and thermal constants of fish muscle, a very fair and in most cases surprisingly good agreement exists between such constants for fish of the same species. Determination of these constants and investigations of the problem into which they enter have therefore a general significance.

Two of the outstanding problems in heat transfer are investigations of the relation of the time of freezing to thickness and thermal conditions, and the time temperature relation in the processing of canned goods.

Okada(1) and Planck(2) have made mathematical analyses of round freezing on the assumption that the fish may be treated as a cylindrical body. A mathematical analysis of the freezing of a rectangular block of fish muscle, from one or both sides, has been made by the writer(3), and yields an expression for the time of freezing in terms of the bath temperature, the thickness the freezing point of the muscle, and the physical and thermal constants.

A summary of the analysis of heat flow in the processing of canned goods has been given by Thompson(4). The analysis given in this paper gives the relation between the processing temperature the initial temperature, the temperature at any point in the can, the time, and the thermal constants. Experiments have shown that practical conditions are very similar to those of the ideal case in both these instances.

The thermal and physical constants which enter these problems have been determined for various species of fish, and their substitution in the expressions developed by mathematical analyses furnishes a solution to heat transfer problems. The values of such constants and a discussion of the problems mentioned above form the basis of the present paper.

Physical Constants

The only physical constant which enters these problems is the density of the fish muscle, and the values given for it are approximate. The water content, and the fat content have also been determined (3), since they shed some light on the freezing problem. The values are given in table 1.

TABLE I

Species of Fish	Density gms./cc.	Water Content	Fat Content
Haddock	1.06	80%	0.3%
Cod	1.06	80	0.3
Herring (fat)	1.06	65	13.
Skate	1.10	79	1.8
Salmon	1.06	--	--
Mackerel (fat)	1.07	58	25.

Thermal Constants

All the necessary thermal constants may be obtained from measurments of two types, (1) heat capacity, and (2) conductivity measurements.

(1) Heat capacity curves over the range -40° to 60° C. have been determined with the aid of a adiabatic calorimeter (5). The freezing points may be obtained to an accuracy of about 0.5°C by extrapolation of the curve for the freezing muscle, while the latent heat (for all purposes), and the specific heats are readily determined. A typical heat capacity curve is shown in Figure 1.

