Image Cover Sheet

CLASSIFICATION	SYSTEM NUMBER 151078
UNCLASSIFIED	

TITLE

NOISE ANALYSIS OF AN INSTRUMENTATION AMPLIFIER

System Number:

Patron Number:

Requester:

Notes:

DSIS Use only:

Deliver to: FF

Defence Research
Establishment Pacific

Centre de recherches
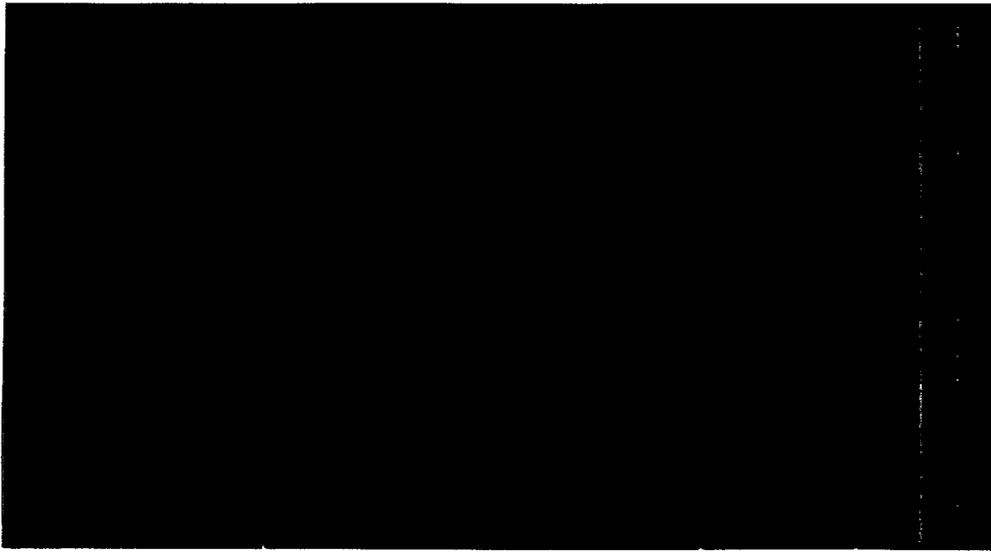pour la défense pacifique

DEFENSIO PER SCIENTIAM

DEFENCE RESEARCH ESTABLISHMENT PACIFIC
Research and Development Branch
Department of National Defence

Canada

▌✺ Defence Research
Establishment Pacific

Centre de recherches
pour la défense pacifique

DEFENCE RESEARCH ESTABLISHMENT PACIFIC

CFB Esquimalt, FMO Victoria, B.C. V0S 1B0

DREP Tech Memo 94-153
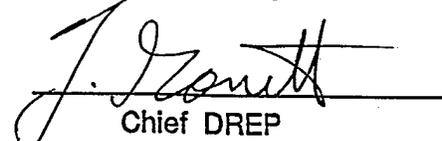
Noise Analysis of an Instrumentation Amplifier

T.C. Richards

Defence Research Establishment Pacific
FMO Victoria B.C. V0S 1B0

March 7, 1995

Approved By:

Chief DREP

Chief DREP

Research and Development Branch
Department of National Defence

Canada

Abstract

A noise analysis of a three opamp instrumentation amplifier is presented. The results, although not surprising, do give a theoretical basis for designing low noise instrumentation amplifiers. Using MAPLE, a symbolic computation package, known sources and loads are connected to the amplifier and the outputs are determined. These outputs are then passed to MAPLE subroutines which have been developed to determine the amplifier noise characteristics by manipulating the sources and loads. Experimental measurements to confirm the analysis and recommendations to minimize the noise are also provided.

