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EVALUATION OF TWO RISK ASSESSMENT METHODS

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EVALUATION OF TWO RISK ASSESSMENT METHODS

by

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ABSTRACT

This paper evaluates the mathematical implementations of the risk assessment methods in the Life Cycle Costing (LCC) model of the Logistics Analyzer (LOGAN) program. The two risk assessment methods were compared against a software simulation which generated a distribution based upon the same data used in the LCC model.

RÉSUMÉ

Ce document évalue l'application mathématique de méthodes d'estimation du risque dans le modèle de Comptabilisation du Coût du Cycle de Vie (CCCV) du programme LOGAN. Les deux méthodes d'estimation du risque sont comparées à un logiciel de simulation qui génère une distribution sur la base des mêmes données utilisées dans le modèle de CCCV.

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EVALUATION OF TWO RISK ASSESSMENT METHODS

I. INTRODUCTION

1. Risk derives from our inability to predict the future. It indicates a degree of uncertainty that is significant enough to deem it noticeable and alter the path envisioned. Risk is categorized either objective or subjective, both requiring personal judgement and frequent assessment to determine acceptability. Objective risk can be described precisely based upon theory, experiment, or common sense. The perfect analogy is the flipping of a coin. It is understood that the probability of achieving either heads or tails is balanced. Subjective risk, however, is very open-ended in the sense that you are able to always refine your assessment with new or more detailed information, further studies or by giving weight to consistent or opposing opinions. Subjective risk is more difficult to judge or prepare for as even the slightest changes can affect the validity of the risk assessment. Predicting the weather follows this loose illustration of subjective risk. In most cases, the risks analyzed are subjective.

2. The first step in assessing and quantifying risk is recognizing a need for risk analysis and modeling. Quantifying risk implies researching all possible values an uncertain variable can undertake and determining the relative likelihood of each value. There is no set mathematical model that is able to solve the risk associated with the possible outcomes and develop a single solution. There are, however, equations and formulae that have been proven to work effectively but by no means give the "perfect" or only solution. Assessing risk implies utilizing the best information made available, always remembering that any quantifying of risk is an estimate. Risk quantification involves judgement as you may not have the complete information necessary or the problem may be too complex or possess numerous uncontrolled variables.

3. After quantifying risk, the set of determined outcomes and probabilities of occurrence for the model can be utilized. People often mistakenly assume that these techniques "are magic black boxes that unequivocally arrive at the correct answer or decision."[1] The distributions have been mathematically and experimentally proven but

they should be recognized as mere quantitative tools which never should represent a replacement for personal judgement.

4. Risk analysis using probability distributions of uncertain parameters has become the standard alternative to using single point estimate for input data [2]. Risk analysis is any method, qualitative and/or quantitative, for assessing the impacts of risk on decision situations. A number of techniques are used that blend both qualitative and quantitative approaches. The goal of any of these methods is to help the decision-maker choose the appropriate course of action, given a better understanding of the possible outcomes that could arise.

5. The Directorate of Logistics Analysis (D Log A) conducts diverse, analytical research and provides decision assistance through the design, development and evaluation of specific decision support systems [3]. The model examined in this paper is part of the Logistics Analyzer (LOGAN) shell which integrates a level of repair analysis, spare parts analysis, and the focus of this report, life cycle costing (LCC). The LOGAN (LCC) program is written in C code with a graphical user interface so that the users can input the data in a clear, concise manner.

6. Life Cycle Costing is defined as the study of all the costs of an equipment or system arising over its entire life. LCC estimates the "cradle to grave" cost of a system [3]. LCC evaluates various alternatives to determine the optimum method to employ resources. The reason for a risk assessment in the LCC model, is to provide the user with a range of values and their respective probabilities around the approximated life cycle cost. Although the life cycle cost is an estimate, it is not the answer, the risk assessment provides a range of likely values around that estimate.

7. An experimental LOGAN (LCC) model houses two risk assessment methods: Edgeworth's Truncated Series Method, the one currently available in the standard LCC model, and the Rational Polynomial Method, earlier implemented for testing. Both are measured against a standard provided by the @RISK software which produces a simulated distribution based solely on the addition or multiplication of known distributions. This will determine which risk assessment method provides a closer representation of the "real" risk distribution.

II. THE STOCHASTIC APPROACH

8. The term stochastic is synonymous with uncertainty or risk and is associated here with cost estimation. The user inputs the most likely value of a given parameter along with its respective lower and upper values. The stochastic approach considers the parameter (i.e. cost or a parameter contributing to the calculation of a cost) as a random variable whose probability density function is triangular. The moments of the distribution can be easily computed. The life cycle cost of a system is the summation and multiplication of these costs and its cumulative distribution function can be approximated assuming independent costs [3].

9. The assumption of independence between costs is unrealistic as costs are not the result of a controlled experiment and usually are mere estimates or extrapolations of other data. However, the assumption is often necessary to simplify or even permit the estimation to be calculated [3].

MOMENTS OF A DISTRIBUTION

10. This section presents a brief description of the moments required for the mathematics involved in the two implemented risk assessment methods. A more complete discussion and mathematical representation of the moments of a distribution can be found in [3, 4].

11. The first raw moment of a distribution is called the mean. The mean gives a descriptive measure of the location of the distribution.

12. The second moment about the mean is known as the variance and is denoted as μ_2. The variance is a measure of dispersion about the mean. The square root of the variance is called the standard deviation.

13. The third moment about the mean is related to the asymmetry of the distribution and is denoted as μ_3. Consider the quantity

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} \qquad (1)$$

This $\sqrt{\beta_1}$ quantity measures the skewness of the distribution relative to its spread. $\sqrt{\beta_1}$ permits the comparison of the symmetry of two distributions whose scales of measurement differ. Skewed distributions have more values to one side of the peak or most likely value. If the skewness of a distribution is zero, the distribution is symmetrical. If $\mu 3$ is greater than 0, the distribution is skewed to the right and less than 0, to the left.

14. The fourth moment about the mean is related to the kurtosis of the distribution and is denoted as $\mu 4$. The kurtosis is a measure of the degree of flatness or peakedness of the distribution. Consider the quantity

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} \qquad (2)$$

β_2 is a relative measure of the kurtosis that is independent of scale. The higher the kurtosis value, the more peaked the distribution.

THREE POINT COST ESTIMATION

15. Under the stochastic approach in LOGAN (LCC), each parameter supporting estimates of lower, most likely and upper values is understood to follow a triangular distribution. This assumption was designed to simplify the encumbrance of data gathering. The triangular distribution provides a rough representation of the actual distribution, and is simple and convenient. LOGAN (LCC) determines the statistical distribution of the life cycle cost of the system on the basis of these triangular distributions.

16. It is important to note that the lower and upper values provided by the user are the 5th and 95th percentiles respectively as opposed to the lower bound (L) and the upper bound (H). The values of L and H are derived iteratively from the percentiles as suggested in [2]. The @RISK program utilizing the TRIGEN function, is able to produce the same distribution using the 5th and 95th percentiles which facilitates the comparison.

17. The height of the triangular distribution is given by

$$h = \frac{2}{H - L} \tag{3}$$

For further notes on the triangular distribution itself or the derivation of its moments refer to [3].

III. RISK ASSESSMENT METHODS

EDGEWORTH'S METHOD

18. As mentioned previously, life cycle cost calculations are a combination of summations and products. Each cost is seen as a random variable following a triangular distribution with the assumption that there is independence between costs. By using the first four moments, the risk assessment can be calculated effectively.

19. Using the Gram-Charlier Type A series, the cumulative distribution function of the life cycle cost can be determined [4]. The distribution function f(x) may be expanded formally as:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \left[1 + \frac{(\mu_2 - 1)}{2} H_2(x) + \frac{\mu_3}{6} H_3(x) + \frac{(\mu_4 - 6\mu_2 + 3)}{24} H_4(x) + \mathrm{K} \right] \quad (4)$$

20. Here, $H_r(x)$ is the Chebyshev-Hermite polynomial of order r, defined by the relationship:

$$\left(-\frac{d}{dx}\right)^r e^{\frac{-x^2}{2}} = [H_r(x)] e^{\frac{-x^2}{2}} \quad (5)$$

It follows that

$H_0(x) = 1$ by convention
$H_1(x) = x$
$H_2(x) = x^2-1$
$H_3(x) = x^3-3x$
$H_4(x) = x^4-6x^2+3$
$H_5(x) = x^5-10x^3+15x$
...

21. The cumulative distribution function associated with a known set of moments can then be expressed as:

$$\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$
$$-\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \left[\frac{(\mu_2 - 1)}{2} H_1(x) + \frac{\mu_3}{6} H_2(x) + \frac{(\mu_4 - 6\mu_2 + 3)}{24} H_3(x) + \mathrm{K} \right] \quad (6)$$

In LOGAN(LCC), the Edgeworth's form of the Type A series depicted in [4] is used. The cumulative distribution function, with a standardized x, is approximated by:

$$\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$
$$-\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \left[\frac{\sqrt{\beta_1}}{6} H_2(x) + \frac{(\beta_2 - 3)}{24} H_3(x) + \frac{\beta_1}{72} H_5(x) \right] \quad (7)$$

where $\sqrt{\beta_1}$ and β_2 are determined by equations (1) and (2). This is the cumulative distribution function of a Normal(0,1) adjusted by an error term.

22. Using this approximation, we will be able to evaluate the probability that the project does not exceed a specified level of life cycle cost.

23. The C programming code for this method is rather easy to understand but confusing to interpret (please refer to Annex A for the complete function). The binary search through the normal table array is very fast which does not hinder the performance of this risk assessment method and until extensive testing was done, this method seemed to be the most outstanding.

24. There are limitations to this method. The points generated by both risk assessment methods create a cumulative distribution graph. The Edgeworth's series will sometimes provide a negative cumulative probability for some life cycle cost values, which is of course impossible. Through further testing, it was observed that, as the $\sqrt{\beta_1}$ value increased, so did the occurrence of the negative probabilities. Moreover, the cumulative

graph begins declining at the upper echelon of values. Both anomalies are probably caused by the truncation of the series.

RATIONAL POLYNOMIAL METHOD

25. The Pearson family of distributions has been used extensively to fit frequency distributions of empirical data. As a tool for approximating percentile points, based upon the four moments of a statistic, the Pearson system, with tables developed by Pearson & Hartley (1972), has few equals [5]. The formulae which are rational polynomials in $\sqrt{\beta_1}$ and β_2 with coefficients of $\alpha_{r,s}^{(i)}$, are established for eleven percentile points. The percentile points approximated are 1%, 2.5%, 5%, 10%, 25%, 50%, 75%, 90%, 95%, 97.5% and 99% so that the tails of the distribution are well represented. Although there is less flexibility in this method, since the Edgeworth's series computes values at any percentile point, these eleven points appear to provide sufficient accuracy for risk analysis purposes [2]. Please note that under this method $\sqrt{\beta_1}$ is taken as the absolute value of (1).

26. The standardized value for a percentile α is given by:

$$Z_\alpha(\sqrt{\beta_1},\beta_2) = \frac{\pi_1(\sqrt{\beta_1},\beta_2)}{\pi_2(\sqrt{\beta_1},\beta_2)} \qquad (8)$$

where for i = 1,2

$$\pi_i(\sqrt{\beta_1},\beta_2) = \sum_{0 \le r+s \le 3} \alpha_{r,s}^{(i)}(\sqrt{\beta_1})^r(\beta_2)^s \qquad (9)$$

with $\alpha_{0,0}^{(2)} = 1$ (see Figure 1). The coefficients are tabulated in Annex B.

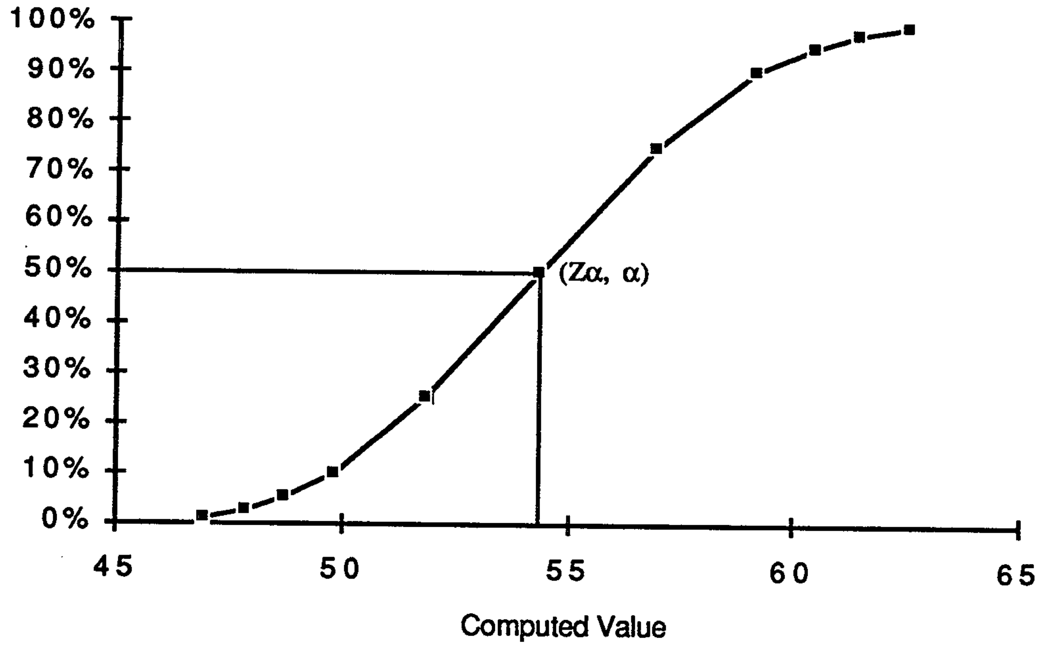

Figure 1: Cumulative Distribution

27. Bowman and Shenton papers [5,6] provide coefficients for $\sqrt{\beta_1}$ values between 0 and 2 inclusively with maximum errors in the calculated percentiles of 0.5%. Bowman and Shenton papers provide no coefficients for $\sqrt{\beta_1}$ values exceeding 2.00. Thorough tests of the LOGAN(LCC) model shows that such values are extremely unlikely (refer to the Testing Results Chart in Annex C).

28. The algorithm and programming code for the rational polynomials is rather simple. The entire logic behind this risk assessment method is in the percentile tables outlined in [5,6] (refer to Annex B).

IV. @RISK SOFTWARE

29. @RISK is a software system developed by Palisade Corporation for the analysis of business and technical situations impacted by risk. The @RISK software runs under Microsoft Excel 5.0. To compare the distributions, both the Edgeworth and rational polynomial methods were contrasted with an @RISK simulation using only triangular distributions.

30. The @RISK software uses a technique called "simulation" to combine all the uncertainties identified in the model being assessed. The software has the ability to incorporate various factors surrounding the variable, including its full range of possible values and some measure of likelihood of occurrence for each possible value. With multiple simulations varying certain attributes, @RISK allows the user to access a full range of solutions.

31. Sampling is the process by which values are randomly drawn from input probability distributions [1]. Sampling within a simulation is done repetitively, with one sample drawn every iteration from each input probability distribution. With ample iterations, the sampled values for a probability distribution will approximate the true statistics of the input data. The dominant factor is to examine the number of iterations required to accurately recreate an input distribution through sampling. By varying the iterations using @RISK, the optimum number which both represented the actual distribution with few skewed points and possessed an acceptable runtime was 1000. Accurate results for resulting distributions depend on a complete sampling of input distributions.

32. The @RISK package contains two methods of sampling: Monte Carlo or Latin Hypercube. The method used to compare against the LOGAN (LCC) models was the Latin Hypercube as it forces the samples drawn to correspond more closely with the input distribution and thus converges faster on the true statistics of the input distribution. Latin Hypercube sampling is a recent development in sampling technology designed to accurately recreate the input distribution through sampling in fewer iterations when compared against the Monte Carlo simulations. The key is the stratification of the input probability distributions. Stratification divides the cumulative curve into equal intervals on the cumulative probability scale between 0 and 1. A sample is then randomly taken from each

interval or stratification. Sampling is forced to represent values in each interval, recreating the input probability distribution and more accurately reflect the dispersion of values in the distribution.

33. The distribution created using the Latin Hypercube sampling in @RISK is as close as possible to the true distribution and therefore was chosen as the best method to compare the implemented risk assessments against. Although the triangular distributions were summed and multiplied like in the two risk assessment methods, the @RISK distribution needed a constant to be added, after the calculations were made to account for other calculations made in LOGAN (LCC) that do not pertain to risk. The @RISK simulation only looked at the extra fixed cost and its accompanying boundaries before any summations or products were done. The twelve mandatory costs were not examined in the @RISK simulation and thus created the constant necessary to align the three generated distributions. The simulation was based upon this @RISK distribution formula: RiskTrigen(Lower, Mode, Upper, Lower Percentile, Upper Percentile). This made the comparison of the three methods possible because the 5^{th} and 95^{th} percentiles could be used and thus similar distributions would be created.

V. CONTRASTING THE THREE DISTRIBUTIONS

34. Both the LOGAN (LCC) model and the @RISK software create a set of points from which the distribution is plotted. All three sets of distributions were brought into Microsoft Excel for each $\sqrt{\beta_1}$ increment of approximately 0.1 and compared until $\sqrt{\beta_1}$ values were unattainable for one fixed cost with one or two multipliers. The $\sqrt{\beta_1}$ values ranged from 0.1 to the extreme value of 3.3. Although these extremes are unlikely, they were tested to determine if the integrity of the distributions would hold if a scenario was created where such extreme values would be necessary.

35. As hoped, throughout the range of $\sqrt{\beta_1}$ values, the three distributions are relatively comparative, in their natural shape. For the entire range of $\sqrt{\beta_1}$ values, both implemented risk assessment methods where frequently plotted over or were extremely close to each other through the 30th to 80th percentile (All output graphs referenced are in Annex C). For the low to middle $\sqrt{\beta_1}$ values, ranging from 0.1 to 1.1, the three distributions remained consistent relative to each other. They had similar end points and seemed to follow the same cumulative curve. Past the 1.1 value, the Edgeworth's method began to breakdown with the appearance of numerous negative values (not shown on the graphs). The distribution had a very skewed beginning, consisting of points below the starting point of the Rational polynomial and the @RISK method. However, the Edgeworth's series continued to follow the other two distributions as the life cycle cost values climbed, even with the distant starting point. With the high range of $\sqrt{\beta_1}$ values, between 1.8 and 2.5, the Edgeworth's series began to decline at the upper values of the distribution. In the high values, it is very apparent that the Edgeworth's series skews from our control. Both the @RISK and Rational polynomial seem to have similar ending points. Although there is no documented behaviour of the Rational polynomial method with $\sqrt{\beta_1}$ values exceeding 2.0, the distribution seemed to perform well until values exceeding 2.5. At $\sqrt{\beta_1}$ values beyond 2.5, all three distributions began to fail, creating jagged and rough distributions.

VI. RECOMMENDATIONS

36. Due to the limitations of the Edgeworth's truncated series, the Rational polynomials method seems to be the obvious choice for LOGAN (LCC). The Rational polynomials seemed to have no obvious limitations between the extremely low to high $\sqrt{\beta_1}$ values, a range of 0.1 to 2.5. The series performed very consistently with the @RISK software. Also, because the algorithm deals only with the eleven predetermined percentile points, calculations could be performed quickly if runtime is a concern. The Rational polynomial method is a simple and solid method for computing the risk assessment of a given scenario.