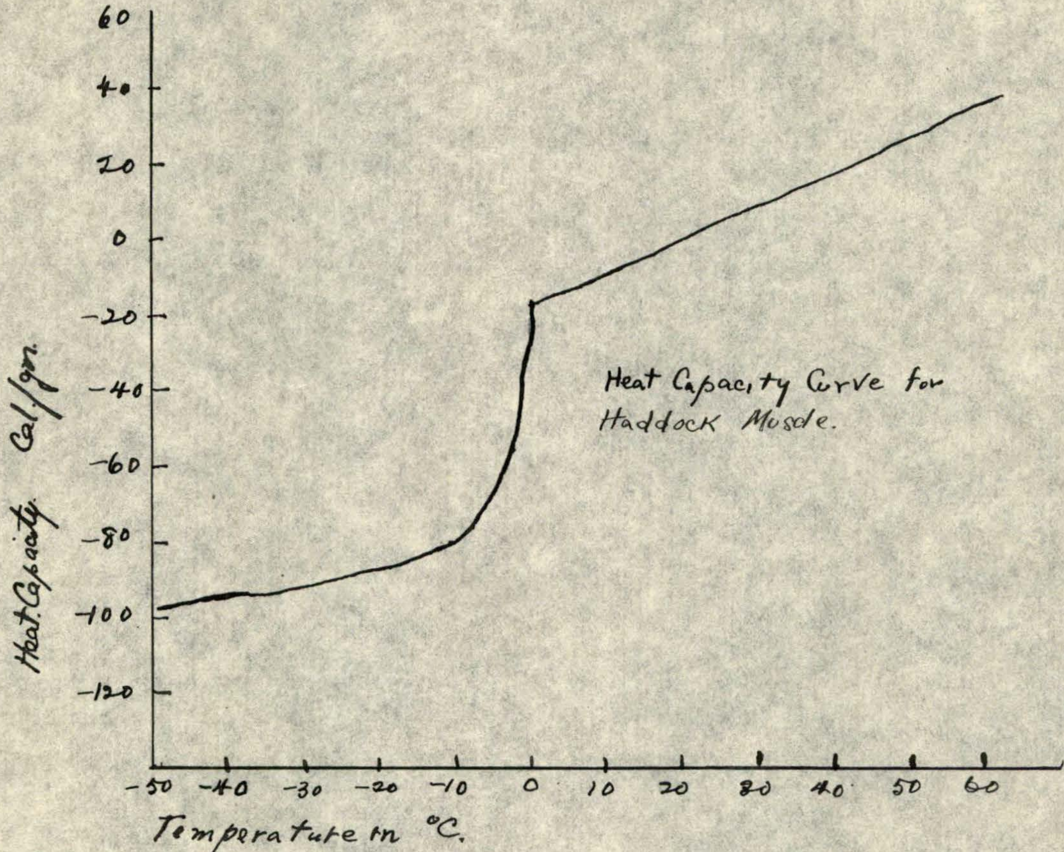


Figure 1

(2) The conductivities are more difficult to determine. A hot plate method (6) gave values for the average conductivity over the range 25° to 100°C. The average conductivity over the range -15° to 0°C was found by substituting experimental freezing times in the therоretical expression for the freezing of a rectangular block of muscle (7). Proof of the validity fof this procedure is given later in this paper. The value in table 2 called processing conductivity was found by applying the process analysis, which is given in the following pages, to experimental data. This is roughly the mean of the average conductivitfes from x° to 100°, where x takes on all values from 15° to 100°.

TABLE II

Species	Freezing Point °C	Latent Heat cal/gm.	Specific Heats cal/gm. °C.	K 25 to 100 cal/gm.sec°C	K -15to 0 cal/cm. sec °C	Frosess k cal/cm sec °C
Haddock	-0.8	63	0 to 60 0.89 -20"-30 0.63 -30"-40 0.56	0.00164	0.0024	------
Cod	-0.8	62	0" 60 0.89 -20"-30 0.63 -30"-40 0.55	-------	0.0023	0.0016
Skate	-1.8	60	0" 60 0.89 -20"-30 0.63 -30"-40 0.56	-------	0.0021	------
Herring	-1.0	56	0" 60 0.89 -20"-30 0.40 -30"-40 0.26	-------	0.0018	------
Burbot	-0.7	64	0" 60 0.89 -20"-30 0.63 -30"-40 0.56	-------	------	------
Brook Trout	-0.8	62	0" 60 0.89	-------	------ K 0.000022 L	------ -----
Salmon	----	--	---------	-------		-----
Mackerel	-0.8	48	0" 36 0.78	-------	0.0012	-----

The specific heats of most species appear to be identical. The freezing points and latent heats, however, vary. This variation in the latent heat is, in direction and magnitude, in accord with the data of table 1.

The conductivity of the frozen (and probably of the unfrozen) fish varies with the species to a great extent. For non-oily fish it appears to be practically constant for all species, but as the fat content increases and the water content decreases the conductivity decreases.

The Freezing Problem

An application of the theory of heat flow (3) to the freezing of a rectangular block of fish muscle gives as a first approximation, the expression,

$$t = \epsilon^2 L \rho / 2k (T - \theta) \text{ ....(1)}$$

where t is the time of freezing, ϵ is the thickness frozen, ρ is the density. T is the temperature of the bath, θ is the freezing point of the fish, and L and k are the latent heat and conductivity respectively. All constants refer to the fish in the frozen state. If freezing takes place from both sides then the thickness frozen in a given time is just four times that given by (1).

Since the above equation was developed for an ideal case its usefulness depends upon how closely practical conditions approach those of the ideal case. In order to ascertain this, the times of freezing of thickness of fish were measured with different bath temperatures. If equation (1) is applicable then k should be a constant for conditions varying over a fairly wide range.

The method of measurement of the time of freezing consisted in determining the temperature of a small plate in thermal contact with the fish muscle by means of a thermocouple. The muscle was in turn in contact with a plate kept at a constant low temperature. By taking temperature readings at intervals it was possible to follow the history of the layer of fish farthest from the freezing plate and hence to determine the time of freezing of the whole thickness of fish (7).