Contents

List of Figures

List of Tables

List of Listings

1 Introduction

The signals generated by multi-turn coils and electrode pairs for the measurement of magnetic and electric fields must be amplified before they can be digitally recorded and analysed. The most common preamplifier configuration for this task is the three opamp instrumentation amplifier shown in Figure 1. This preamplifier consists of two stages, a differential input–differential output (DIDO) stage followed by a differential input–single ended output (DISO) stage. A thorough literature search eg. [1–8], revealed no reference that provided a theoretical noise analysis of the instrumentation amplifier. Section 2 presents the models required for the analysis, which are then used to create MAPLE procedures that perform the necessary algebraic manipulations. The results from the MAPLE worksheets, which are provided in the Appendix, are summarized in Section 3. Measurements to verify the analysis and to show the contribution of each noise source are given in Section 4.
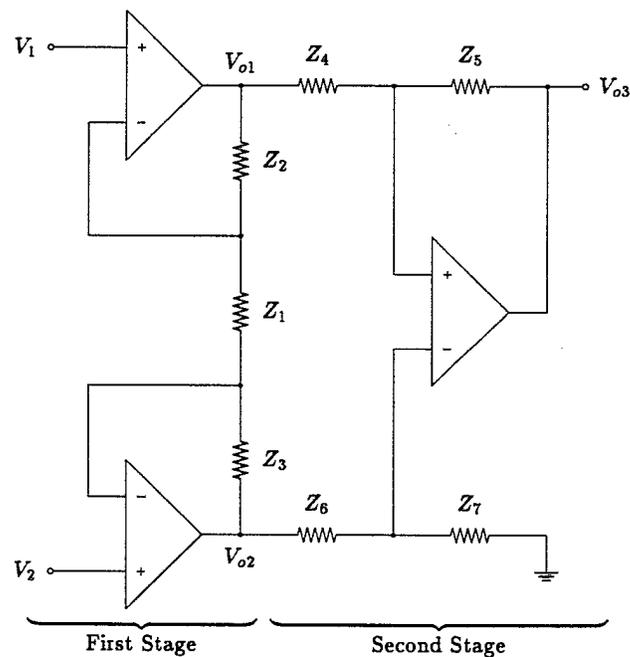
Figure 1: The Three Opamp Instrumentation Amplifier

1

2 Noise Models

This section presents the tools to perform the noise analysis of an instrumentation amplifier. First a brief review of the response of linear systems to noise signals is given. Then the thermal noise of a resistor and the equivalent noise of a complex valued impedance are discussed. Next a Power Spectral Density (PSD) model for the noise of an opamp is developed based on the manufacturer's noise specifications. Finally the DISO and DIDO noise models along with the mathematical techniques to determine their parameters are given.

2.1 Response of Linear Systems to Noise Signals

To determine the response of a linear system when the inputs are noise signals, consider the following two input case. The Laplace transform input/output relationship can be represented as

$$V_o(s) = G(s) V_1(s) + H(s) V_2(s) \tag{1}$$

where $V_1(s)$ and $V_2(s)$ are input voltages, and $G(s)$ and $H(s)$ are transfer functions. The PSD of $V_o(s)$ can be shown to be

$$P_{V_o}(\omega) = |G(j\omega)|^2 P_{V_1}(\omega) + |H(j\omega)|^2 P_{V_2}(\omega) + 2G(j\omega) H(j\omega) P_{V_1 V_2}(\omega) \tag{2}$$

where $P_{V_1}(\omega)$ and $P_{V_2}(\omega)$ are the PSDs of the inputs and $P_{V_1 V_2}(\omega)$ is their cross power spectral density [9]. For this work all noise sources are assumed independent so their cross spectral density is zero and

$$P_{V_o}(\omega) = |G(j\omega)|^2 P_{V_1}(\omega) + |H(j\omega)|^2 P_{V_2}(\omega), \tag{3}$$

which can be extended to any number of inputs.

A MAPLE routine to perform this calculation processes each input in turn, first computing the transfer function with respect to the current input and then creating a running sum of the product of the transfer function squared and a new variable which is the concatenation of the letter P with the current input (see the MAPLE routine NOISESUM found in Listing A.4). As an example if the output is given as Vo = G*V1 + H*V2 and the input list is vars = {V1 V2} then NOISESUM(Vo,vars) = G^2*PV1 + H^2*PV2. (Note that MAPLE is performing symbolic algebra).

Function arguments are omitted henceforth. Thus the spectral density $P_{V_o}(\omega)$ is written as P_{V_o}, the Laplace transform of a signal $V_o(s)$ is represented as V_o, and the transfer function $G(j\omega)$ is represented as G. In addition, the circuit diagrams show the Laplace transform of all noise sources and impedances. And, since the square root of PSDs are often quoted, all quantities that represent the *root spectral density* will be identified as $\check{P}$ (thus $\check{P}_{V_o} = \sqrt{P_{V_o}}$).

2.2 Impedance Noise Model

Every resistor has an associated thermal noise, which, when expressed in terms of its PSD can be written as

$$P_{V_R} = 4kTR \ \left(V^2/Hz \right) \tag{4}$$

where

k – Boltzmann's Constant in Joules per Kelvin $(1.38 * 10^{-23} J/K)$
T – Absolute Temperature in Kelvin (K)
R – Resistance in ohms (Ω)

2

Table 1: Typical voltage and current noise specifications for the OP10 and OP227 opamps, with the maximum expected values in parentheses.