REFERENCES

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4. Kendall, M. and Stuart, A., The Advanced Theory of Statistics, Volume I - Distribution Theory, Fourth Edition, New York, MacMillan Publishing Co. Inc., 1977

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ANNEX A
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EDGEWORTH'S TRUNCATED METHOD

1. The code breaks the Type A series into two parts, the "availability"

$$\left(\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx\right)$$

and the error term, $$\left(\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\left[\frac{\sqrt{\beta_1}}{6} H_2(x) + \frac{(\beta_2 - 3)}{24} H_3(x) + \frac{\beta_1}{72} H_5(x)\right]\right).$$

The availability is the standard normal cumulative distribution. The availability is found by a binary search in the normal table array (NORM) which locates two points that straddle the deviate and then interpolates the real value from these four numbers in the array (two x and two y coordinates). If the standardized x variable was negative, the availability is subtracted from one. The error term is computed simply by applying the x, $\sqrt{\beta_1}$ and β_2 values into the method. Their resulting difference returns the value for the risk and this function is repeated until the risk is at 99.9% or until 51 points are plotted in the distribution.

```
float pgen(float xtemp, fuzzy *xlcc0)
   {
       float    x,          /* Could also be called Z                      */
                x0,         /* Work variables                                      */
                x1          /* Used to find the x and y coordinates in     */
                f0,         /* Array  table to determine the availability  */
                f1,
                aval,       /* Availability from array in CONST.H          */
                sigma,   /* Standard Deviation                             */
                err,        /* Error term of Edgeworth's formula           */
                b1,         /* √β1                                                 */
                b2,         /* β2                                                  */
                risk;       /* Edgeworth's formula                         */

       int      i,          /* i, j, k are variables used in binary division */
```

```
            j,                          /* to find the position of the CTAB matrix  */
            k,
            l_xneg = 0,                 /* Boolean expression (value = FALSE)       */
            l_xpos = 1;                 /* Boolean expression (value = TRUE)        */

sigma = sqrt (xlcc0 -> arr[2])          /* Finds sigma from root of variance */

/*          If sigma > 0.0, it standardizes the x variable                                  */
if (sigma > 0.0)
    {
            b1 = xlcc0 -> arr[3] / pow(sigma, 3.0);
            b2 = xlcc0 -> arr[4] / pow(sigma, 4.0);
            x  = (xtemp-xlcc0 -> arr[5]) / sigma;

/*          Sets up boolean expression if x value is negative so it can do the      */
/*          the correct calculations because all values are absolute                */
            if (x < 0.0)
                {
                        l_xneg = 1;
                        l_xpos = 0;
                }

            x = fabs(x);                        /* Finds absolute value of x                */

            x = min(x, 6.50);
            i = 0;
            j = TSIZE -1;                       /* TSIZE = 236                              */

/*          Perform binary search in normal table find two points straddling       */
/*          the deviate                                                                     */
            k = (j - i) / 2;

/*          This loop repeats until it finds, using the N[0,1] table, the exact     */
/*          value of the probability.  It keeps determining whether it should       */
/*          move up or down the table until it finds the correct table value        */
            do
                {
                        if (NORM[k][0] < x)
                            {
                                    i = k;
                                    k = (j + k) / 2;
                            }
                        else
                                    if (NORM[k][0] >= x)
                                        {
                                                j = k;
                                                k = (i + k) / 2;
                                        }
                }
            while (!((x >= NORM[k][0])&&(x <= NORM[k+1][0])));
```

```
        /*      If k = 235, assign these certain values to x0, x1, f0, f1          */
        /*      Otherwise, use k and (k+1) to locate the numbers surrounding real x*/
                if (k == TSIZE -1)
                    {
                        x0 = NORM[TSIZE-2][0];
                        x1 = NORM[TSIZE-1][0];
                        f0 = NORM[TSIZE-2][1];
                        f1 = NORM[TSIZE-1][1];
                    }
                else
                    {
                        x0 = NORM[k][0];
                        x1 = NORM[k+1][0];
                        f0 = NORM[k][1];
                        f1 = NORM[k+1][1];
                    }

        /*      If x was negative, must subtract from 1 the probability value  */
        /*      Interpolates to find the availability                          */
                if (l_xneg)
                        aval = 1.0 - (f0 + ((x - x0) / (x1 - x0)) * (f1 - f0));
                if (l_xpos)
                        aval = f0 + ((x - x0) / (x1 - x0)) * (f1 - f0);

        /*      Edgeworth's error calculation                                  */
                err = ((pow(x,2) - 1.0) * (b1 / 6.0));
                err = err+(((pow(x,2) - 3.0) * x) * ((b2 - 3.0) / 24.0));
                err = err+(((((pow(x,2) - 10.0) * (pow(x,2))) + 15.0) * x) * (pow(b1,2) /
                                72.0));
                err = err * exp(-pow(x,2) / 2.0) / sqrt(2.0 * PI);
                risk = aval-err;
        }
        /*      If the standard deviation is 0, no standardizing is needed     */
        /*      and a risk value of 50% is assessed                            */
        else if (sigma == 0)
                risk = 0.5;
        return ( risk );
}
```

ANNEX B
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RATIONAL POLYNOMIAL METHOD

1. The function must first determine if the $\sqrt{\beta_1}$ value is less than one, due to the nature of the tables as there are differing coefficient values for ranges of [0,1] and [1,2]. Next it simply sums the products of the corresponding $\sqrt{\beta_1}$ and β_2 values to their respective exponents for both the numerator and denominator. It then divides the two, multiplying the fraction by the standard deviation and adding the mean. The function is looped through until all eleven percentile points have a computed life cycle cost value.

Table of Coefficients in Approximation

```
/* r     s        N    RANGE[0,1]     D             N     RANGE[1,2]    D            */

                         /*      1%              */
{ 0.0, 0.0,-0.19336E+01, +0.10000E+01, +0.25597E+00, +0.10000E+01 },
{ 1.0, 0.0,-0.16061E+01, +0.16042E+00, +0.32520E+00, -0.41444E+01 },
{ 0.0, 1.0,+0.26955E+01, -0.10616E+01, -0.94571E-01, +0.13884E+00 },
{ 2.0, 0.0, -0.17036E+01, +0.43929E+00, +0.20418E+01, +0.46625E+01 },
{ 1.0, 1.0, +0.17236E+01, -0.35799E+00, -0.25786E+01, +0.37661E+00 },
{ 0.0, 2.0, -0.13209E+01, +0.51040E+00, +0.61148E+00, -0.20447E+00 },
{ 3.0, 0.0, -0.25464E+01, +0.10958E+01, -0.64242E+00, -0.19508E+01 },
{ 2.0, 1.0, +0.16812E+01, -0.63453E+00, +0.11968E+01, -0.20214E+00 },
{ 1.0, 2.0, -0.11812E+00, +0.53175E-01, -0.33145E+00, +0.11921E+00 },
{ 0.0, 3.0, +0.50875E-01, -0.19945E-01, +0.15508E-01, -0.85268E-02 },

                         /*      2.5%            */
{ 0.0, 0.0, -0.16453E+01, +0.10000E+01, -0.15063E+01, +0.10000E+01 },
{ 1.0, 0.0, -0.18494E+01, +0.89117E+00, +0.44876E+01, -0.19526E+01 },
{ 0.0, 1.0, +0.23915E+01, -0.12700E+01, -0.60765E+00, +0.21332E+00 },
{ 2.0, 0.0, -0.15844E+01, +0.37653E+00, -0.65584E+01, +0.21317E+01 },
{ 1.0, 1.0, +0.18290E+01, -0.81542E+00, +0.28944E+01, -0.11996E+01 },
{ 0.0, 2.0, -0.11167E+01, +0.56815E+00, -0.42381E+00, +0.21033E+00 },
{ 3.0, 0.0, -0.36091E+01, +0.16562E+01, +0.22664E+01, -0.52154E+00 },
{ 2.0, 1.0, +0.27150E+01, -0.11908E+01, -0.13425E+01, +0.53597E+00 },
{ 1.0, 2.0, -0.45857E+00, +0.24574E+00, +0.25806E+00, -0.12355E+00 },
{ 0.0, 3.0, +0.48102E-01, -0.24375E-01, -0.10421E-01, +0.60658E-02 },
```

```
                        /*      5%            */
{ 0.0, 0.0, -0.11044E+01, +0.10000E+01, -0.14645E+01, +0.10000E+01 },
{ 1.0, 0.0, -0.11300E+01, +0.74941E+00, +0.45349E+01, -0.20999E+01 },
{ 0.0, 1.0, +0.17681E+01, -0.13174E+01, -0.71053E+00, +0.30754E+00 },
{ 2.0, 0.0, +0.10947E+00, +0.23974E+00, -0.79213E+01, +0.36865E+01 },
{ 1.0, 1.0, +0.30566E+00, -0.12433E+00, +0.40018E+01, -0.23140E+01 },
{ 0.0, 2.0, -0.90598E+00, +0.59105E+00, -0.61932E+00, +0.39961E+00 },
{ 3.0, 0.0, -0.98814E+00, +0.85714E+00, +0.27320E+01, -0.11530E+01 },
{ 2.0, 1.0, +0.49355E+00, -0.47292E+00, -0.17752E+01, +0.10257E+01 },
{ 1.0, 2.0, +0.30625E+00, -0.16028E+00, +0.35919E+00, -0.22981E+00 },
{ 0.0, 3.0, -0.18707E-01, +0.12684E-01, -0.14944E-01, +0.11032E-01 },

                        /*      10%           */
{ 0.0, 0.0, -0.85842E+00, +0.10000E+01, -0.92873E+00, +0.10000E+01 },
{ 1.0, 0.0, +0.78929E+00, -0.63153E+00, +0.22039E+01, -0.29758E+01 },
{ 0.0, 1.0, +0.12196E+01, -0.12697E+01, +0.25828E+00, -0.50738E-01 },
{ 2.0, 0.0, -0.55088E+00, +0.64649E+00, +0.74312E+00, +0.67069E-01 },
{ 1.0, 1.0, -0.96385E+00, +0.92712E+00, -0.24307E+01, +0.20835E+01 },
{ 0.0, 2.0, -0.57312E+00, +0.49088E+00, +0.49934E+00, -0.45211E+00 },
{ 3.0, 0.0, -0.76974E-01, +0.48118E+00, +0.29833E-01, -0.28149E+00 },
{ 2.0, 1.0, +0.34882E+00, -0.46407E+00, +0.68249E+00, -0.58012E+00 },
{ 1.0, 2.0, +0.47311E+00, -0.39753E+00, -0.19687E+00, +0.17621E+00 },
{ 0.0, 3.0, -0.56120E-01, +0.52413E-01, +0.68136E-02, -0.67195E-02 },

                        /*      25%           */
{ 0.0, 0.0, -0.24031E+01, +0.10000E+01, -0.16092E+01, +0.10000E+01 },
{ 1.0, 0.0, -0.14891E+01, +0.16616E+02, +0.81385E+00, +0.43836E+00 },
{ 0.0, 1.0, +0.75590E+00, -0.49826E+01, -0.59847E-01, +0.27965E+00 },
{ 2.0, 0.0, -0.23309E+01, +0.10626E+01, -0.18180E+00, +0.12516E+01 },
{ 1.0, 1.0, +0.92314E+01, -0.18888E+02, +0.12679E+01, -0.30536E+01 },
{ 0.0, 2.0, -0.38930E+01, +0.72830E+01, -0.36205E+00, +0.65936E+00 },
{ 3.0, 0.0, -0.70192E+01, +0.14642E+02, +0.29683E+00, -0.10579E+01 },
{ 2.0, 1.0, +0.23648E+01, -0.26394E+01, -0.83310E+00, +0.17886E+01 },
{ 1.0, 2.0, +0.44308E+00, -0.12739E+01, +0.26630E+00, -0.47962E+00 },
{ 0.0, 3.0, -0.29437E-01, +0.60862E-01, -0.15081E-01, +0.25311E-01 },

                        /*      50%           */
{ 0.0, 0.0, -0.20711E-03, +0.10000E+01, -0.97078E-01, +0.10000E+01 },
{ 1.0, 0.0, +0.68277E-02, +0.25087E+01, +0.58192E+00, -0.18383E+01 },
{ 0.0, 1.0, -0.75482E-05, -0.26169E+01, -0.19936E-01, -0.54124E+00 },
{ 2.0, 0.0, +0.47086E+00, +0.94103E+00, -0.70250E+00, +0.68041E+00 },
{ 1.0, 1.0, -0.21213E+00, -0.23261E+01, +0.11526E+00, +0.17654E+01 },
{ 0.0, 2.0, +0.11003E-03, +0.16482E+01, +0.13959E-02, -0.37606E+00 },
{ 3.0, 0.0, -0.22462E+00, +0.23024E+01, +0.15950E+00, -0.11130E+01 },
{ 2.0, 1.0, +0.11733E+00, -0.14821E+01, -0.62382E-01, +0.44447E-01 },
{ 1.0, 2.0, -0.79682E-01, -0.20294E-01, +0.13380E-01, +0.42456E-01 },
{ 0.0, 3.0, -0.10565E-04, +0.99168E-03, +0.18557E-03, -0.17448E-02 },

                        /*      75%           */
{ 0.0, 0.0, +0.39672E+00, +0.10000E+01, +0.45223E+00, +0.10000E+01 },
{ 1.0, 0.0, -0.17140E+00, -0.60818E+00, -0.24004E+01, -0.43837E+01 },
{ 0.0, 1.0, -0.74246E+00, -0.19005E+01, +0.38372E+00, +0.67456E+00 },
{ 2.0, 0.0, +0.48553E+00, +0.45451E+00, +0.36817E+01, +0.63029E+01 },
{ 1.0, 1.0, +0.19650E+00, +0.79612E+00, -0.10796E+01, -0.18042E+01 },
{ 0.0, 2.0, +0.58942E+00, +0.10544E+01, +0.16619E-01, +0.23516E-01 },
{ 3.0, 0.0, -0.15768E+01, -0.19010E+01, -0.16829E+01, -0.29320E+01 },
{ 2.0, 1.0, +0.37082E+00, +0.29849E+00, +0.81147E+00, +0.14537E+01 },
{ 1.0, 2.0, -0.32481E+00, -0.51728E+00, -0.10376E+00, -0.20680E+00 },
{ 0.0, 3.0, +0.25296E-01, +0.50926E-01, +0.52116E-02, +0.10634E-01 },
```

```
                    /*      90%           */
{ 0.0, 0.0, +0.77212E+00, +0.10000E+01, +0.22548E+01, +0.10000E+01 },
{ 1.0, 0.0, +0.28272E-01, +0.17133E+00, -0.17005E+01, +0.30478E+00 },
{ 0.0, 1.0, -0.13932E+01, -0.15330E+01, -0.65377E+00, -0.58544E+00 },
{ 2.0, 0.0, +0.95628E+00, +0.12721E+01, -0.29256E+01, -0.40524E+01 },
{ 1.0, 1.0, -0.37918E+00, -0.58446E+00, +0.21397E+01, +0.18239E+01 },
{ 0.0, 2.0, +0.85487E+00, +0.78993E+00, +0.93244E-01, +0.83952E-01 },
{ 3.0, 0.0, +0.38225E+00, +0.33412E+00, +0.32094E+01, +0.34072E+01 },
{ 2.0, 1.0, -0.13009E+01, -0.12988E+01, -0.22896E+01, -0.20163E+01 },
{ 1.0, 2.0, +0.42400E+00, +0.39115E+00, +0.24580E+00, +0.20345E+00 },
{ 0.0, 3.0, -0.51933E-01, -0.46683E-01, -0.20536E-02, -0.75748E-03 },

                    /*      95%           */
{ 0.0, 0.0, +0.11504E+01, +0.10000E+01, +0.21265E+01, +0.10000E+01 },
{ 1.0, 0.0, +0.22674E+00, +0.17074E+00, -0.87599E+01, -0.55206E+01 },
{ 0.0, 1.0, -0.17407E+01, -0.12892E+01, +0.23560E+01, +0.16024E+01 },
{ 2.0, 0.0, +0.11356E+01, +0.11653E+01, +0.80491E+01, +0.61077E+01 },
{ 1.0, 1.0, -0.58596E+00, -0.46699E+00, -0.33844E+01, -0.24508E+01 },
{ 0.0, 2.0, +0.88839E+00, +0.58479E+00, -0.28416E-01, -0.12514E-01 },
{ 3.0, 0.0, +0.34743E+00, +0.33975E+00, -0.43522E+01, -0.37719E+01 },
{ 2.0, 1.0, -0.12531E+01, -0.10005E+01, +0.39741E+01, +0.28877E+01 },
{ 1.0, 2.0, +0.42319E+00, +0.25271E+00, -0.92840E+00, -0.59134E+00 },
{ 0.0, 3.0, -0.14362E-01, -0.91547E-02, +0.55908E-01, +0.34299E-01 },

                    /*      97.5%   */
{ 0.0, 0.0, +0.15989E+01, +0.10000E+01, +0.10803E+02, +0.10000E+01 },
{ 1.0, 0.0, +0.15353E+01, +0.42581E+00, -0.56739E+01, +0.28678E+01 },
{ 0.0, 1.0, -0.22124E+01, -0.11636E+01, -0.83559E+01, -0.30580E+01 },
{ 2.0, 0.0, +0.83145E+00, +0.67165E+00, -0.12404E+02, -0.14864E+02 },
{ 1.0, 1.0, -0.12543E+01, -0.47849E+00, +0.29846E+02, +0.16489E+02 },
{ 0.0, 2.0, +0.92247E+00, +0.46159E+00, -0.77439E+01, -0.40334E+01 },
{ 3.0, 0.0, +0.22076E+01, +0.97601E+00, -0.62673E+01, -0.49139E+00 },
{ 2.0, 1.0, -0.14147E+01, -0.61165E+00, -0.13612E+01, -0.17927E+01 },
{ 1.0, 2.0, +0.53879E-01, -0.54400E-01, +0.13966E+01, +0.98545E+00 },
{ 0.0, 3.0, +0.58513E-01, +0.29952E-01, -0.22122E+00, -0.10118E+00 },

                    /*      99%           */
{ 0.0, 0.0, +0.24201E+01, +0.10000E+01, -0.15787E+02, +0.10000E+01 },
{ 1.0, 0.0, -0.19281E+01, -0.78628E-01, -0.39798E+01, -0.14830E+02 },
{ 0.0, 1.0, -0.33357E+01, -0.12092E+01, +0.23933E+02, +0.85161E+01 },
{ 2.0, 0.0, +0.23720E+01, +0.43924E+00, +0.24332E+02, +0.23701E+02 },
{ 1.0, 1.0, +0.17318E+01, +0.43093E+00, -0.46762E+02, -0.19419E+02 },
{ 0.0, 2.0, +0.14149E+01, +0.53223E+00, +0.60862E+01, +0.24239E+01 },
{ 3.0, 0.0, -0.64616E+00, +0.34235E+00, +0.15874E+02, -0.18457E+01 },
{ 2.0, 1.0, -0.27558E+01, -0.10741E+01, +0.52360E+01, +0.48007E+01 },
{ 1.0, 2.0, +0.28474E+00, +0.80494E-01, -0.24644E+01, -0.12525E+01 },
{ 0.0, 3.0, -0.34471E-01, -0.13004E-01, +0.28404E+00, +0.99997E-01 };
```