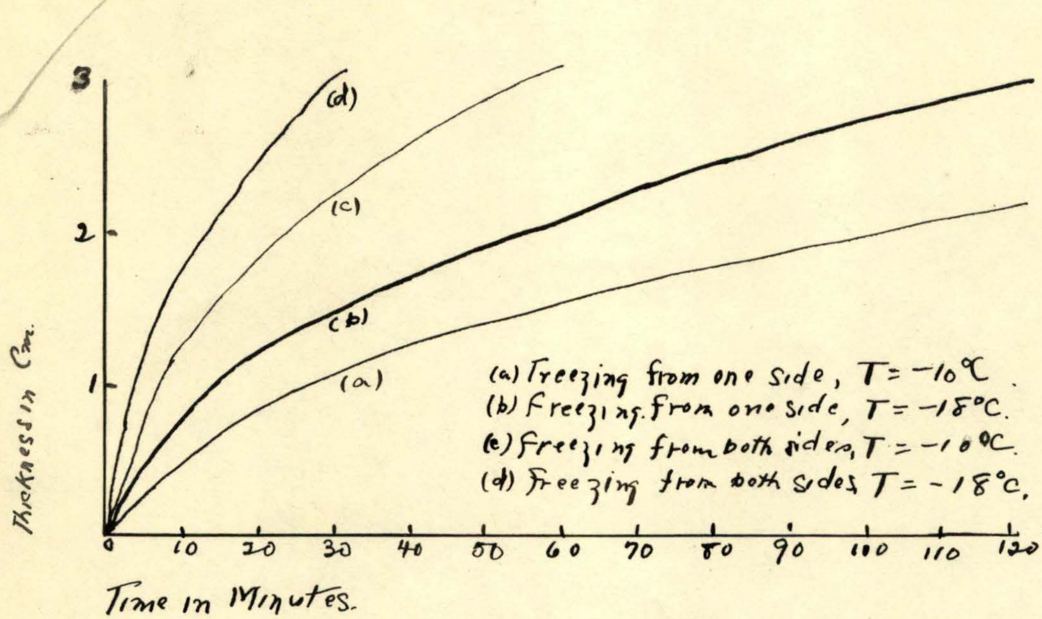
Table 3 gives the results of the application of equation (1) to the experimental data for Haddock. In columns four and five is given a comparison of experimental and theoretical freezing times using the average value of k in the calculations.

TABLE 3

Thickness cm.	Bath temperature ° C.	k cal/cm. °C sec.	Exp. Time min.	Cal. Time min.
1.30	-7.1	0.0023	66	63
2.31	-7.4	0.0025	184	191
0.78	-9.1	0.0022	20	18
1.20	-9.2	0.0023	41	39
1.38	-10.5	0.0027	40	45
1.44	-10.8	0.0023	50	48
1.18	-10.9	0.0025	31	32
1.49	-11.9	0.0023	50	48
1.33	-12.4	0.0023	33	32
1.72	-14.1	0.0025	49	51
1.69	-14.8	0.0025	46	48
1.42	-15.5	0.0022	36	33

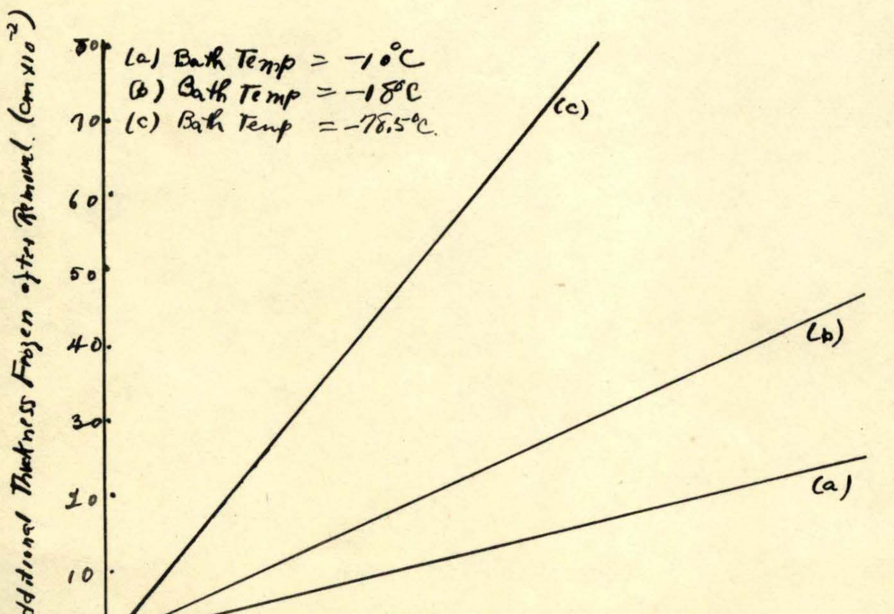
It will be noted that the mathematical analysis applies exceedingly well under experimental conditions. The times of freezing of different thicknesses of haddock have been plotted in figure 2. Equation (1) can be applied in the same way to construct curves for the freezing of any fish whose constants are known, when the conditions approximate one dimensional freezing. i.e. Such types of freezing as used by the Atlantic Coast Fisheries, Company, Kolbe or the Fisheries Experimental Station at Halifax.

An interesting point is illustrated in figure 3. The additional thickness of fish which is frozen after removal from the bath, due to temperature redistribution, is plotted against the thickness which is frozen before removal. The analysis of this point is included in a former paper (3). The effect is small at high bath temperatures byt increases with lowered bath temperature, until at -78.5 °C. it contributes approximately one quarter of the whole effect.