OP10			OP227		
f (Hz)	$\overset{\vee}{P}_{V'_N}$ $\left(nV/\sqrt{Hz}\right)$	$\overset{\vee}{P}_{I_N}$ $\left(pA/\sqrt{Hz}\right)$	f (Hz)	$\overset{\vee}{P}_{V'_N}$ $\left(nV/\sqrt{Hz}\right)$	$\overset{\vee}{P}_{I_N}$ $\left(pA/\sqrt{Hz}\right)$
10	10.3 (18.0)	0.32 (0.80)	10	3.5 (5.5)	1.7 (4.0)
100	10.0 (13.0)	0.14 (0.23)	30	3.1 (4.5)	1.0 (2.3)
1000	9.6 (11.0)	0.12 (0.17)	1000	3.0 (3.8)	0.4 (0.6)

Note that this expression is independent of frequency and is therefore considered a white noise source. The noise model of a resistor is then just the series connection of an ideal (noiseless) resistor and a voltage noise source which is represented as V_R on circuit diagrams. At room temperature $(T = 290K)$, the root spectral density of the thermal noise of a resistor with resistance in $k\Omega$ can be written as

$$\frac{\overset{\vee}{P}_{V_R}}{\sqrt{R}} = 4.0 \left(\frac{nV}{\sqrt{Hz}\sqrt{k\Omega}}\right) \tag{5}$$

which provides a useful rule of thumb for computing the thermal noise of resistors. For complex valued impedances the thermal noise is given as

$$P_{V_Z} = 4kT \, \Re\left\{Z\right\} \tag{6}$$

where $\Re\left\{Z\right\}$ is the real part of the impedance Z. This result can also be obtained by representing each resistor in the impedance by its noise model and then computing the Thévenin equivalent circuit.

2.3 Opamp Noise Model

For these analyses the opamps are assumed to be ideal with infinite gain, infinite input impedance, zero output impedance and no $1/f$ noise. To account for the noise, a voltage and current source is connected to each input. Opamp manufacturers quote the combined or sum of the voltage noise sources, while the current noise sources are assumed equal and a single value for both is given. The specifications from [10] for PMI's OP10 and OP227 dual opamps are listed in Table 1. The OP10 is the industry standard in terms of noise specifications, and as evident from the OP227 data a lower voltage noise will usually be at the expense of higher current noise. Unfortunately opamp manufacturers only provide data at a few selected frequencies and usually do not give parameters that can be used to model the noise over all frequencies.

A simple $1/f$ noise model requires the corner frequency and the minimum value of the noise. For example if the corner frequency is f_V and the minimum value is $\mathcal{V}'_m$, the spectral density of the noise can be written as