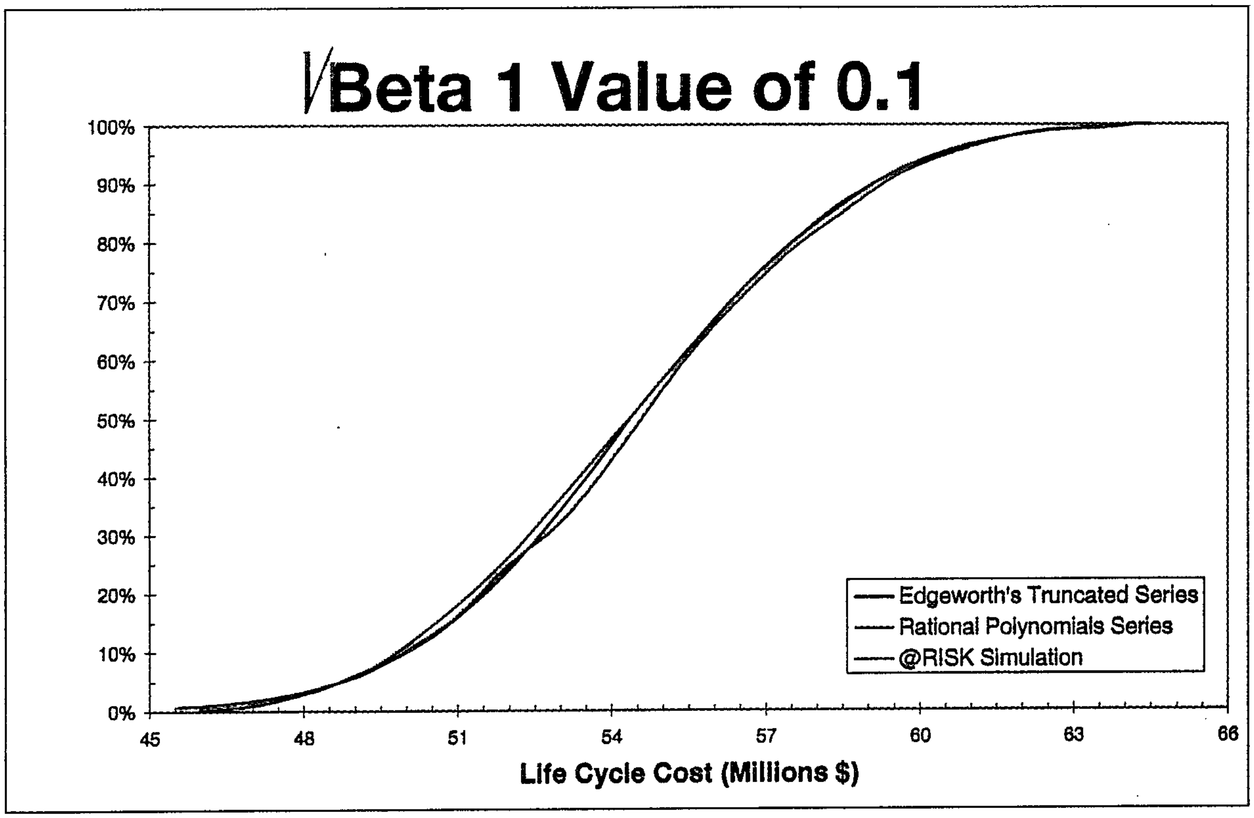
```
float npgen( int j, fuzzy *xlcc0)
   {
        float     sigma,          /* Standard deviation                         */
                  b1,                 /* β1                                         */
                  b2,                 /* β2                                         */
                  PN = 0.0,           /* Numerator of Rational Number               */
                  PD = 0.0,           /* Denominator of Rational Number             */
                  xtemp;          /* X co-cordinate in distribution             */

        int       n,                  /* Numerator column in CTAB matrix            */
                  d,                  /* Denominator column in CTAB                 */
                  i,                  /* Loop variable                              */

        sigma = sqrt(xlcc0 -> arr[2]);
        b1 = xlcc0->arr[3]/pow(sigma,3.0);
        b1 = fabs(b1);

        /*        if β1 [0,1] use column 2 and 3 otherwise use columns 4 and 5 */
        if ( b1 <= 1.0 )
           {
                  n = 2;
                  d = 3;
           }
        else
           {
                  n = 4;
                  d = 5;
           }

        /*        Calculates the Numerator and Denominator terms of the Rational   */
        /*        Polynomial by using one block of the CTAB matrix at a time       */
        /*        and summing the results                                          */
        /*        Formula:  PN or PD = (β1^r × β2^s)                               */

        for (i = ((j - 1) * 10); i <= (((j - 1) * 10) + 9); i++)
           {
                  PN += CTAB[i][n]* pow(b1, CTAB[i][0]) * pow(b2, CTAB[i][1]);
                  PD += CTAB[i][d]* pow(b1, CTAB[i][0]) * pow(b2, CTAB[i][1]);
           }

        /*        Formula of X coordinate:  x = PN/PD × σ + μ1                     */
        xtemp = PN / PD * sigma + xlcc0 -> arr[5];
        return (xtemp);
   }
```

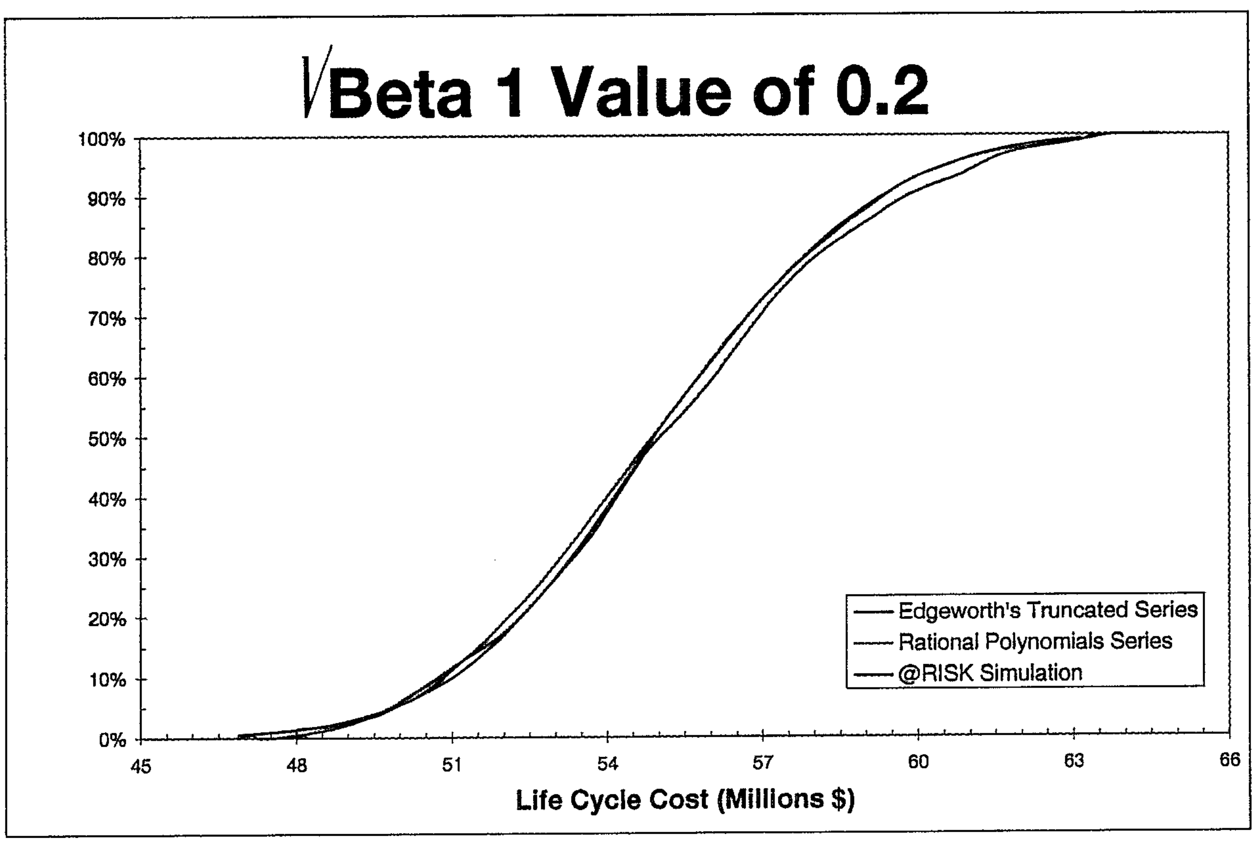
ANNEX C
TO D LOG A RESEARCH NOTE 9505
DATED SEPTEMBER 1995

TESTING RESULTS OF LOGAN (LCC) MODEL

1. This annex contains the graphical data from the testing of the LOGAN (LCC) model. The constant, needed for the @RISK simulation, was calculated from the twelve mandatory costs totaling $ 44,637,684. The extra fixed cost, was a value of $ 10,000,000.00 applied immediately for the life of a ten year system with no inflation. The multipliers were set at 1.25 and 1.50 respectively. The boundaries for the fixed cost and multipliers are the percentages applied to the value in order to skew the distribution and create a triangular distribution. These values provided the 5^{th} and 95^{th} percentiles. The one extra fixed cost, with its accompanying multipliers and applied through different boundaries, was the lone cost where the risk was assessed. The constant dealing with the twelve mandatory costs was not a factor in the risk assessment but merely computed so that the @RISK program created simulations comparable to those of LOGAN (LCC). The LCC percent difference is the relative difference expressed in percentages between the LOGAN (LCC) and @RISK simulated life cycle costs which is nearly insignificant for all values.

Test $\sqrt{\beta_1}$	Actual $\sqrt{\beta_1}$	Life Cycle Cost for LOGAN(LCC)	Fixed Boundary	Mult. 1 Boundary	Mult. 2 Boundary	LCC % Difference
0.1	0.1044	$ 55, 422, 737	80 - 150	30 - 100	—	0.3514
0.2	0.2019	$ 55, 023, 354	80 - 150	40 - 100	—	0.2979
0.3	0.3052	$ 55, 733, 080	80 - 150	52 - 100	—	0.2353
0.4	0.4020	$ 59, 639, 118	80 - 150	98 - 120	—	0.0000
0.5	0.5022	$ 61, 145, 279	100 - 150	72 - 150	—	0.2170
0.6	0.6031	$ 59, 735, 905	80 - 150	70- 150	—	0.0000
0.7	0.7001	$ 63, 212, 097	100 - 200	75 - 135	—	0.4015
0.8	0.8001	$ 65, 487, 106	100 - 200	77 - 165	—	0.4350
0.9	0.9008	$ 69, 280, 304	99 - 200	99 - 200	—	0.0000
1.0	1.0070	$ 71, 254, 669	95 - 225	83 - 225	—	0.0000
1.1	1.1005	$ 78, 293, 251	95 - 250	96 - 275	—	0.0000
1.2	1.2019	$ 87, 817, 451	97 - 300	99 - 325	—	0.0000
1.3	1.3008	$ 88, 989, 487	80 - 325	84 - 350	—	0.0000
1.4	1.4009	$ 92, 464, 035	60 - 375	67 - 375	—	0.0000
1.5	1.5000	$ 137, 986, 420	85 - 525	93 - 550	—	0.0000
1.6	1.6007	$ 131, 432, 816	50 - 550	40 - 575	—	0.0000
1.7	1.7007	$ 161, 114, 197	10 - 700	15 - 700	—	0.0000
1.8	1.8006	$ 246, 961, 243	-50 - 999	-37 - 999	—	0.0000
1.9	1.9028	$ 108, 113, 472	50 - 300	50 - 300	57 - 225	0.0000
2.0	2.0020	$ 122, 622, 253	50 - 325	50 - 325	61 - 250	0.0000
2.1	2.1021	$ 138, 209, 824	50 - 350	50 - 350	61 - 275	0.0000
2.2	2.1998	$ 154, 929, 581	50 - 375	50 - 375	58 - 300	0.0000
2.3	2.3002	$ 172, 001, 022	50 - 400	50 - 400	50 - 325	0.0035
2.4	2.4013	$ 200, 433, 075	50 - 425	50 - 425	46 - 375	0.0000
2.5	2.5020	$ 250, 046, 127	50 - 500	50 - 475	45 - 400	0.0000
2.6	2.6018	$ 362, 196, 564	50 - 600	50 - 575	56 - 450	0.0000
2.7	2.7016	$ 516, 153, 564	50 - 700	50 - 700	57 - 500	0.0000
2.8	2.8008	$ 741, 453, 796	50 - 800	50 - 800	56 - 600	0.0000
2.9	2.9003	$ 1, 202, 178, 711	50 - 950	50 - 950	57 - 750	0.0000
3.0	3.0013	$ 1, 272, 743, 774	0 - 999	0 - 999	67 - 800	0.0000
3.1	3.1017	$ 1, 400, 853, 149	-50 - 999	-50 - 999	71 - 999	0.0000
3.2	3.2011	$ 1, 219, 995, 972	-99 - 999	-99 - 999	34 - 999	0.0000
3.3	3.3006	$ 1, 114, 005, 005	-99 - 999	-99 - 999	-31 - 999	2.6930


√Beta 1 Value of 0.1
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
45
48
51
54
57
60
63
66
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation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80%
70%
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40%
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20%
10%
0%
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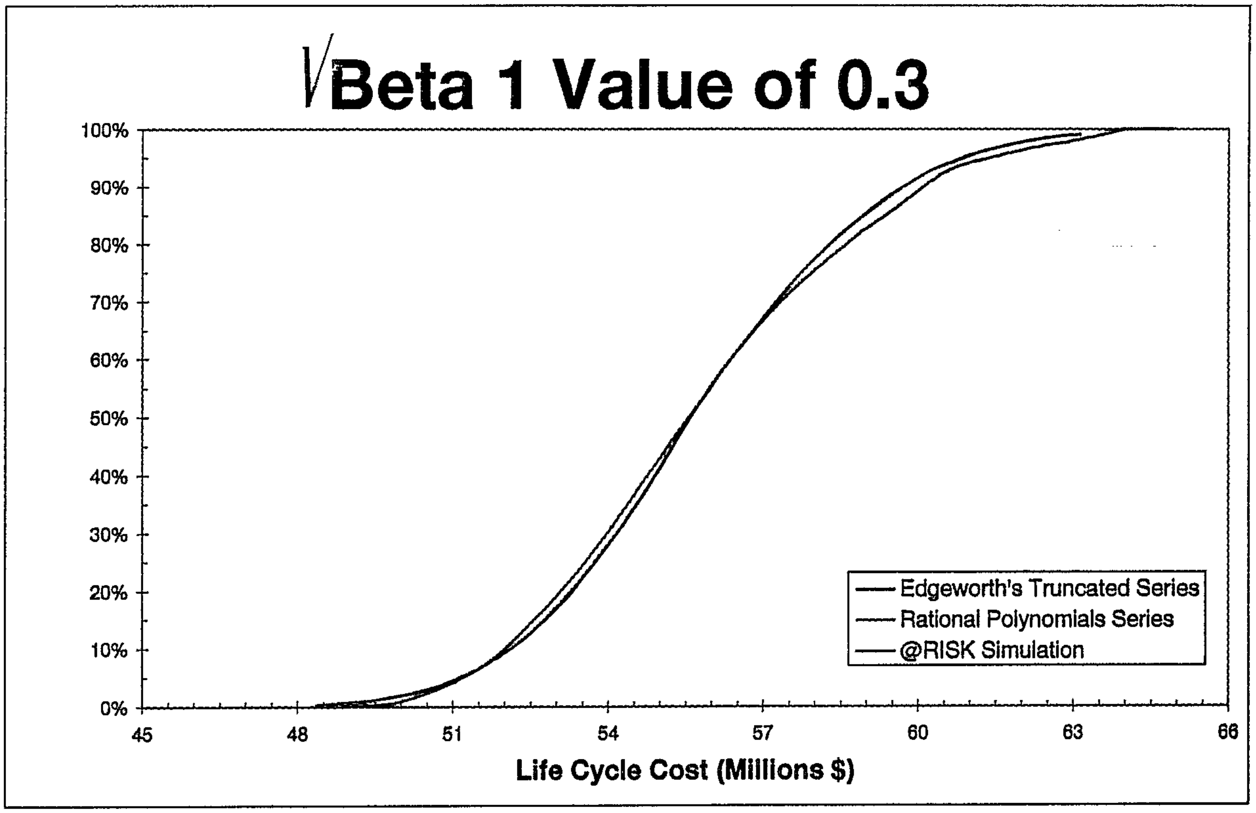
66
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation


√Beta 1 Value of 0.3
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70%
60%
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40%
30%
20%
10%
0%
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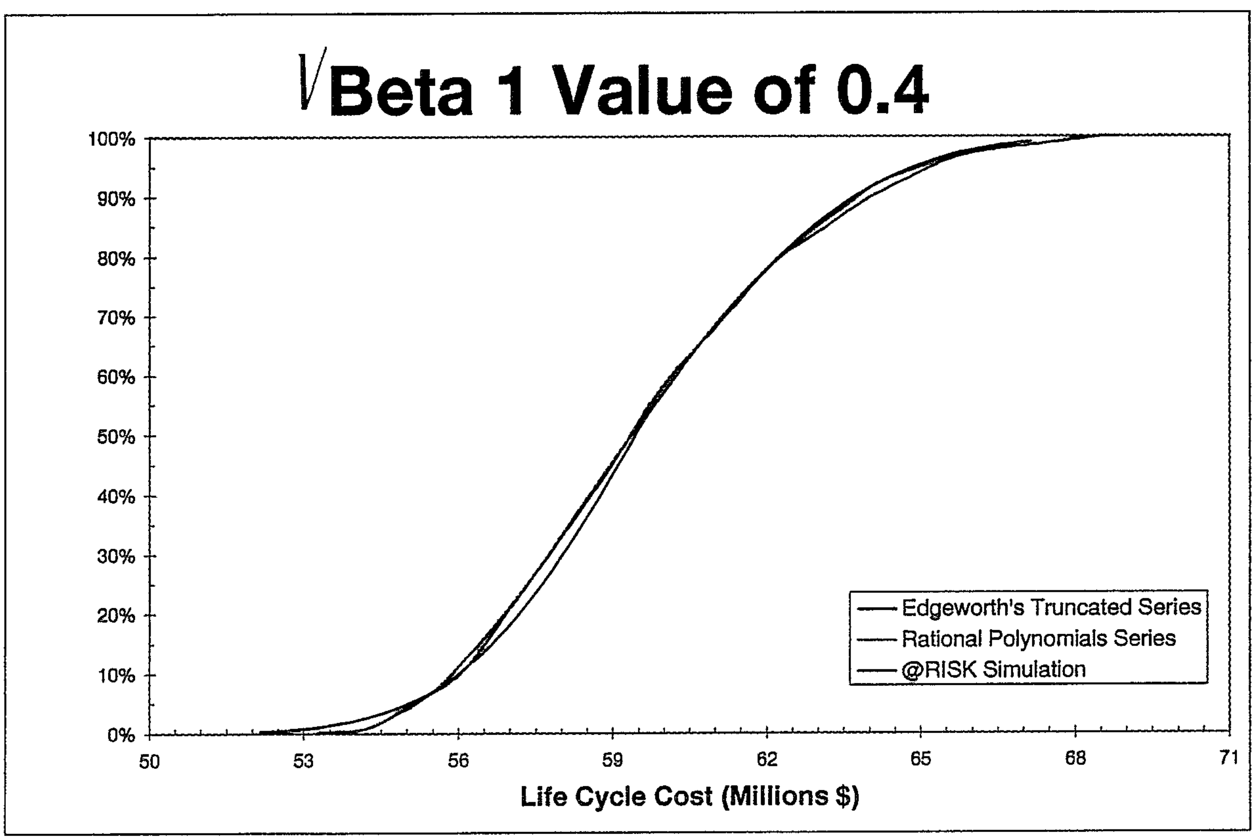
60
63
66
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation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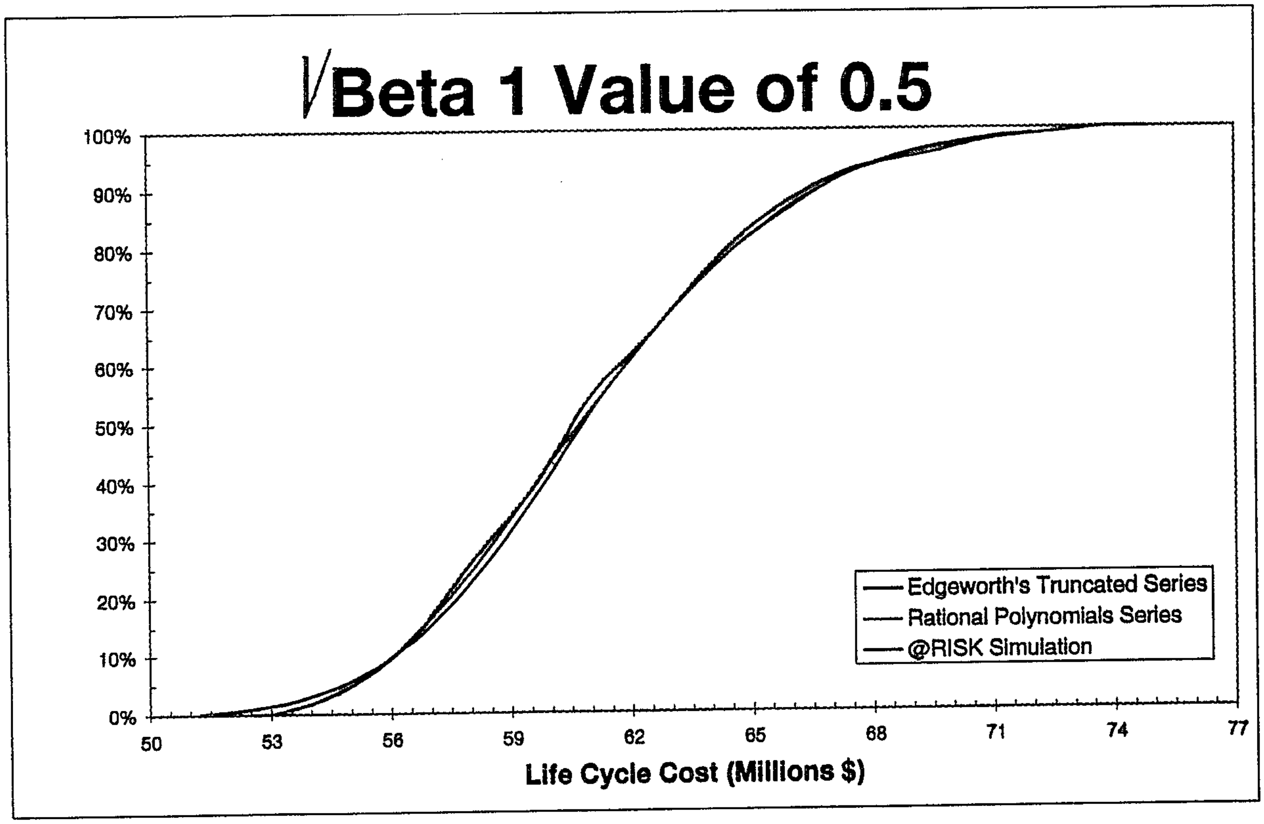
70%
60%
50%
40%
30%
20%
10%
0%
50
53
56
59
62
65
68
71
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation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30%
20%
10%
0%
50
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56
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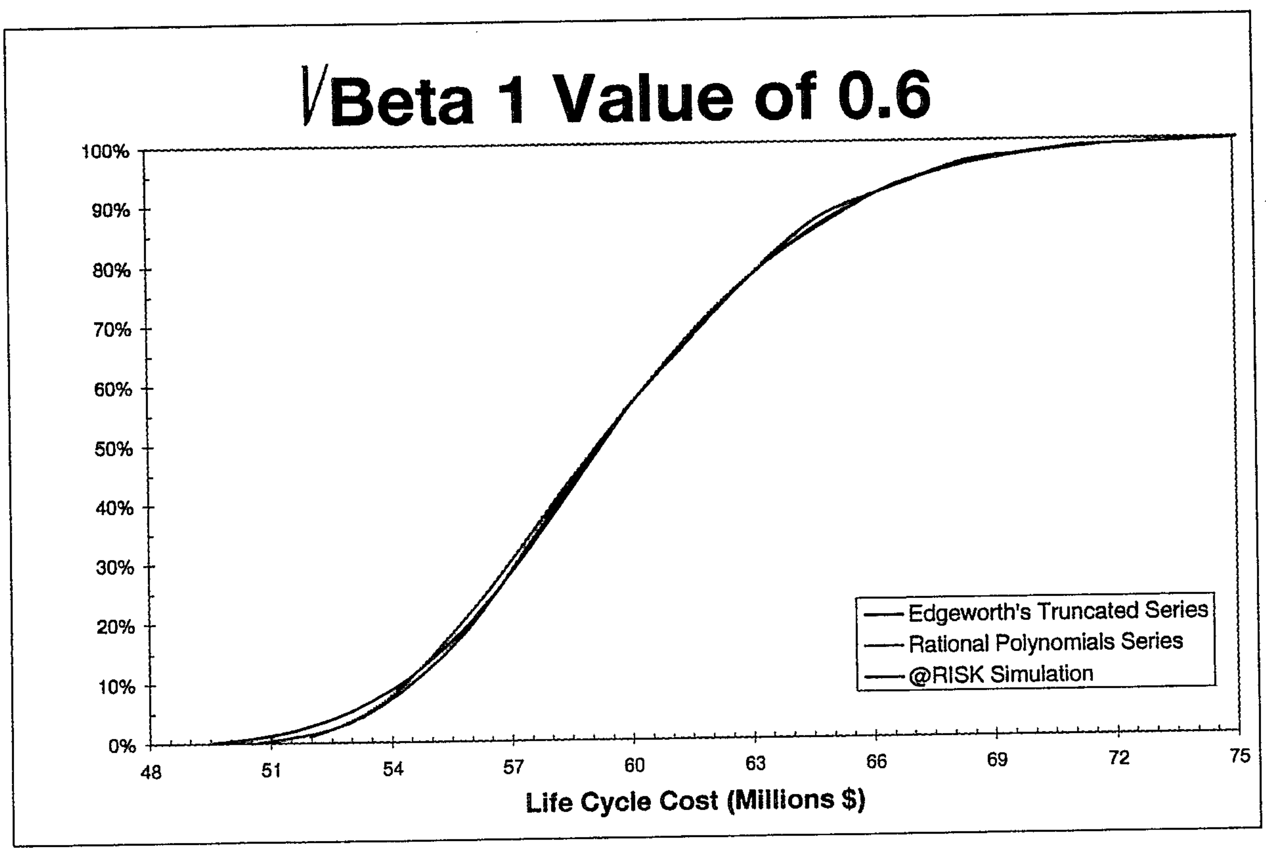
65
68
71
74
77
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation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10%
0%
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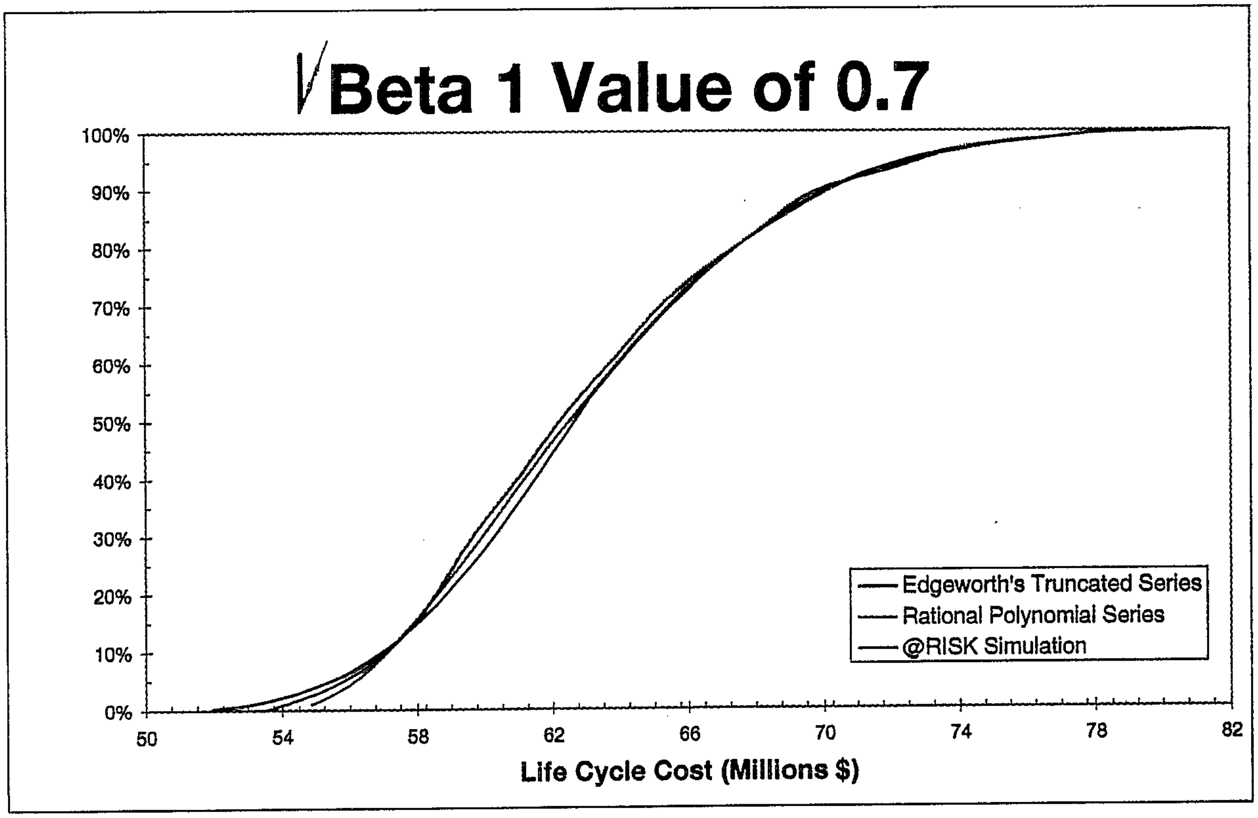
66
69
72
75
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
Rational Polynomials Series
@RISK Simulation