Times of freezing for Haddock

Fig. 2.



'Residual' freezing for Haddock

Fig. 3.

The Processing Problem

In preparing canned fish for the market it is the custom to subject the tins, when sealed, to processing in a steam bath. The object of this procedure is to raise the temperature of the contents to such a degree that harmful bacteria are destroyed. It is obviously essential that the temperature of every point in the contents be raised above the thermal death point of the bacteria, but processing for very long times is not desirable.

An application of the mathematical theory of the flow of heat in a solid cylinder of finite length, permits the calculation of the temperature at any point in the contents at any time, under any processing conditions, and hence enables one to choose an approximate optimum time for processing. (8)

The present application of the theory yields the relation between the variables for a tin of radius (a) and length $2l$ whose initial temperature T°, and processing temperature θ, are in a scale having an arbitrary zero.

The equation for the flow of heat, $KD^2\theta = \frac{\partial\theta}{\partial t}$ becomes in cylindrical polar co-ordinates (r, ϕ, z)

$$\frac{\partial\theta}{\partial t} = k\left[\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\phi^2} + \frac{\partial^2\theta}{\partial z}\right]$$

where $k = \frac{k}{c\rho}$, and θ is the temperature at any point (r, ϕ, z) at any time t. If the flow be symmetrical about the z axis,

$$\frac{\partial\theta}{\partial T} = K\left[\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2}\right] \qquad (1)$$

The solution of this equation must satisfy the conditions,

(a) $0 < r < a$
(b) $z = \pm l, \quad \theta = 0$
(c) $r = a, \quad \theta = 0$
(d) $t = 0, \quad \theta = T$

To obtain a solution of (1) write

$$\theta = \psi(r)\, u(z)\, e^{-\alpha^2 t}$$

The equation which ψ and u must satisfy is then,

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r\psi}\frac{\partial\psi}{\partial r} + \frac{1}{u}\frac{\partial^2 u}{\partial z^2} + \frac{\alpha^2}{K} = 0 \qquad (2)$$

and since the first two terms are independent of the last two,

$$\frac{1}{u}\frac{\partial^2 u}{\partial z^2} + \frac{\alpha^2}{K} = \mu^2 \qquad (3)$$

$$\frac{1}{\psi}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r\psi}\frac{\partial\psi}{\partial r} + \mu^2 = 0 \qquad (4)$$

where μ is a constant.

The solution of (3) is of the form.

$$u = \frac{\sin}{\cos}\sqrt{\frac{\alpha^2}{K} - \mu^2}\cdot z$$

and on consideration of condition (b), and since $\sqrt{\frac{\alpha^2}{K} - \mu^2} = \frac{m\pi}{2l}$, we may write a solution of (3) as,

$$u e^{-\alpha^2 t} = \sum_{m=1}^{a} B_m \sin\frac{m\pi}{2l}(z+l)\, e^{-K\left(\mu^2 + \frac{m^2\pi^2}{4l^2}\right)t} \quad \ldots m = 1, 2, 3 \ldots$$

Equation (4),

$$r^2\frac{\partial^2\psi}{\partial r^2} + r\frac{\partial\psi}{\partial r} + \mu^2 r^2 \psi = 0$$

is a Bessels equation of the zero order. The solution is,

$$\psi = \sum_n c_n J_0(\mu_n r).$$

since μ_n may have an infinite number of values given by the roots of the equation,

$$J_0(\mu_n a) = 0$$

The solution of (1) may then be written

$$\theta \sum_n \sum_{m=1}^{\infty} A_{mn} J_0(\mu_n r) \sin \frac{m\pi}{2l}(z+l) e^{-K\left(\mu_n^2 + \frac{\pi^2 m^2}{4l^2}\right)t} \quad \text{- - - -} \qquad (7)$$

The value of A_{mn} is found in the usual way by multiplying each side of the equation (when $t = 0$) by $J_0(\mu_n r)\, r\, dr \sin \frac{m\pi}{2l}(z+l)\, dz$ and integrating between the limits 0 and a, and $-l$ and $+l$. Since

$$\int_0^a J_0(\mu_n r) J_0(\mu_K r) = 0 \qquad K \neq n$$

and

$$\int_{-l}^{+l} \sin \frac{m\pi}{2l}(z+l) \sin \frac{K\pi}{2l}(z+l)\, dz = 0 \qquad K \neq m.$$

$$A_{mn} = \frac{T \int_0^a J_0(\mu_n r)\, r\, dr \int_{-l}^{+l} \sin \frac{m\pi}{2l}(z+l)\, dz}{\int_0^a [J_0(\mu_n r)]^2 r\, dr \int_{-l}^{+l} \sin^2 \frac{m\pi}{2l}(z+l)\, dz}$$