$$P_{V'_N} = {\mathcal{V}'_m}^2 \left(1 + \frac{f_V}{f}\right) \tag{7}$$

To determine the parameters of this nonlinear model requires an imaginative approach because the standard nonlinear least squares method produces poor fits. Satisfactory results can be obtained if the errors are weighted by their corresponding frequency. Intuitively this makes sense because the
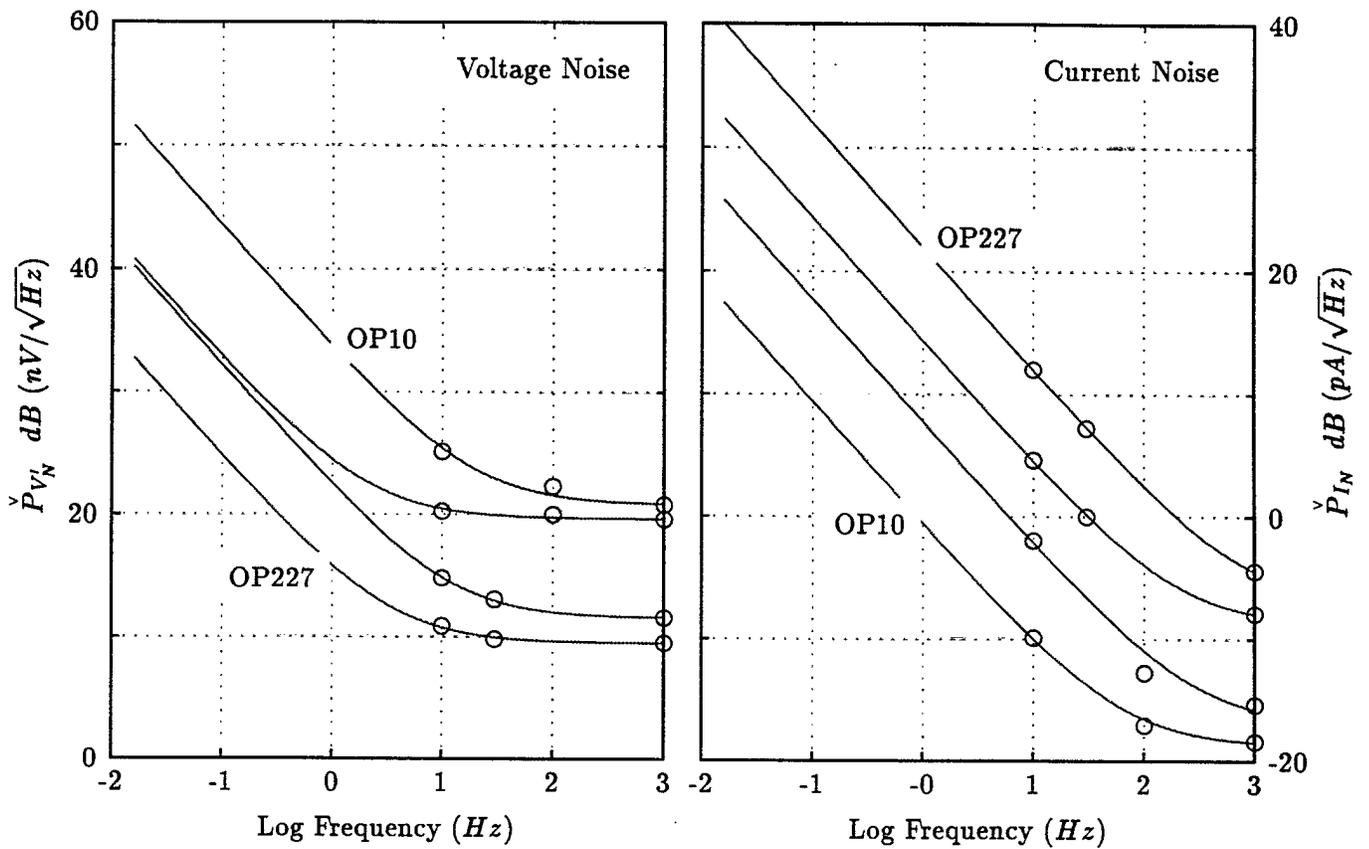
3

Figure 2: Voltage and current noise of the OP10 and OP227; the shaded regions represent the spread between the typical and maximum noise values. The manufacturers specifications, which are the basis for the curves, are shown as circles.

data at higher frequencies is given more emphasis. Applying the weighted nonlinear least squares approach to the OP10 data yields

$$\mathcal{V}'_m = 9.60 \ (11.0) \ nV/\sqrt{Hz} \qquad f_V = 2.03 \ (18.7) \ Hz$$

$$\mathcal{I}_m = 0.12 \ (0.14) \ pA/\sqrt{Hz} \qquad f_I = 65.3 \ (290) \ Hz$$

and for the OP227

$$\mathcal{V}'_m = 2.99 \ (3.78) \ nV/\sqrt{Hz} \qquad f_V = 3.30 \ (11.51) \ Hz$$

$$\mathcal{I}_m = 0.36 \ (0.45) \ pA/\sqrt{Hz} \qquad f_I = 206.0 \ (781.0) \ Hz$$

Note the lower voltage noise, higher current noise and greater corner frequencies of the OP227. Graphs of $\check{P}$ for the voltage and current noise of both the OP10 and OP227, along with the manufacturers specifications are shown in Figure 2. Clearly the weighted nonlinear least squares has provided acceptable results.
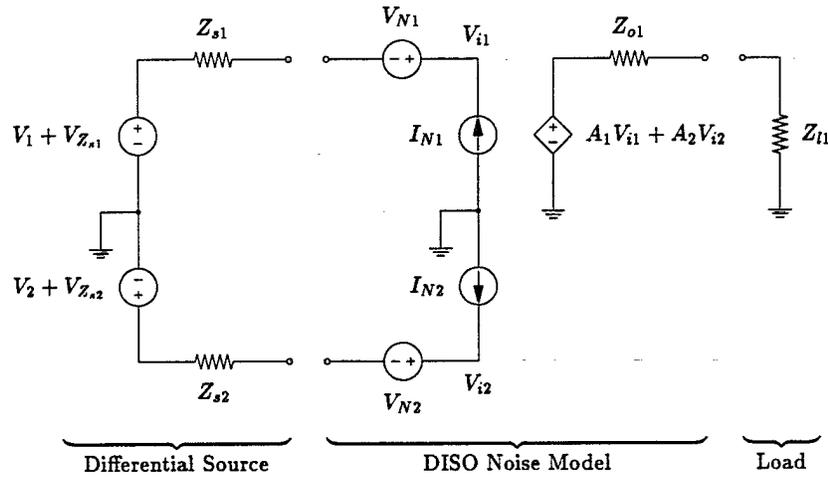
Figure 3: DISO noise model with differential source and single-ended load.