√Beta 1 Value of 0.7
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70%
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50%
40%
30%
20%
10%
0%
50
54
58
62
66
70
74
78
82
Life Cycle Cost (Millions $)
Edgeworth's Truncated Series
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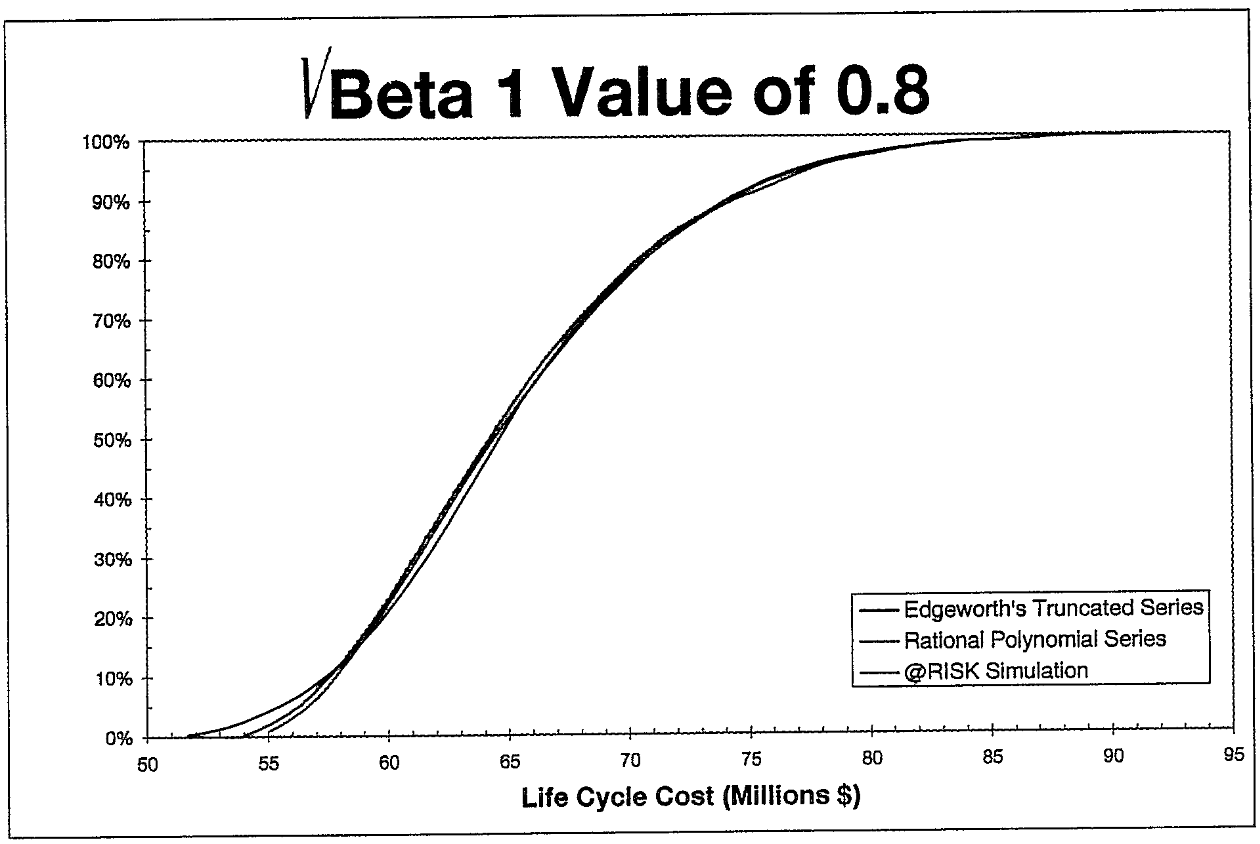
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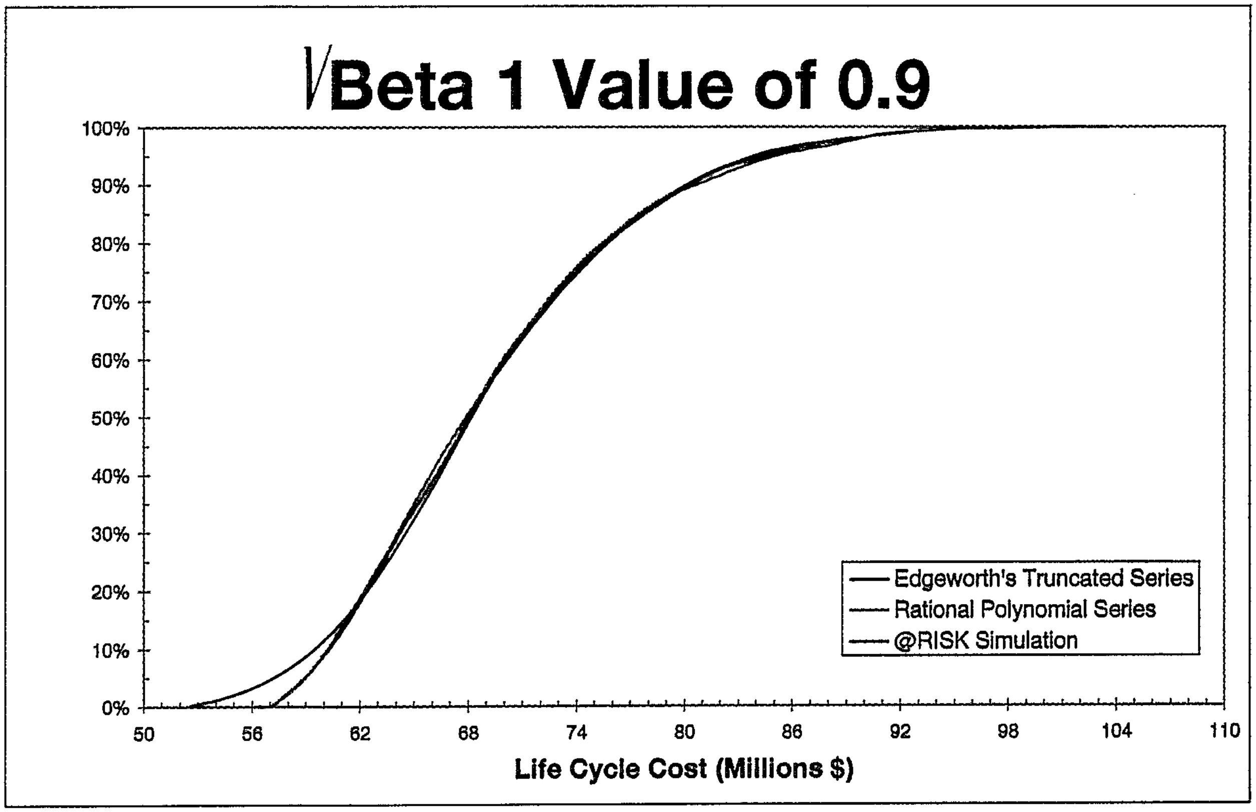
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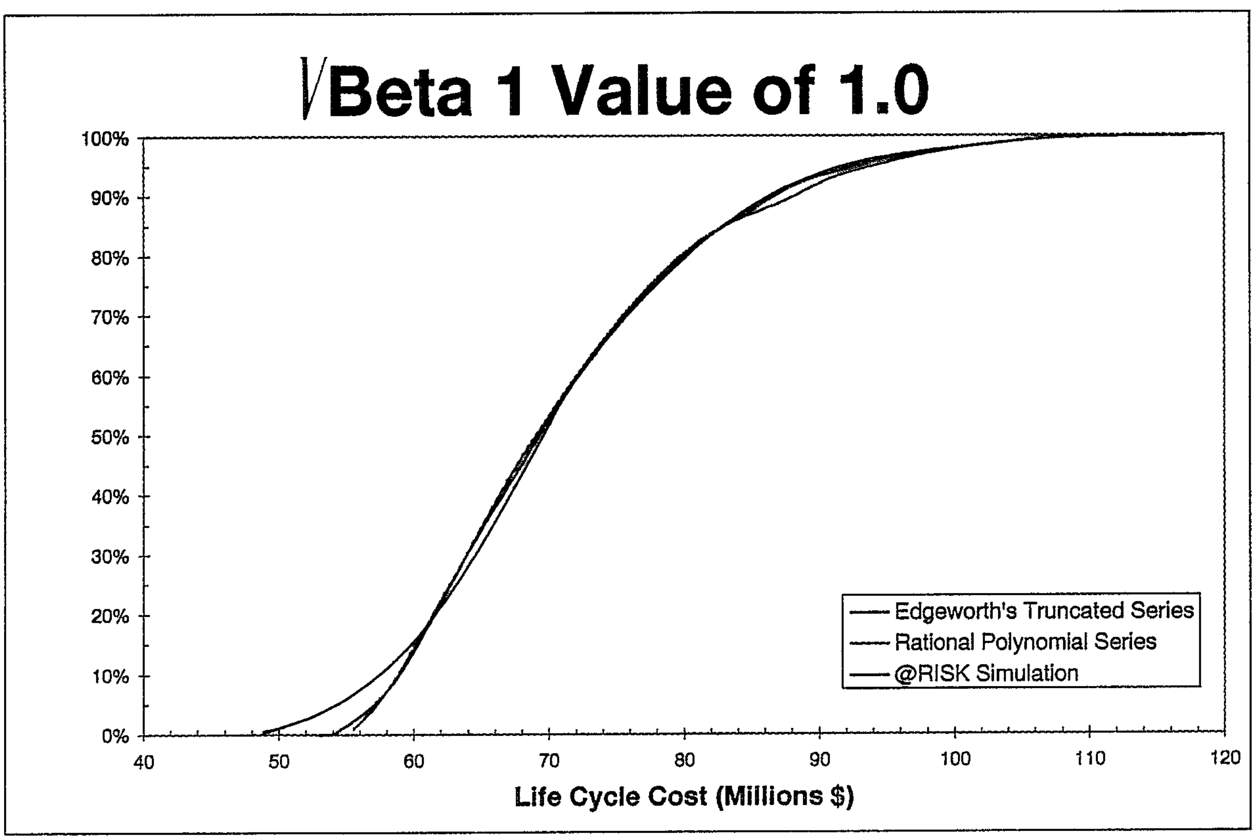
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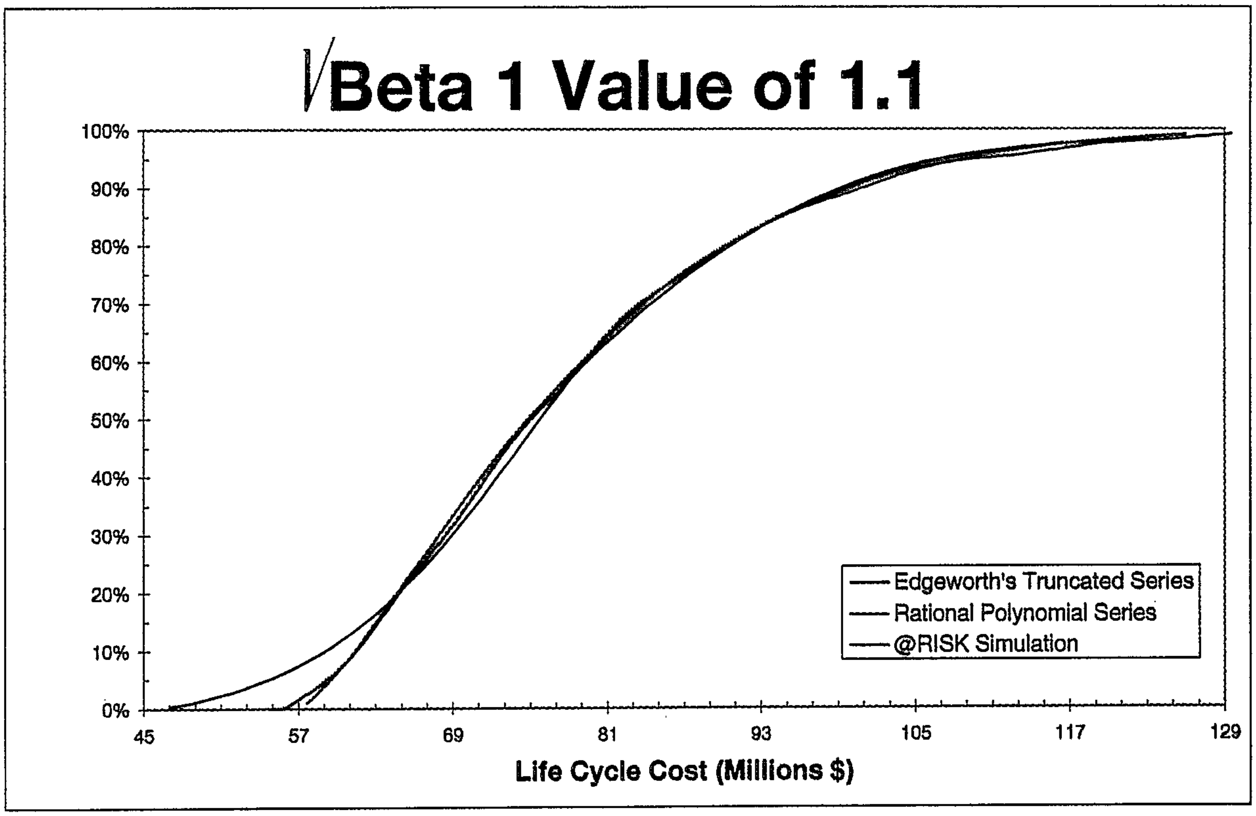
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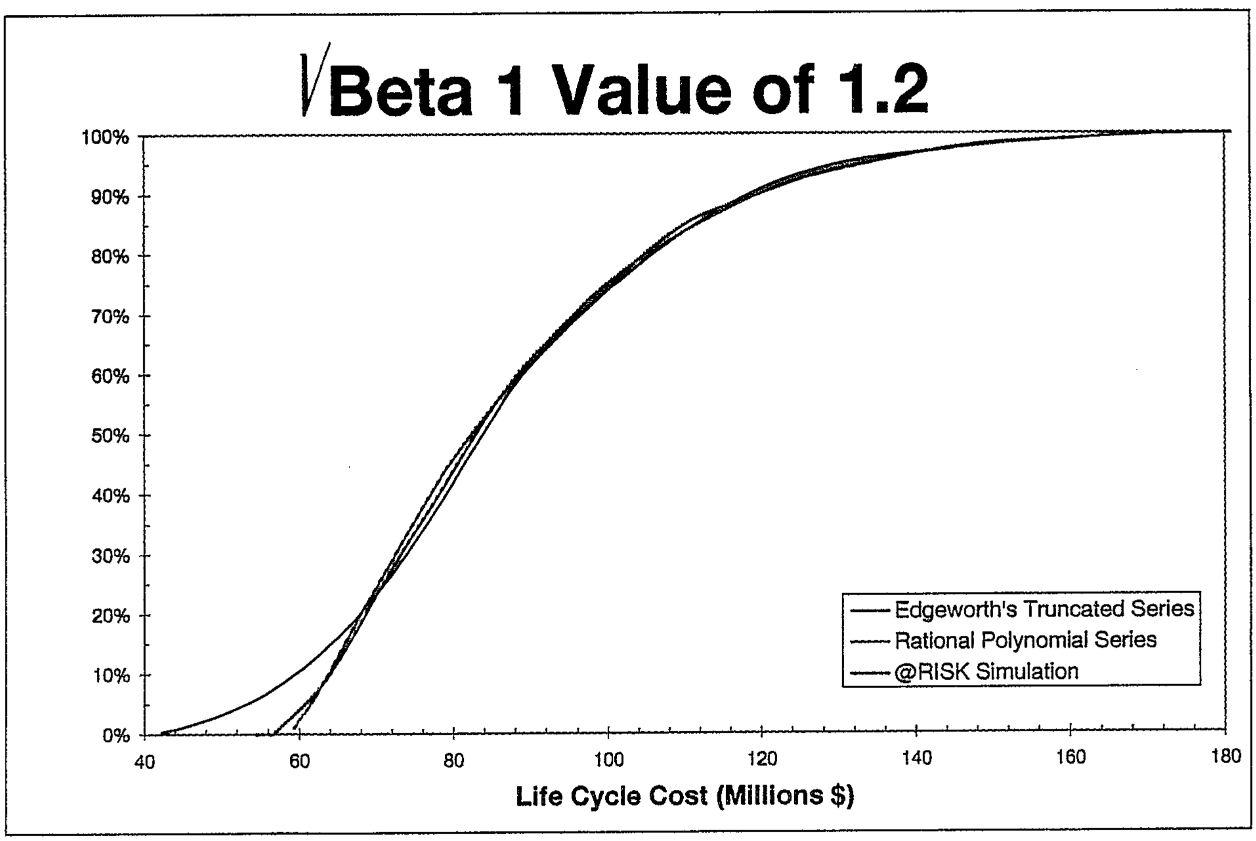
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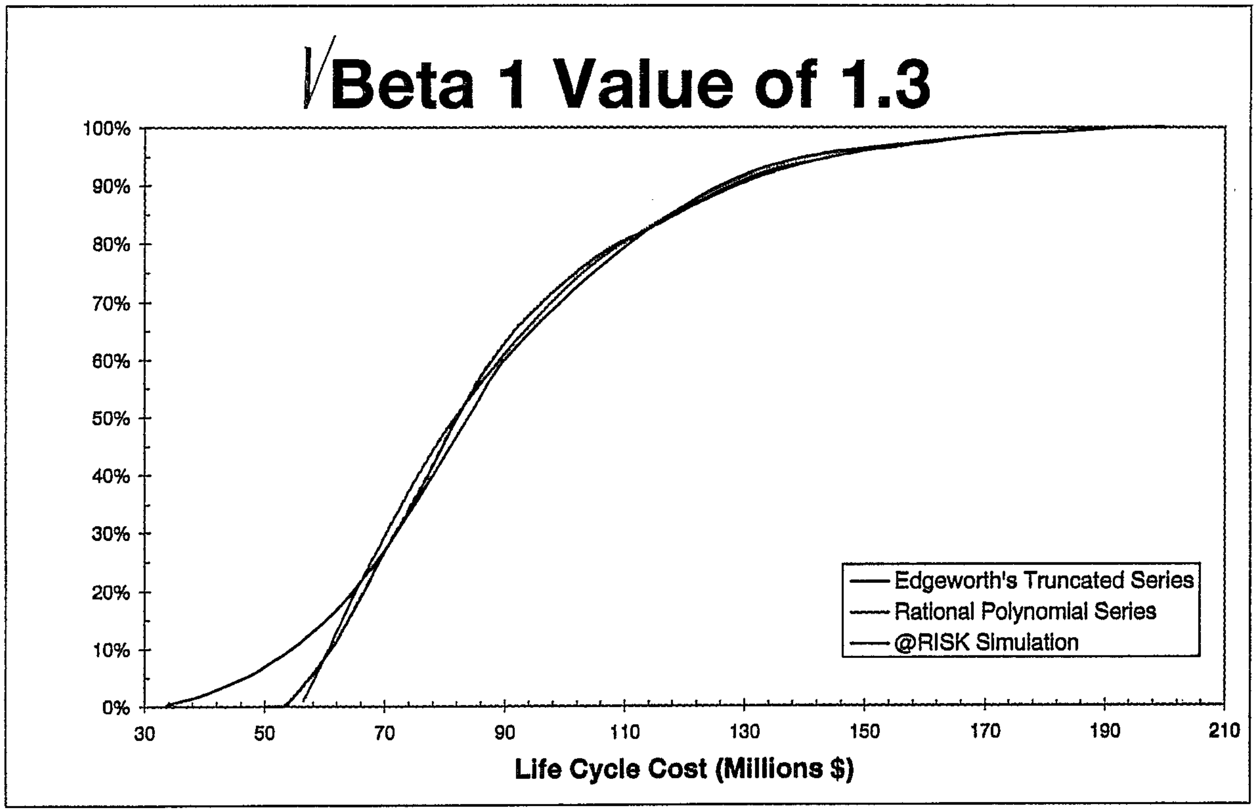