$$= \frac{8T}{m\pi} \frac{1}{\mu_n a\, J_1(\mu_n a)} \qquad m \text{ odd}$$

$$= 0 \qquad m \text{ even.} \qquad (8)$$

The complete solution of (1) is then,

$$\theta = \sum_n \sum_m \frac{8T}{m\pi} \frac{1}{\mu_n a\, J_1(\mu_n a)} J_0(\mu_n r) \sin \frac{m\pi}{2l}(z+l) e^{-K\left(\mu_n^2 + \frac{m^2\pi^2}{4l^2}\right)t} \qquad (9)$$

$m = 1, 3, 5, 7$ - - - -

The values of μ_n may be obtained from the zeros of the function $J_0(x)$ which are given by,

$$x = \left(n\pi - \frac{\pi}{4}\right) + \frac{1}{8\left(n\pi - \frac{\pi}{4}\right)} - \frac{31}{381\left(n\pi - \frac{\pi}{4}\right)^3} + \frac{3779}{15360\left(n\pi - \frac{\pi}{4}\right)^5} \text{ - - -} \qquad n = 2, 3, 4 \text{ - - -}$$

The first forty are listed in Table 1.

In this problem we are interested in the thermal conditions of the centre of the contents. For example consider the point $(\frac{a}{100}, a)$. Equation (9) then becomes,

$$\theta = \sum_m \sum_n \frac{8\pi}{\pi} \frac{J_0\left(\frac{\mu_n a}{100}\right)}{\mu_n a J_1(\mu_n a)} \cdot \frac{\sin \frac{m\pi}{2}}{m} \cdot e^{-K\left(\mu_n^2 + \frac{m^2\pi^2}{4l^2}\right)t} \qquad (10)$$

Table 1 gives the values for $\mu_n a$, $\frac{8}{\mu_n a J_1(\mu_n a)}$, and $\frac{8 J_0\left(\frac{\mu_n a}{100}\right)}{\mu_n a J_1(\mu_n a)}$ for the first forty terms. When t is zero a great number of terms is necessary to express θ accurately, but as t increases, the effect of the first term in both the n and m summations becomes relatively greater, until when t is moderately large it is the only factor in a discussion where an accuracy of one or two percent is required.

TABLE 1

$\mu_n a$	$\frac{8}{\mu_n a J_1(\mu_n a)}$	$\frac{8 J_0\left(\frac{\mu_n a}{100}\right)}{\mu_n a J_1(\mu_n a)}$	$\mu_n a$	$\frac{8}{\mu_n a J_1(\mu_n a)}$	$\frac{8 J_0\left(\frac{\mu_n a}{100}\right)}{\mu_n a J_1(\mu_n a)}$
2.405	+6.42	+6.42	36.92	-1.65	-1.59
5.520	-4.25	-4.24	40.06	+1.58	+1.52
8.654	+3.41	+3.40	43.20	-1.53	-1.46
11.79	-2.95	-2.91	46.34	+1.48	+1.41
14.93	+2.60	+2.59	49.48	-1.43	-1.35
18.07	-2.25	-2.23	52.62	+1.38	+1.29
21.21	+2.13	+2.16	55.77	-1.34	-1.23
24.35	-2.04	-2.01	58.91	+1.31	+1.20
27.49	+1.92	+1.88	62.05	-1.28	-1.15
30.63	-1.81	-1.78	65.19	+1.25	+1.11
33.78	+1.73	+1.58	68.33	-1.21	-1.08
71.47	+1.81	+1.04	99.75	-1.00	-0.769
74.61	-1.16	-1.00	102.9	+0.989	+0.743
77.76	+1.14	+0.970	106.0	-0.972	-0.718
80.90	-1.12	-0.940	109.2	+0.959	+0.695
84.04	+1.09	+0.906	112.5	-0.940	-0.668
87.18	-1.07	-0.878	115.5	+0.930	+0.648
90.32	+1.05	+0.849	118.6	-0.917	-0.620
93.46	-1.04	-0.823	121.7	+0.910	+0.602
96.61	+1.02	+0.790	124.9	-0.899	-0.580

From equation (10) it is possible to calculate the temperature of the contents at the centre of the can under any initial and processing conditions. The number of terms used depends on the values of t, a, and l and can be easily ascertained from inspection. When conditions are such that only the first term in the m and n summations is necessary, the equation reduces to the form,

$$t = \frac{2.302}{K} \frac{1}{\left[\left(\frac{2.40}{a}\right)^2 + \frac{\pi^2}{4l^2}\right]} \log_{10} \frac{6.42\, T}{11\, \theta_{10}} \quad \text{-----} \quad (11)$$

which is quite sufficient for practical purposes, where t is always large.

The following curves (4 and 5) have been included in the paper merely to furnish a graphical representation of the general results of the analysis, and, perhaps, to make more evident the facts contained in equations (10) and (11).

The graphs apply to tins of haddock, for which the constant was determined. It is probable that this constant does differ greatly for non-oily fish, as indicated by the figures of table 2. When the contents contain a great deal of liquid, convection currents are set up and the analysis becomes less applicable, but should still give approximate results providing the value of k is determined by experiment. (Figures 4 and 5).

The value of the constant k for haddock was determined by measuring the temperature at the centre of different sized cans with a properly protected thermocouple, at different times after introduction to the processing bath. It was approximately constant for cans of different sizes and for central temperatures taken at various times thus indicating the close parallel between the practical and the ideal case.

A comparison between the theoretical and the experimental temperatures at the centre of various times after the introduction of the tin to the processing bath, is given in figure 6, for two different sized tins of haddock muscle.

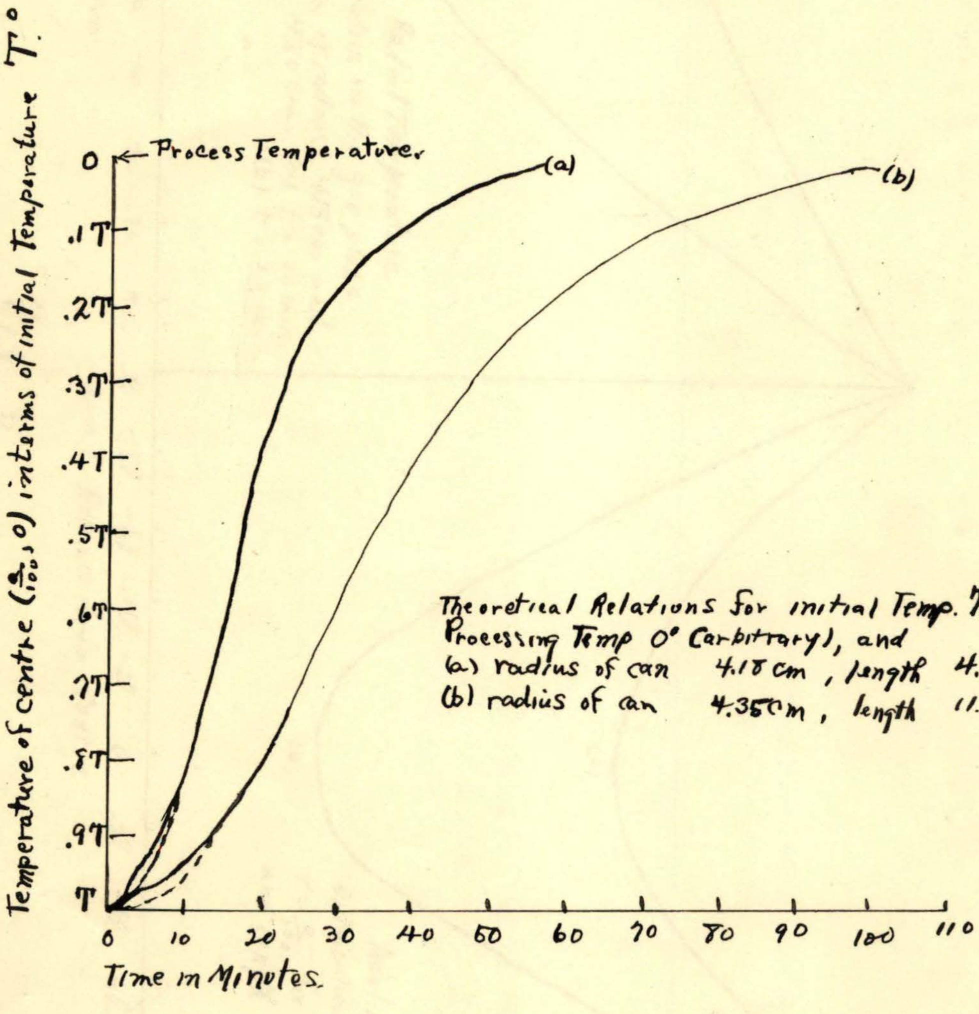


Fig. 4.

Theoretical Time-Temperature Relations.