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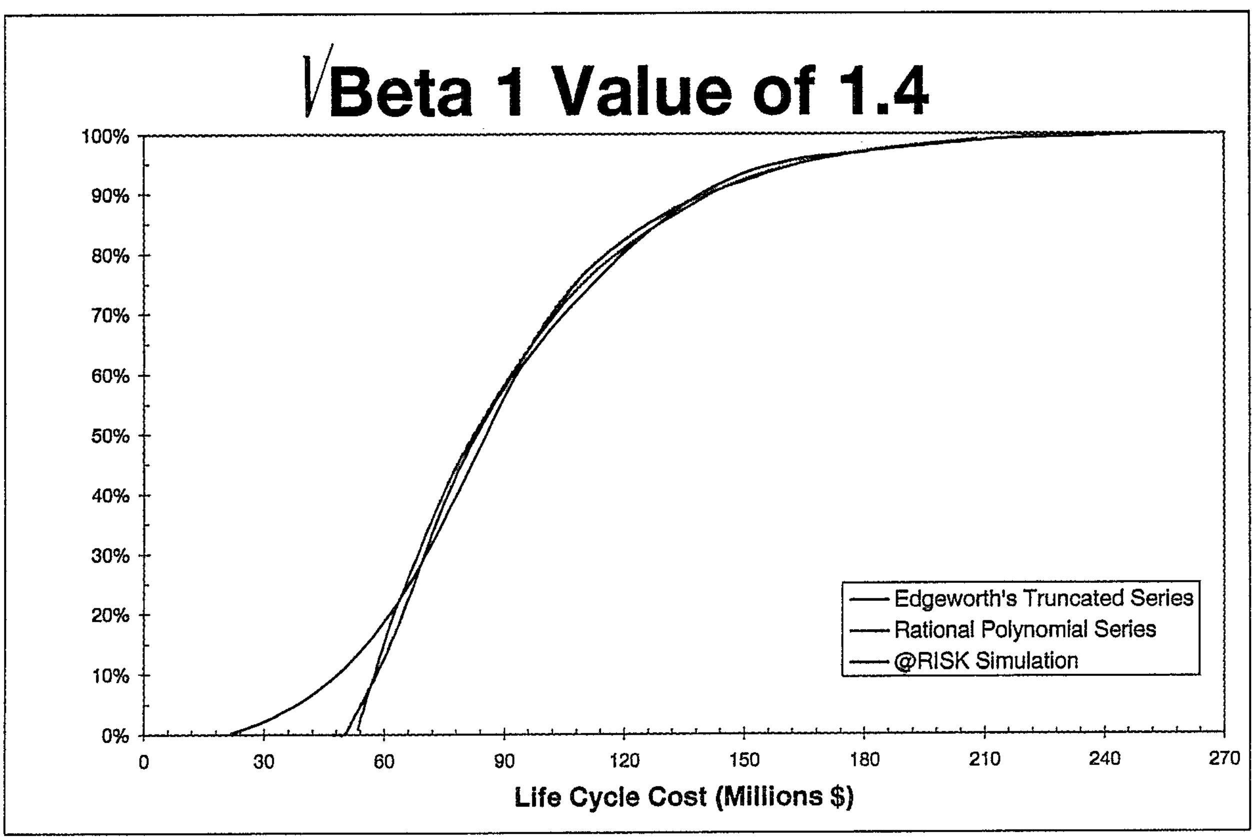
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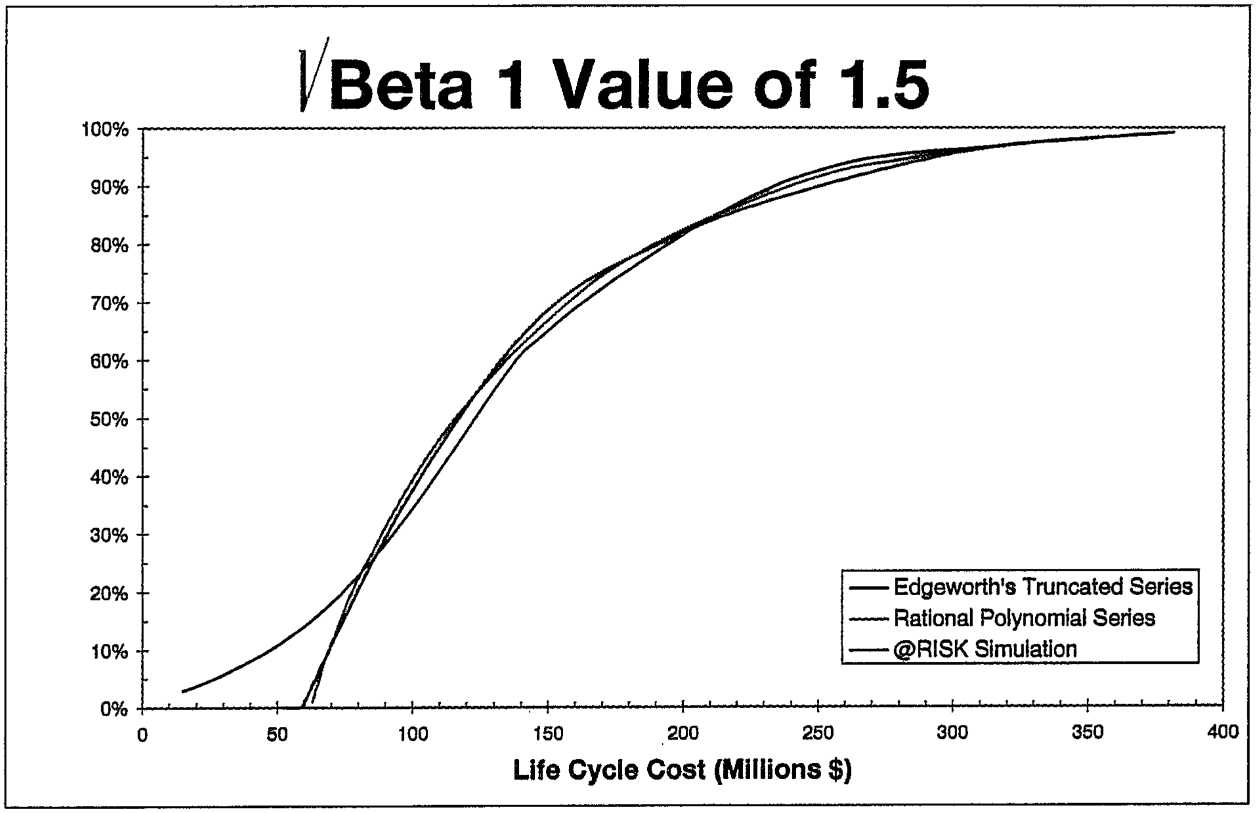
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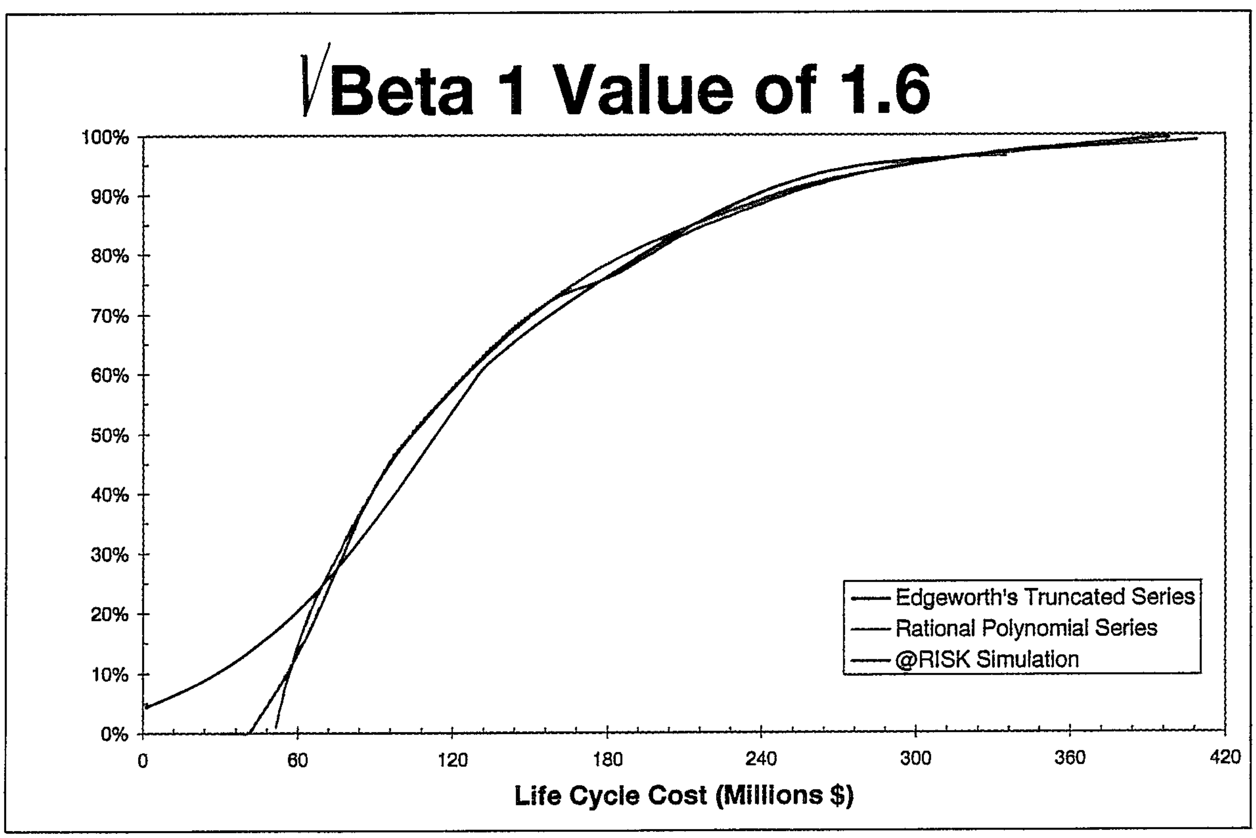
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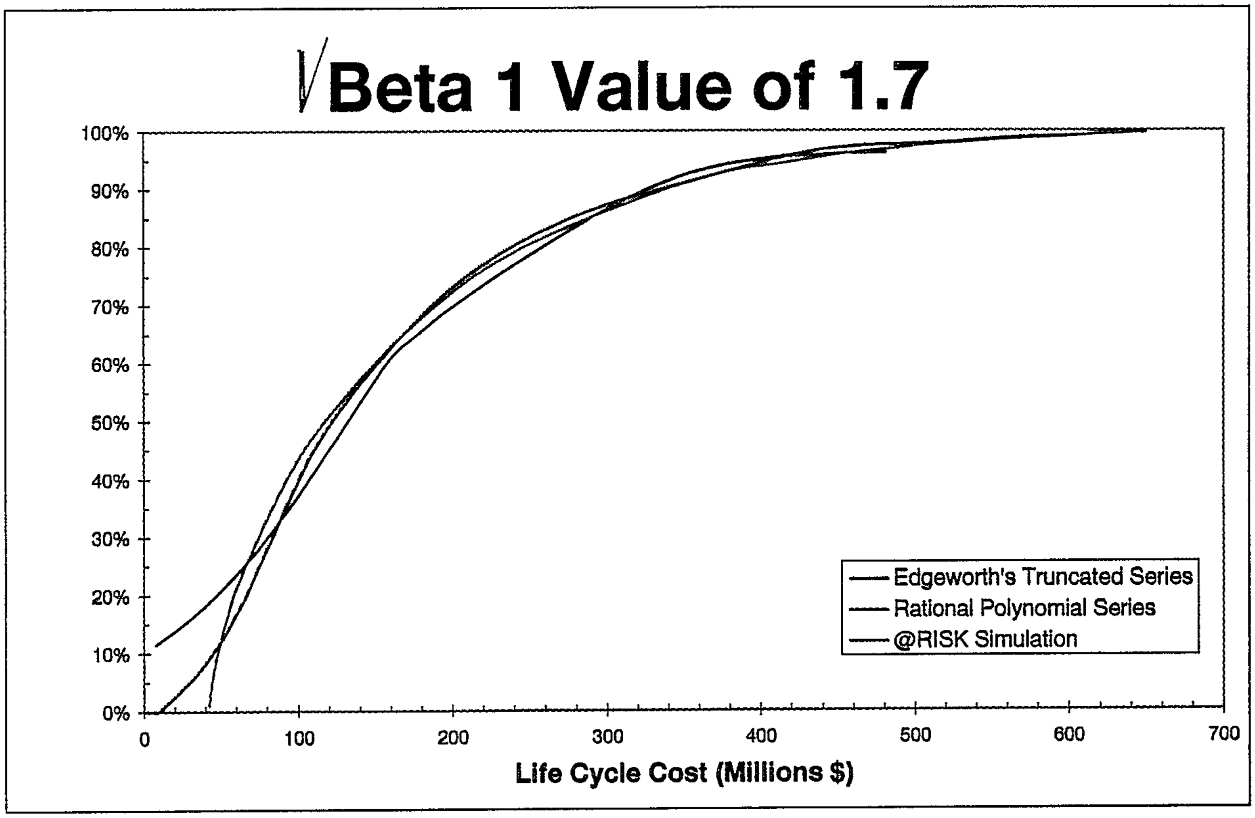
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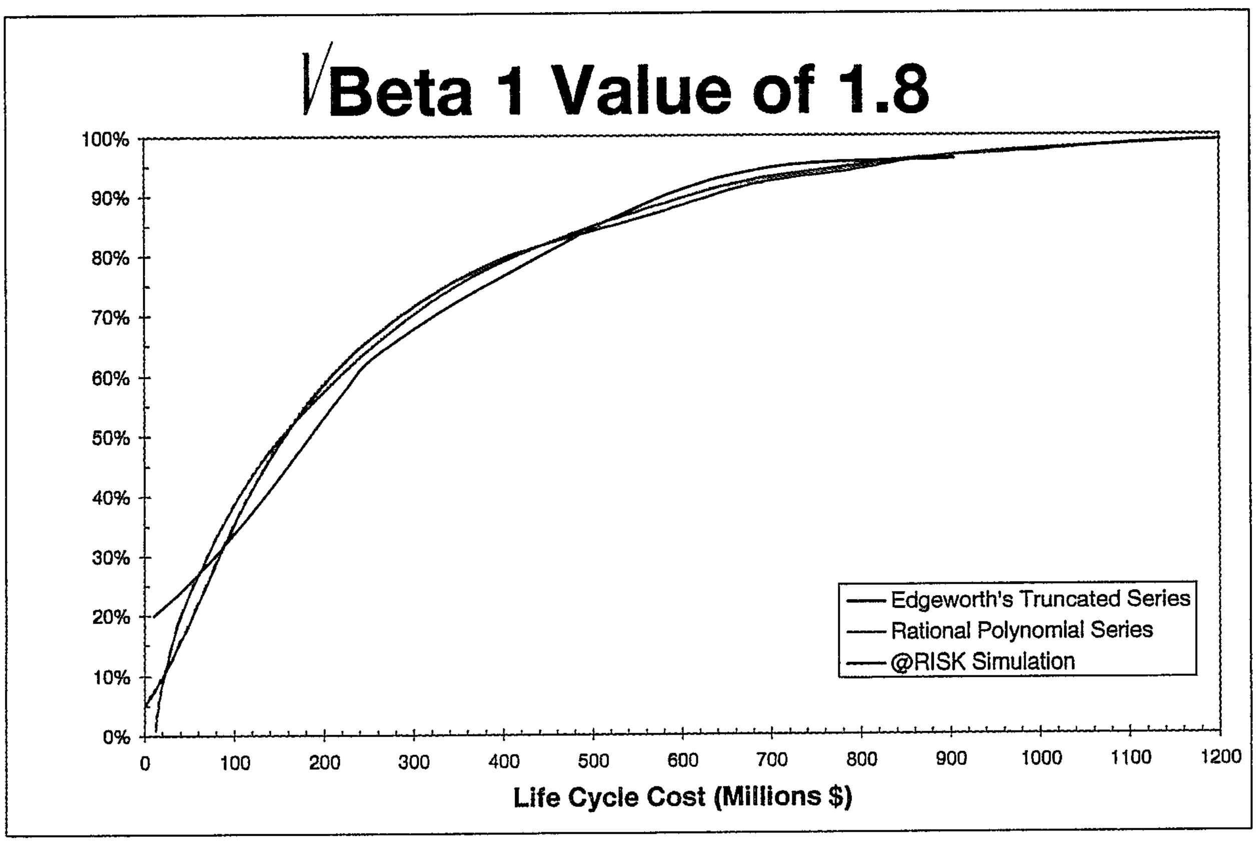
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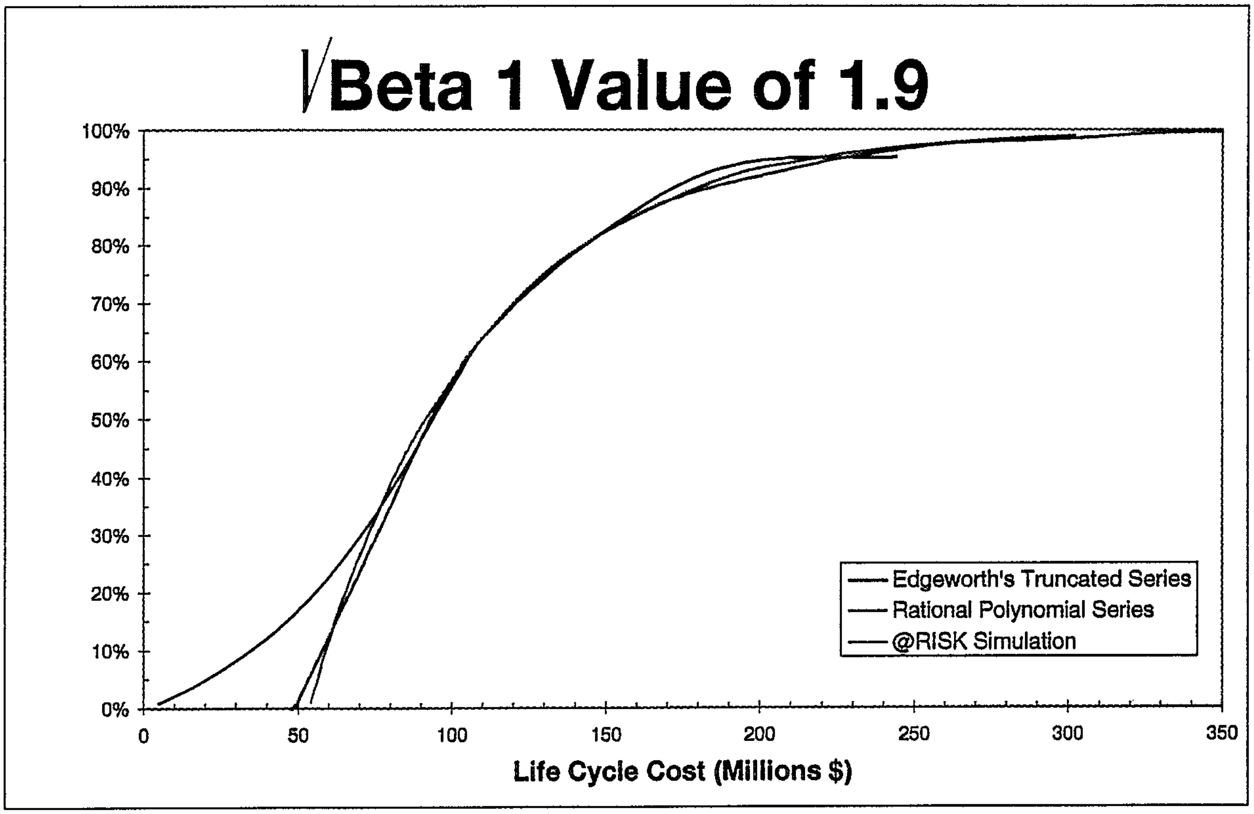
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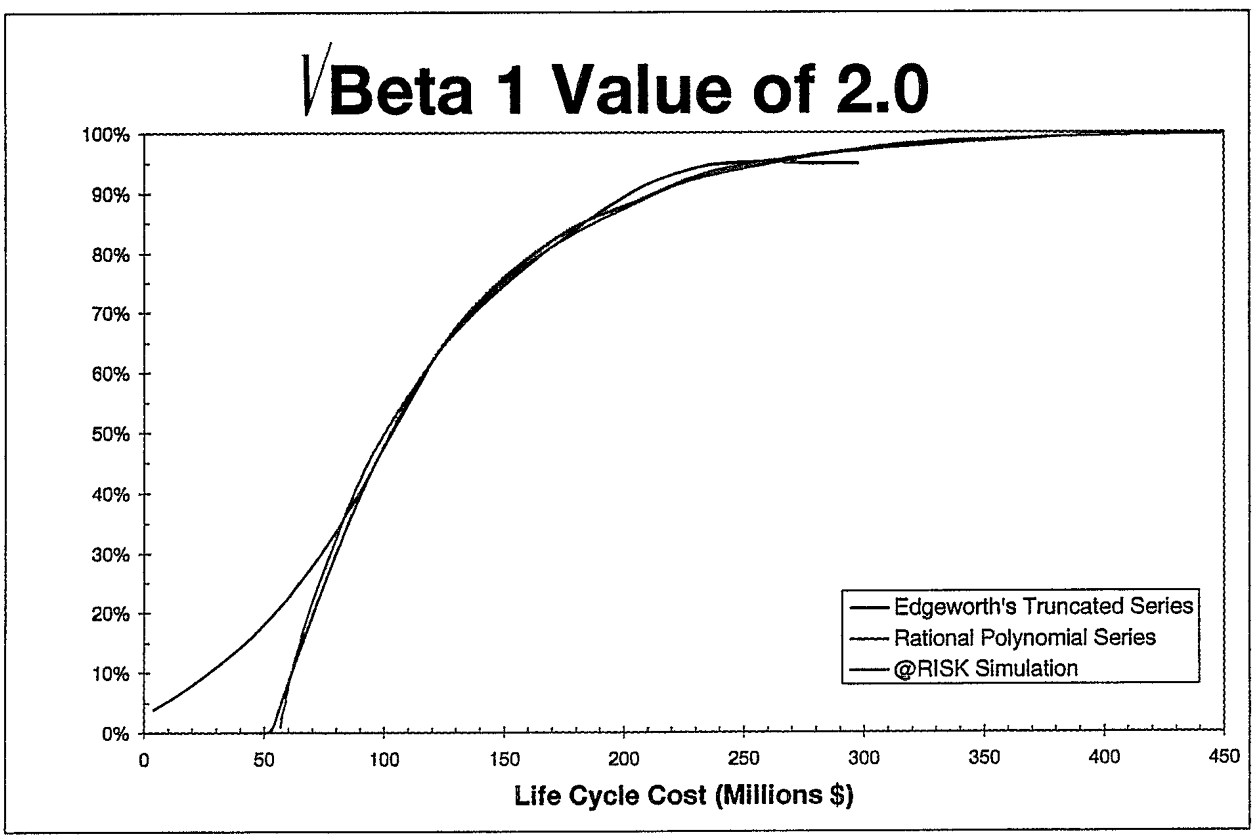
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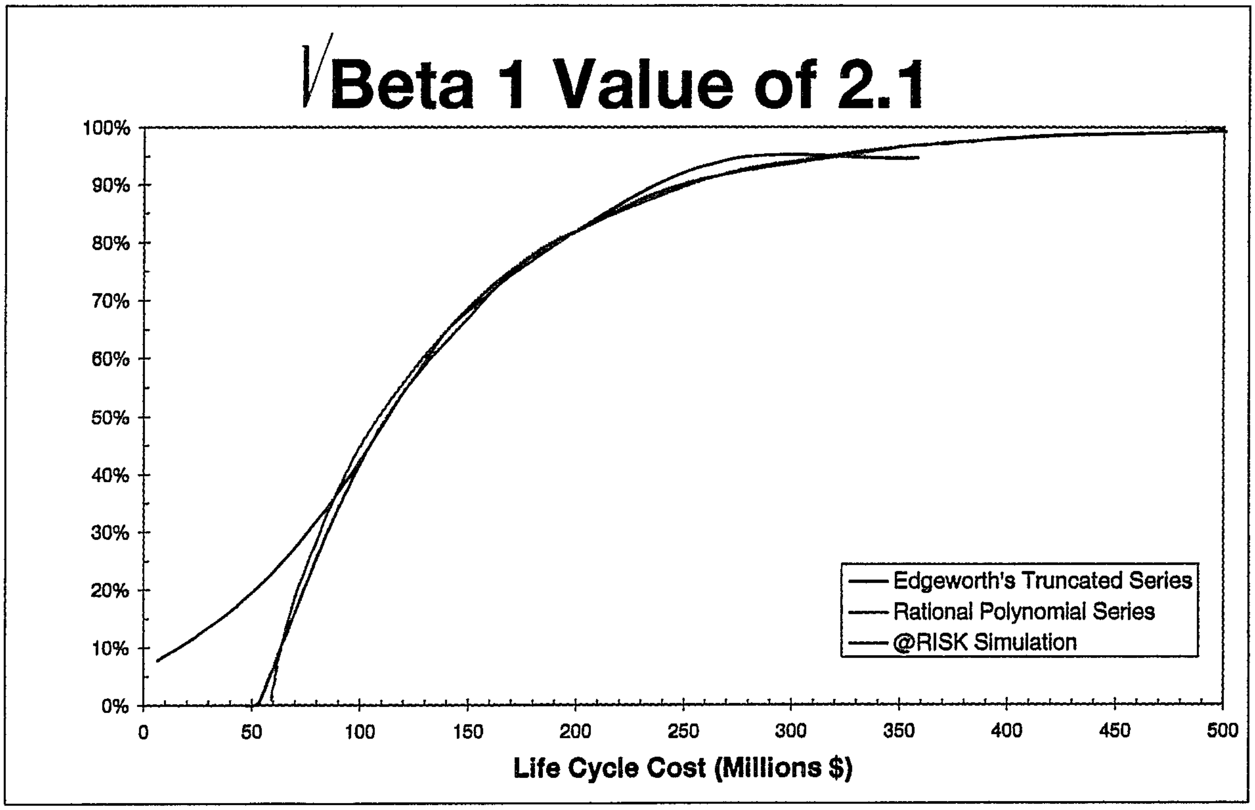