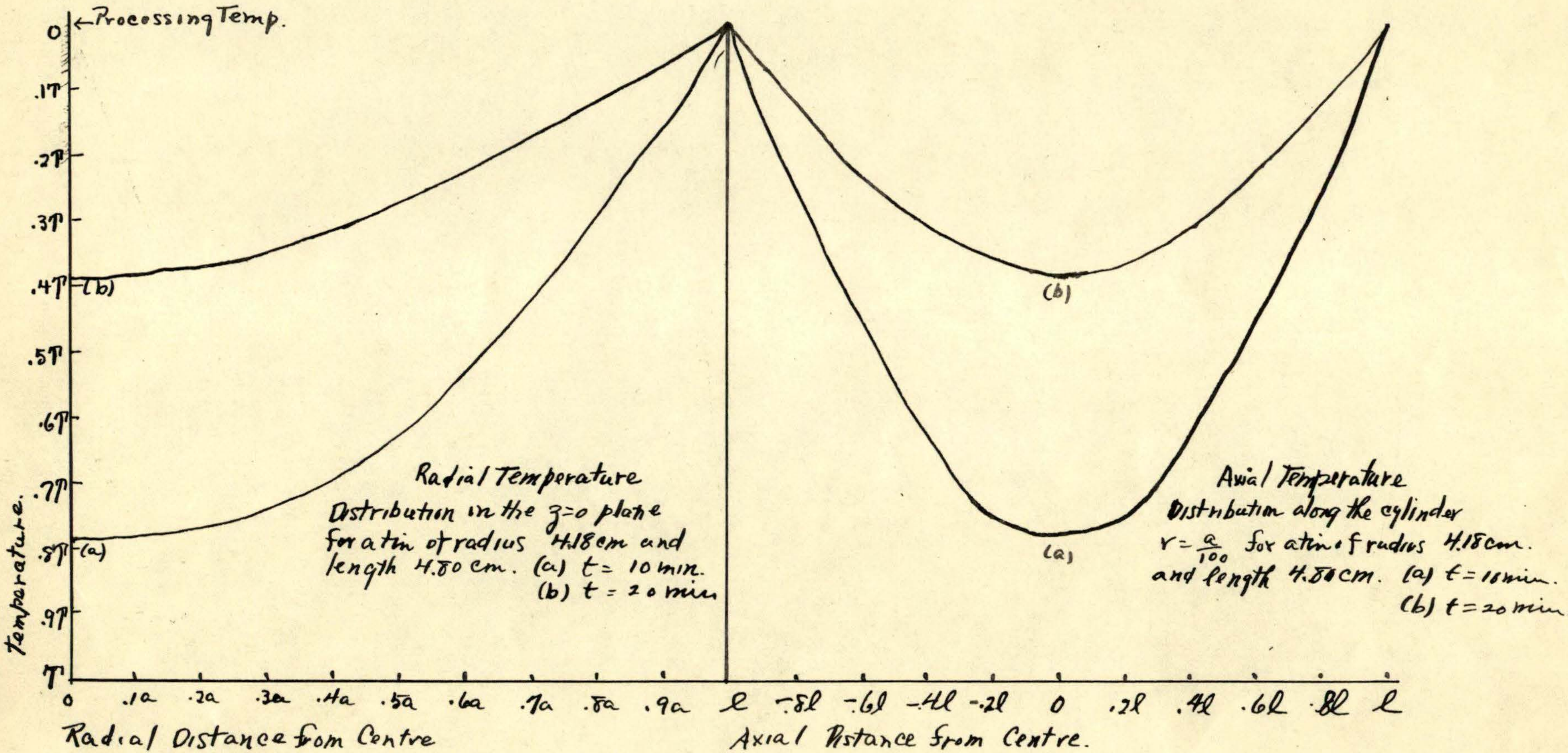

← Processing Temp.
0
.1T
.2T
.3T
.4T
.5T
.6T
.7T
.8T
.9T
T
Temperature.
(b)
(a)
Radial Temperature
Distribution in the z=0 plane
for a tin of radius 4.18 cm and
length 4.80 cm. (a) t = 10 min.
(b) t = 20 min
(b)
(a)
Axial Temperature
Distribution along the cylinder
r = a/100 for a tin of radius 4.18 cm.
and length 4.80 cm. (a) t = 10 min.
(b) t = 20 min.
0
.1a
.2a
.3a
.4a
.5a
.6a
.7a
.8a
.9a
l
-.8l
-.6l
-.4l
-.2l
0
.2l
.4l
.6l
.8l
l
Radial Distance from Centre
Axial Distance from Centre.

Fig. 5.

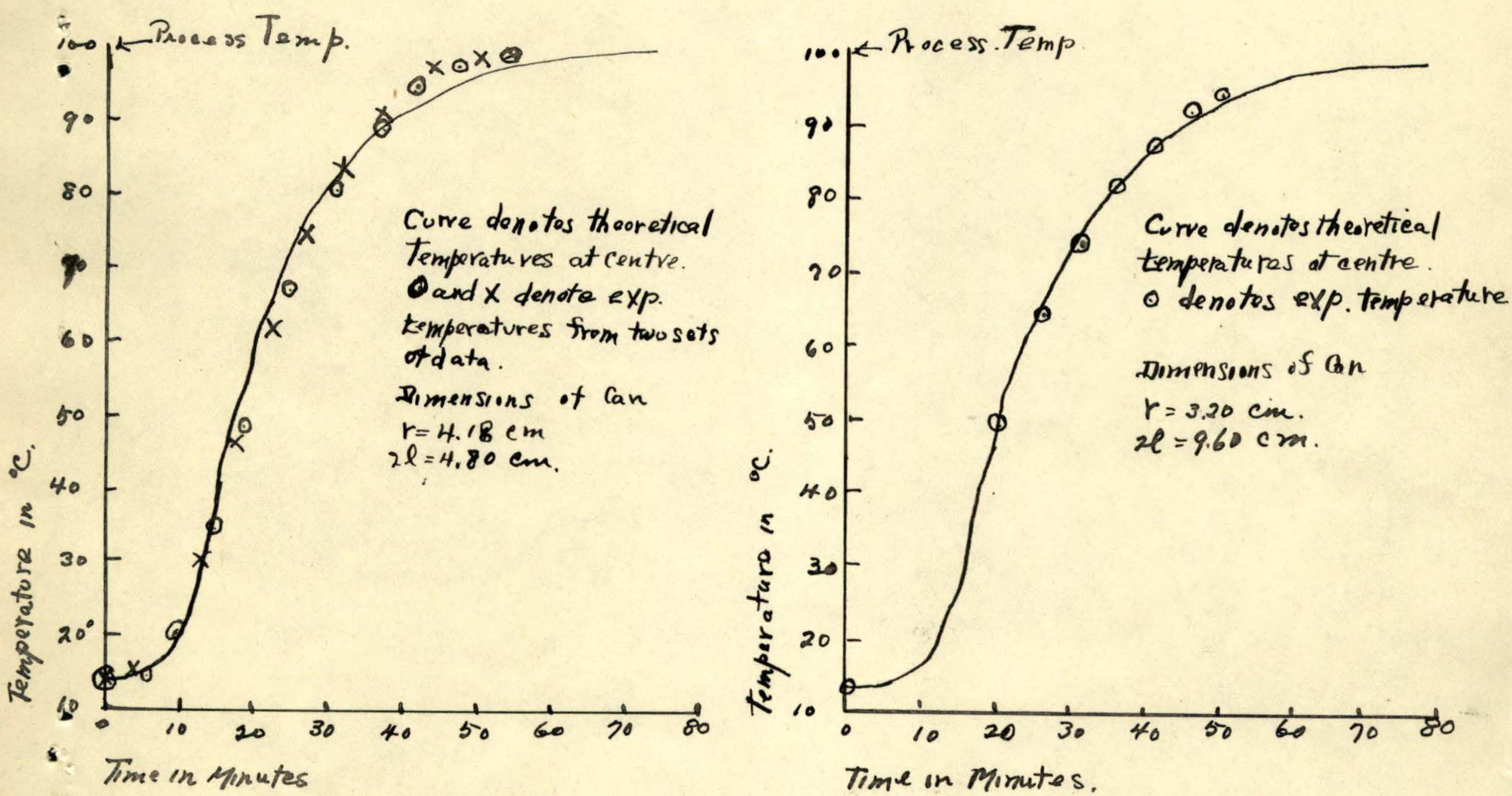


Fig. 6.
Comparisons of the theoretical and experimental temperatures at the centres of two tins of different dimensions.

The average value of the constant k for haddock for a processing temperature of 100°C. was 0.0016 cm.^{2}/sec. This value may be useful over a wide range of processing temperatures, since the value of k, c, or ρ is not likely to vary greatly over the range of temperatures which are apt to be used. Further work on this point is, however, necessary. It will be noted that the value of k calculated from $k = 0.0016 = \frac{k}{c\rho}$ agrees well with that determined by the hot plate method. (0.00164)

Summary

1. Values are given for the physical and thermal constants of the muscle of fish of various species.

2. An equation, developed by the writer in a previous paper, for the rate of freezing of a rectangular block of fish muscle, is shown to hold exceptionally well under experimental conditions.

3. A mathematical analysis of the temperature - time relations in the processing of canned goods is given. The resulting equation holds under experimental conditions.

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