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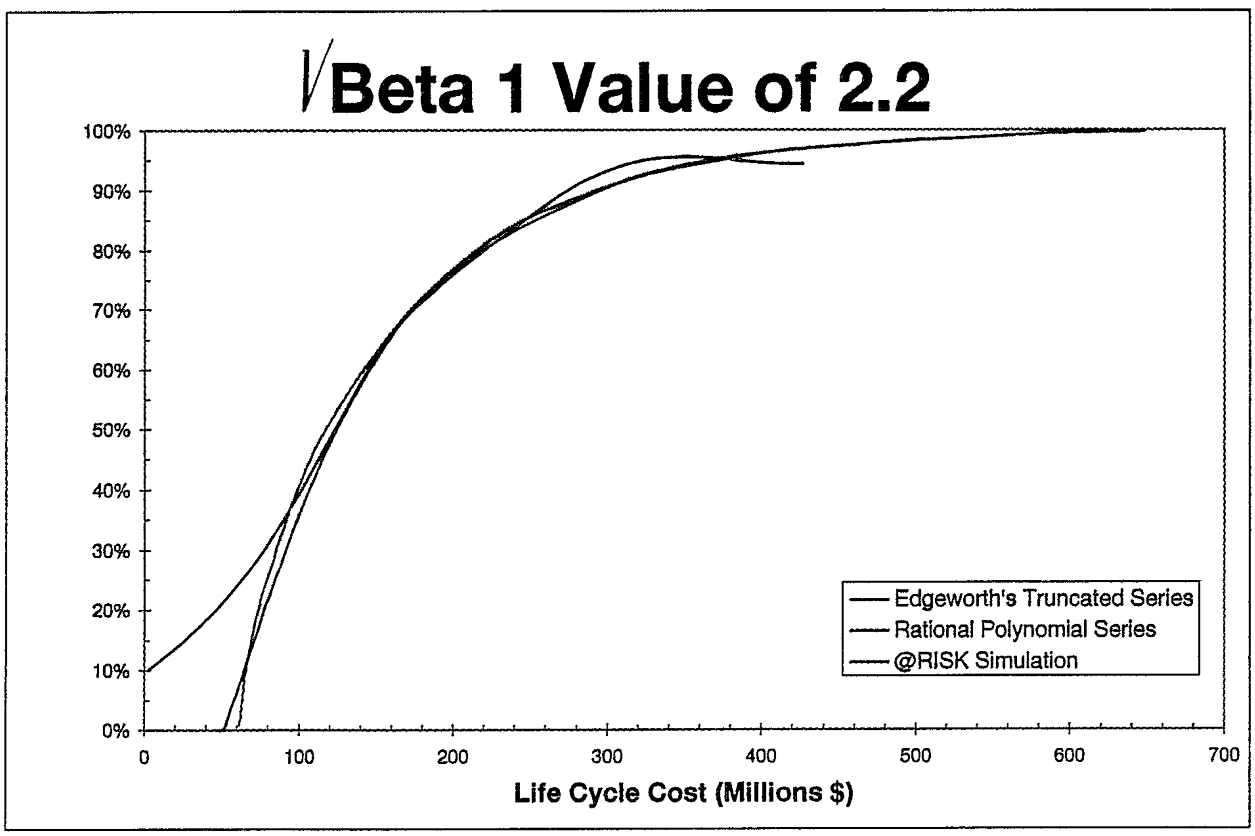

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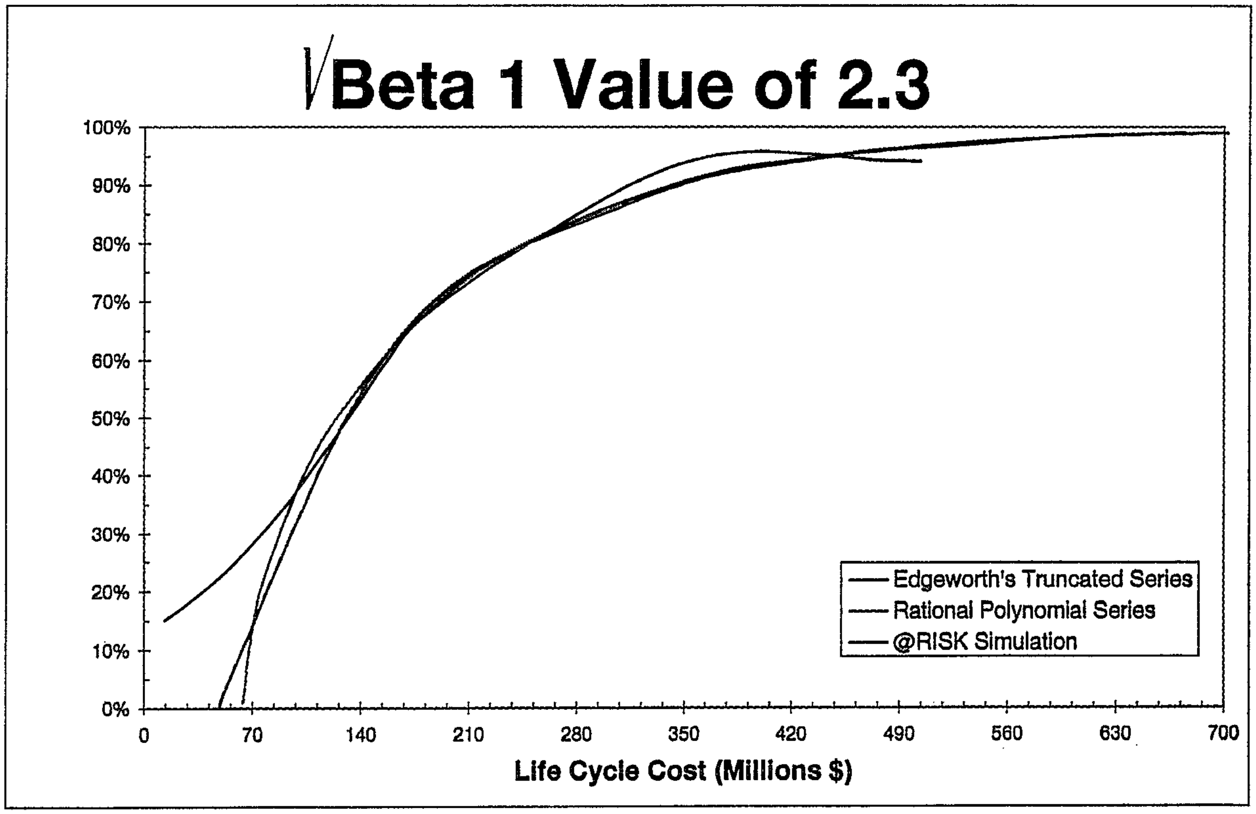
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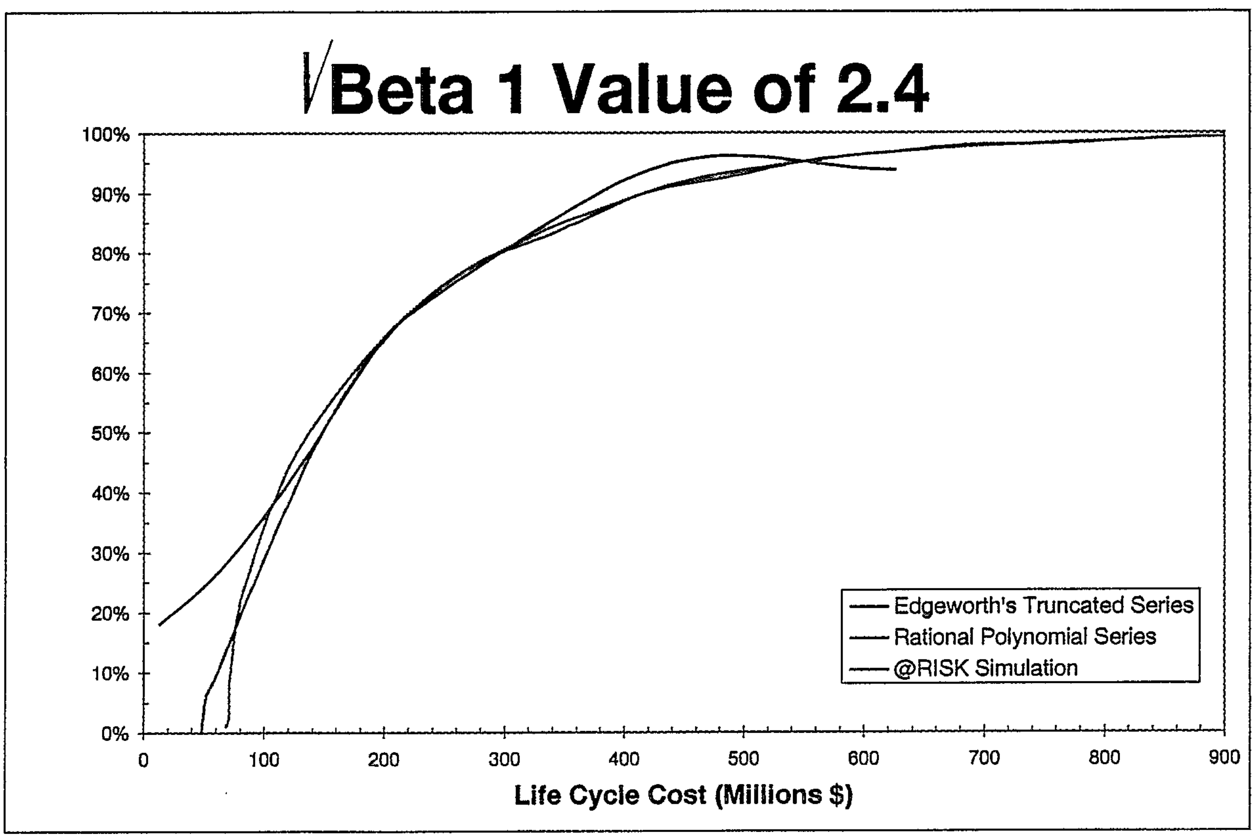
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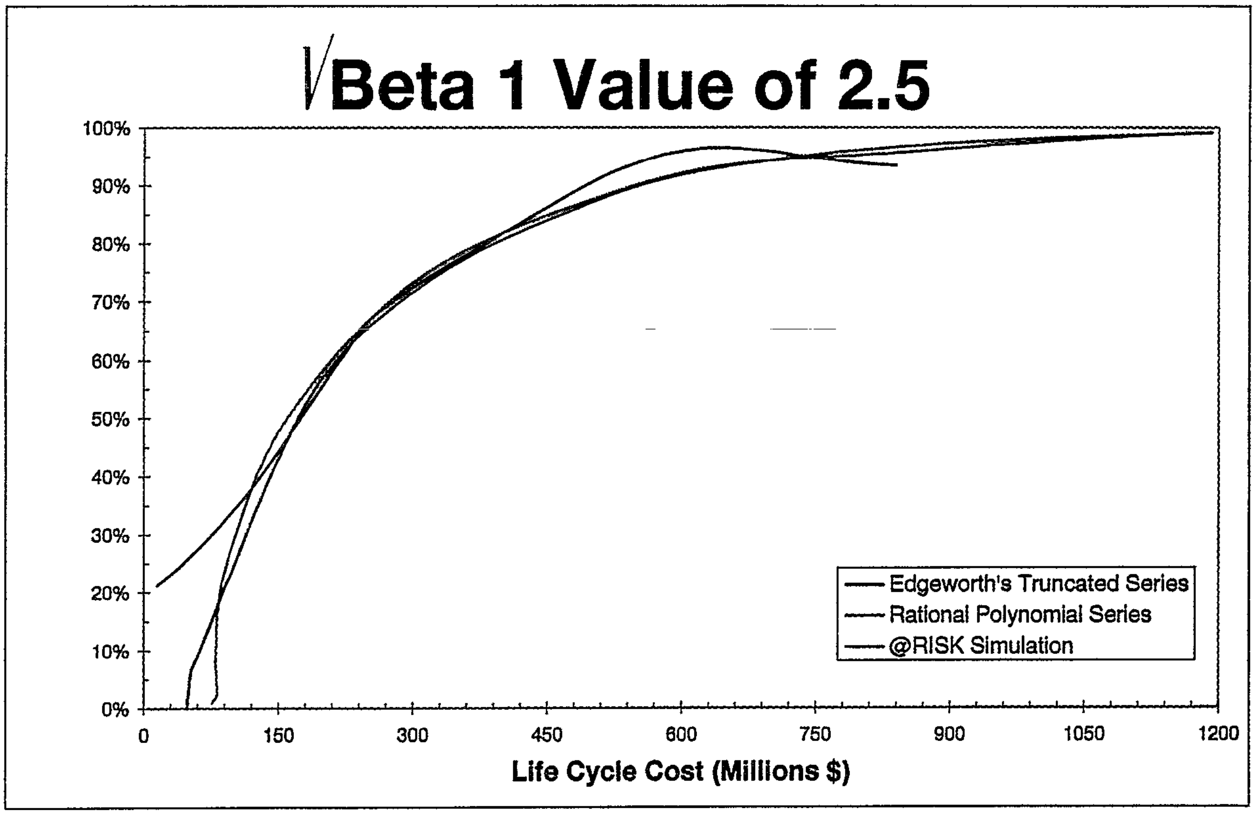
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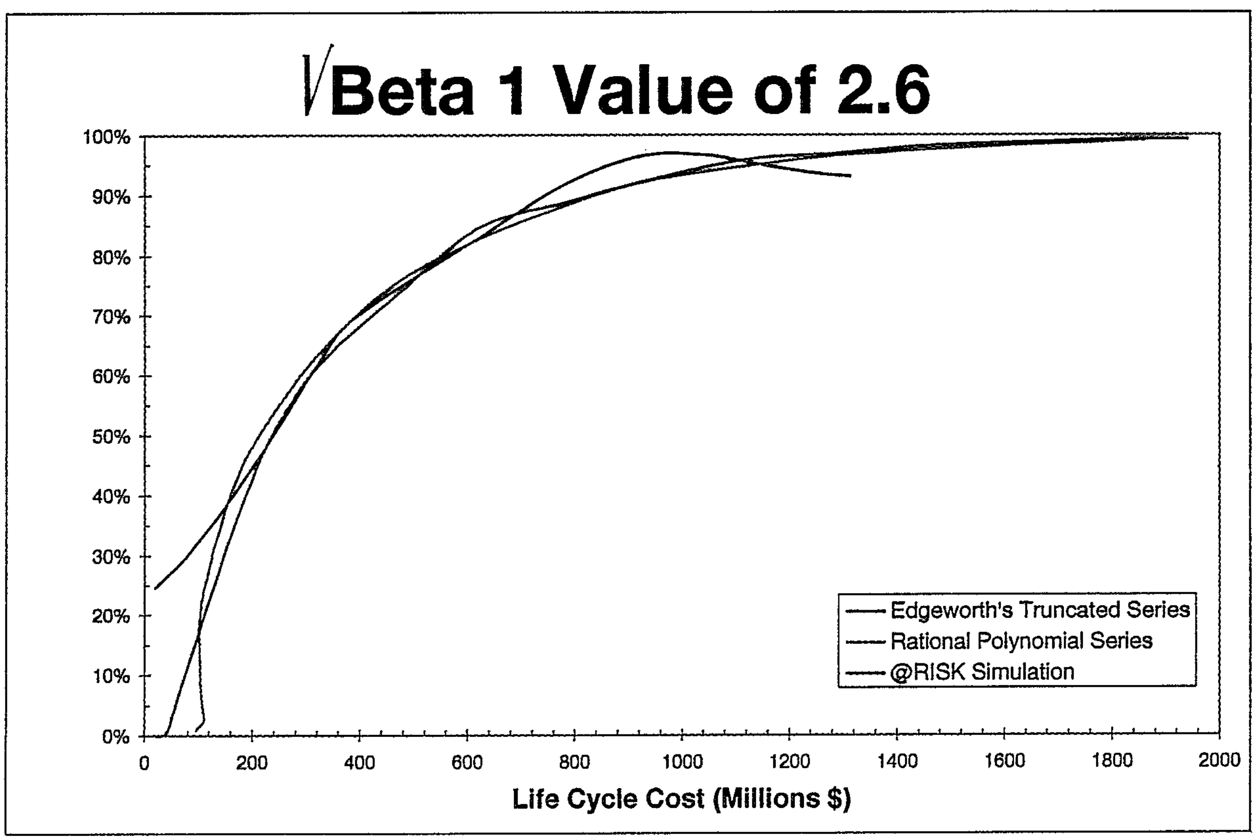
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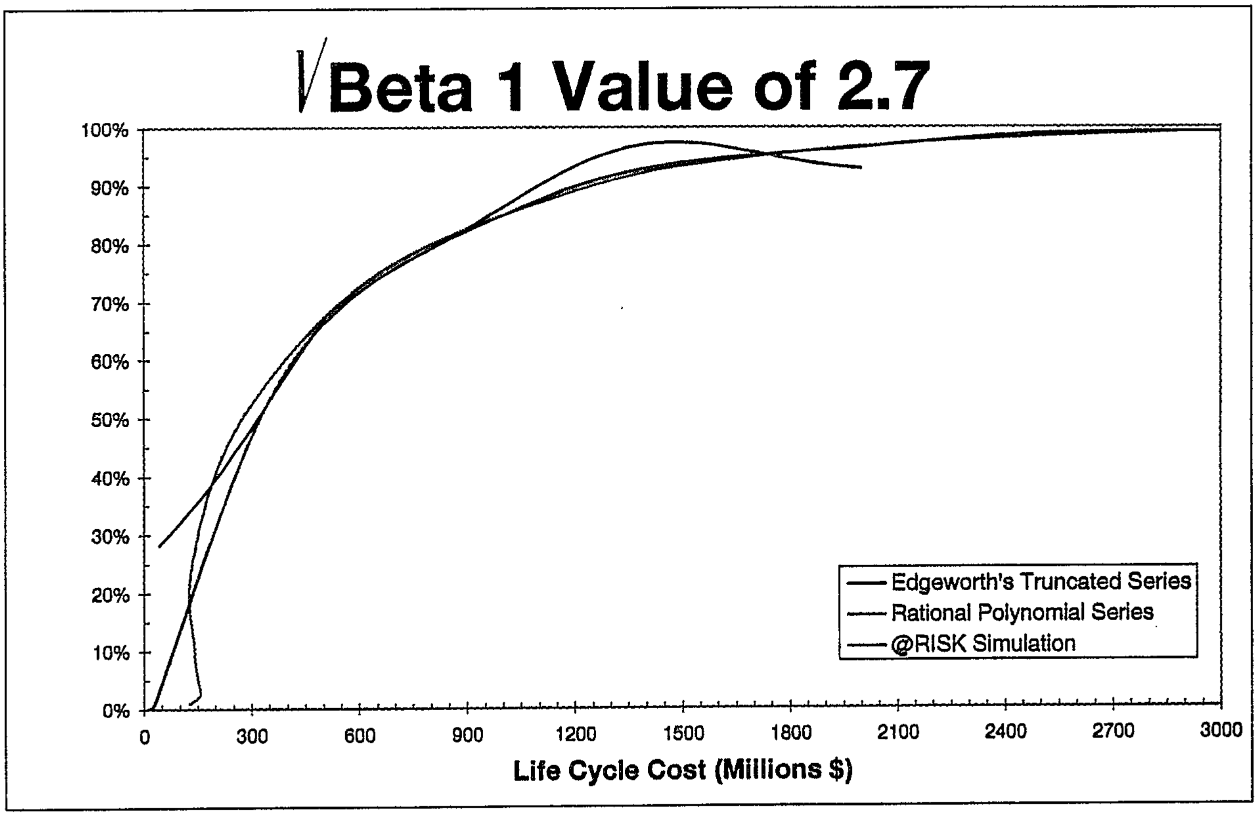
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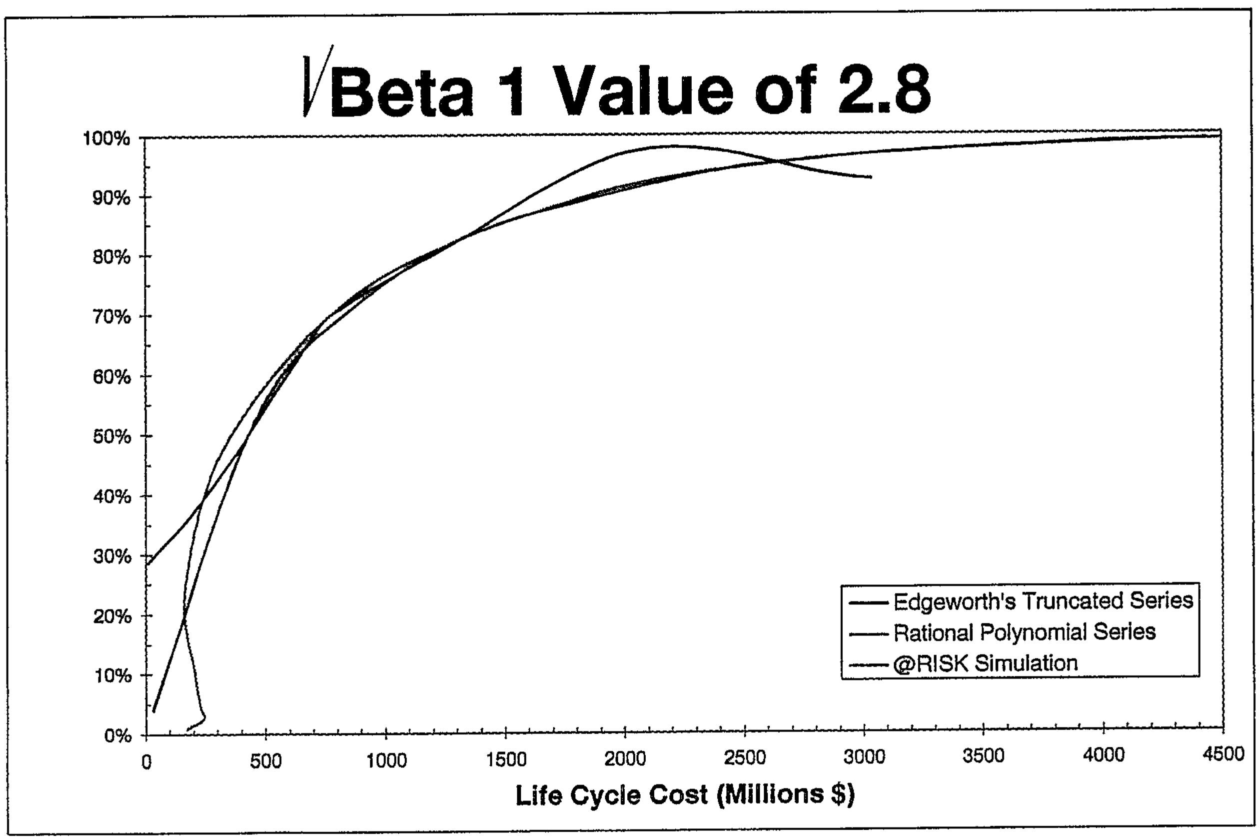
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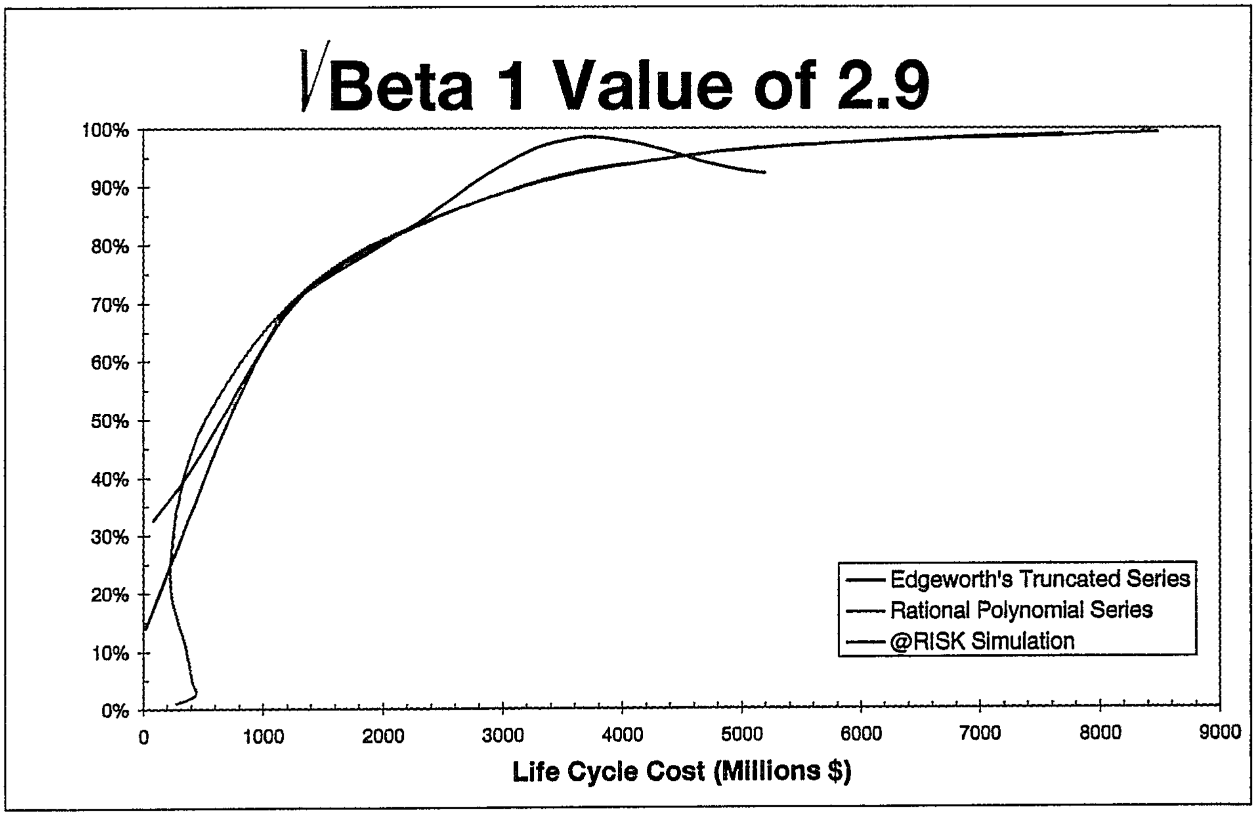
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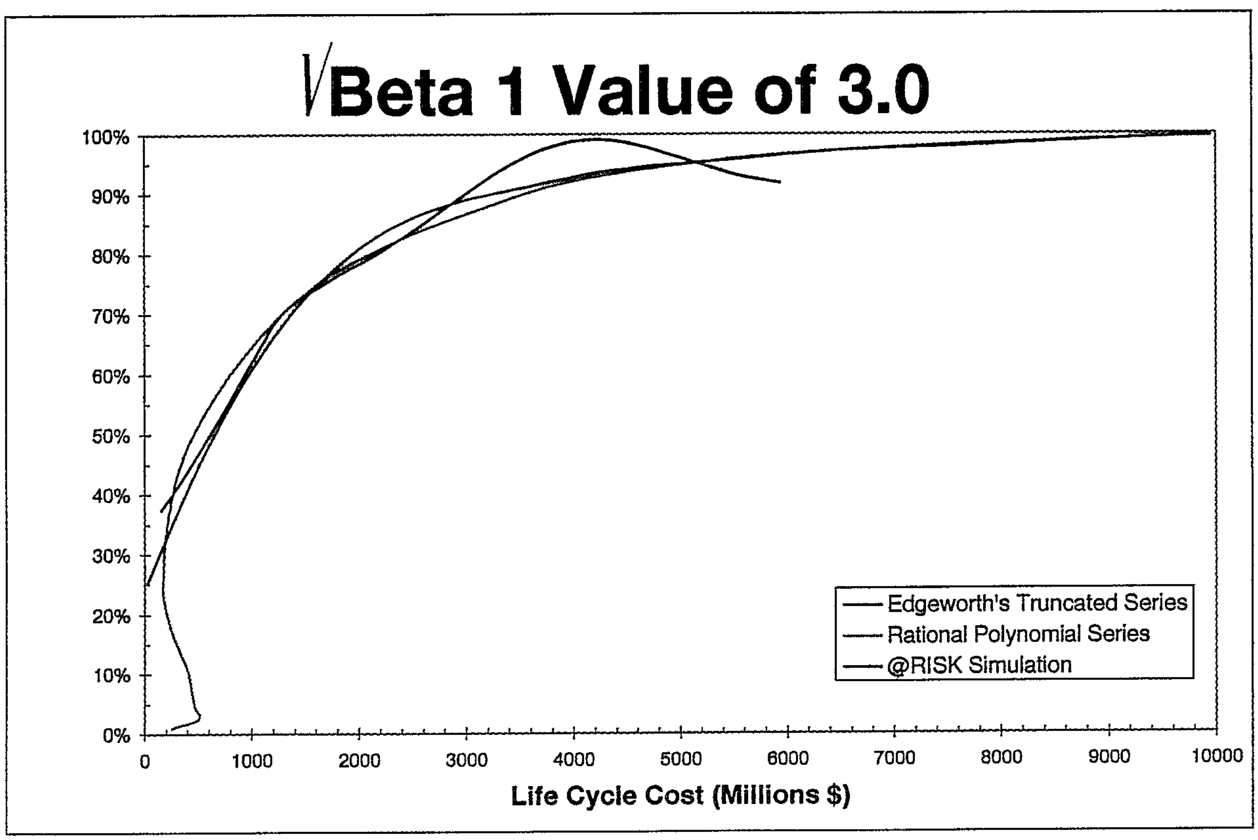
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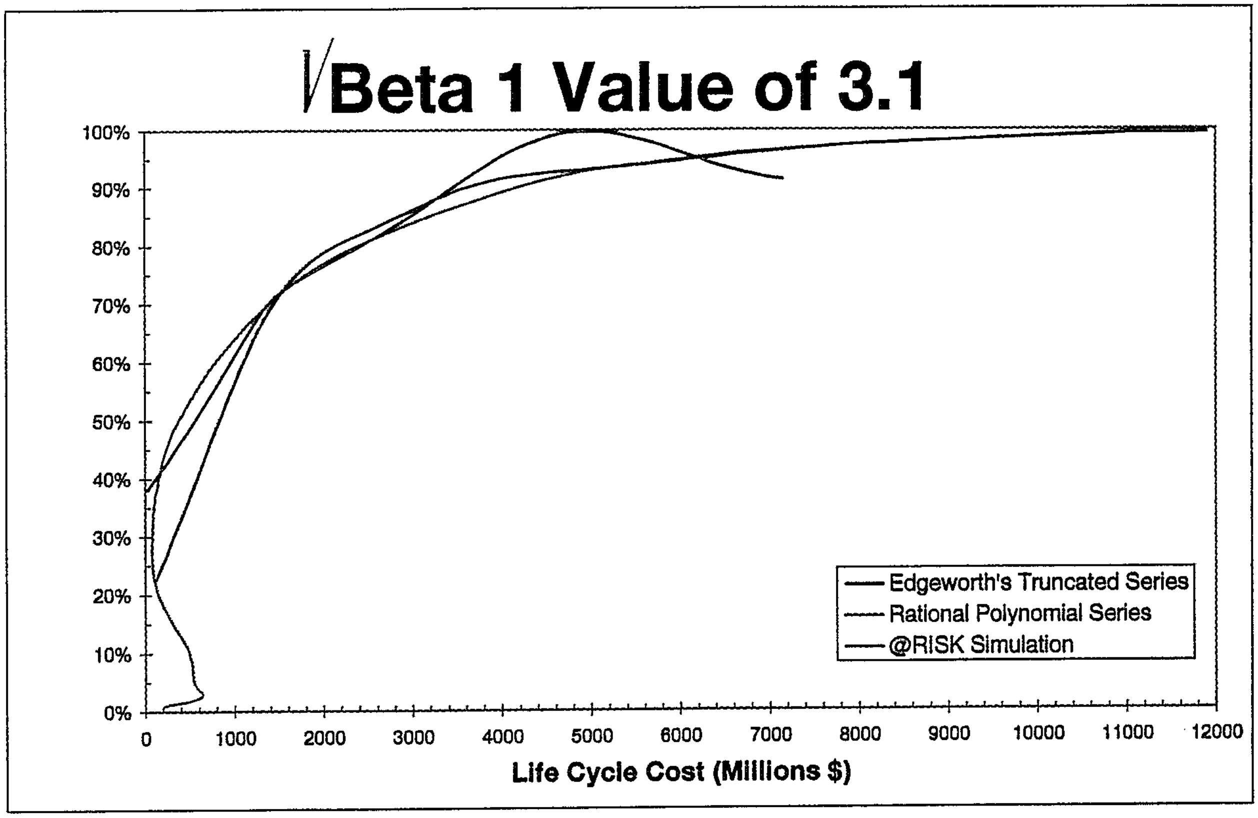
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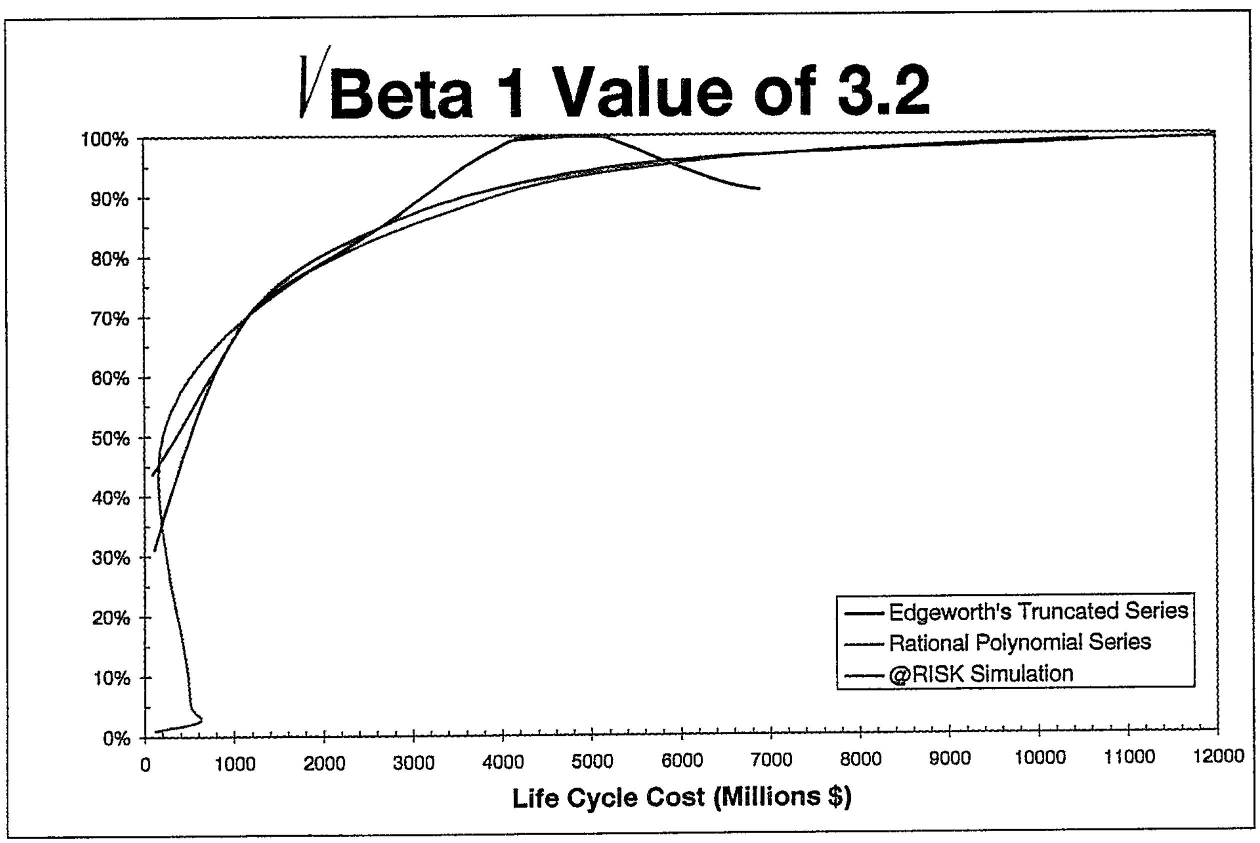
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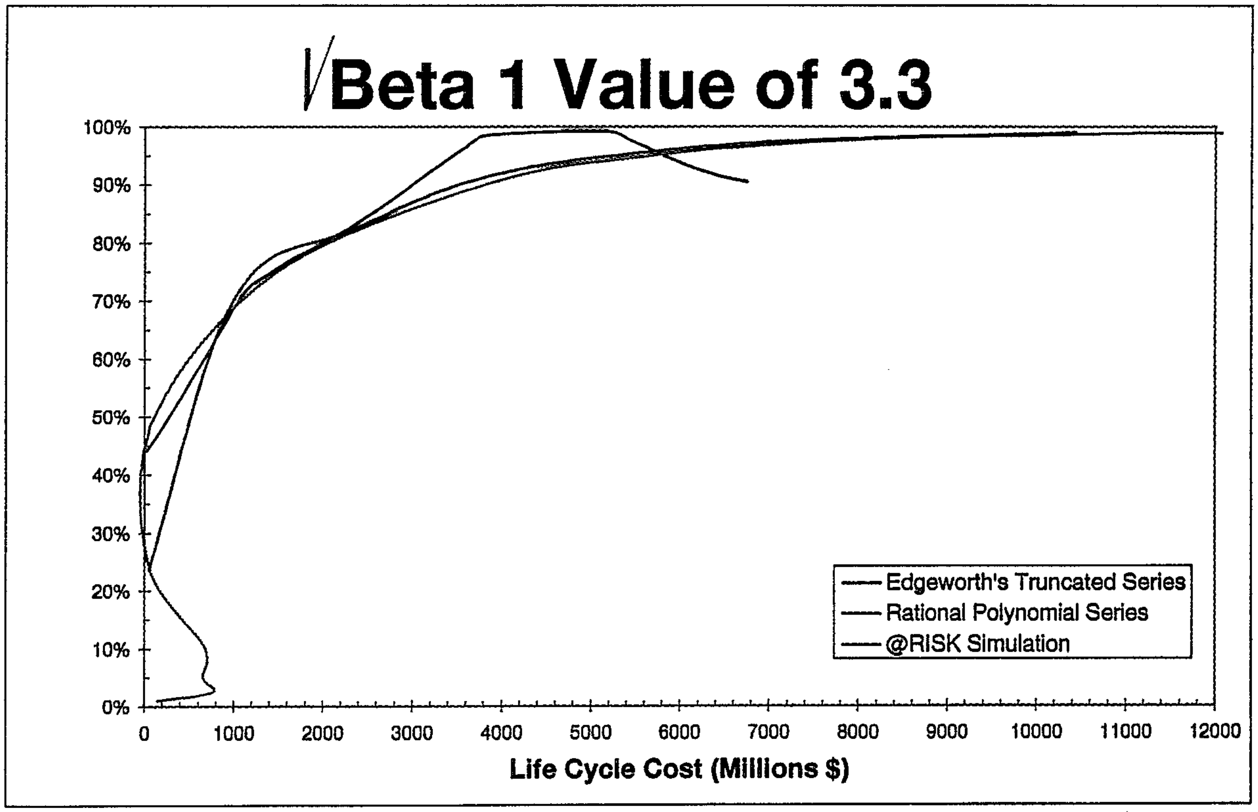
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Life Cycle Cost (Millions $)
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LAU, Tony C.K.

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This paper evaluates the mathematical implementations of the risk assessment methods in the Life Cycle Costing (LCC) model of the Logistics Analyzer (LOGAN) program. The two risk assessment methods were compared against a software simulation which generated a distribution based upon the same data used in the LCC model.

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