2.4 DISO Amplifier Noise Model

The most common noise model for an amplifier is to represent all the noise of the amplifier by voltage and current sources connected to the amplifier's input. Most references [1, 3] deal with the single input, single output case. No reference was found that treated the case where the amplifier has a differential input and/or differential output. As mentioned, the instrumentation amplifier can be considered a DISO amplifier or the cascade connection of a DIDO amplifier and a DISO amplifier. Thus noise models for both the DISO and DIDO amplifiers are needed.

The noise model for the DISO amplifier shown in Figure 3 consists of the following parameters

Z_{o1} – output impedance
A_1, A_2 – gains
$P_{V_{N1}}, P_{V_{N2}}$ – voltage noise
$P_{I_{N1}}, P_{I_{N2}}$ – current noise

Note that like the opamp the input impedance is assumed infinite or more precisely much greater than the output impedance of the previous stage[1].

To develop procedures to calculate the DISO model parameters a known differential source and a known load are connected to the DISO model as shown in Figure 3. The source consists of two input signals V_1 and V_2, source impedances Z_{s1} and Z_{s2}, and their thermal noise $V_{Z_{s1}}$ and $V_{Z_{s2}}$. The single-ended load is just an impedance represented by Z_{l1}. Once the output is determined, the sources and load can be algebraically manipulated and the model parameters can be found. The required manipulations form the basis for the procedure to calculate the parameters for any DISO amplifier. Thus, from the circuit the following equations are deduced

$$\frac{V_{i1} - V_{N1} - V_{Z_{s1}} - V_1}{Z_{s1}} = I_{N1} \tag{8}$$

---

[1]A general model for the input impedance of a differential input amplifier, does not exist. In fact for the four resistor differential amplifier which composes the second stage of the instrumentation amplifier, the input impedance can be shown to be vary with the applied inputs [11].

5

$$\frac{V_{i2} - V_{N2} - V_{Z_{n2}} - V_2}{Z_{s2}} = I_{N2} \qquad (9)$$

$$\frac{V_{o1} - A_1 V_{i1} - A_2 V_{i2}}{Z_{o1}} + \frac{V_{o1}}{Z_{l1}} = 0 \qquad (10)$$

which on solving for the output yields

$$V_{o1} = \frac{Z_{l1}}{Z_{l1} + Z_{o1}} \{A_1 (V_1 + V_{N1} + Z_{s1} I_{N1} + V_{Z_{n1}}) + A_2 (V_2 + V_{N2} + Z_{s2} I_{N2} + V_{Z_{n2}})\} \qquad (11)$$

To generalize this result to include the possibility of a finite input impedance the system gains K_1 and K_2 are introduced such that

$$V_{o1} = K_1 (V_1 + V_{N1} + Z_{s1} I_{N1} + V_{Z_{n1}}) + K_2 (V_2 + V_{N2} + Z_{s2} I_{N2} + V_{Z_{n2}}) \qquad (12)$$

These system gains take into account the loading effect of the output impedance, the loading effect of the input impedance[2] and the amplifier gains.

2.4.1 Output Impedance

To determine the output impedance allow the load impedance to increase to infinity and define the output

$$V'_{o1} = \lim_{Z_{l1} \to \infty} V_{o1} \qquad (13)$$

$$= \mathbf{limit}\{V_{o1}\}\{Z_{l1} \to \infty\} \qquad (14)$$

Then the output voltage can be written as

$$V_{o1} = \frac{Z_{l1}}{Z_{l1} + Z_{o1}} V'_{o1} \qquad (15)$$

which can be easily rearranged to yield

$$Z_{o1} = Z_{l1} \frac{V'_{o1} - V_{o1}}{V_{o1}} \qquad (16)$$

The assumption of infinite input impedance (no loading between stages) means that the output impedance does not affect the overall gain. However, it interacts with with the current noise present in the next amplifier stage. The above operations can easily be performed by MAPLE (Note the introduction of a simpler notation for the limit operation).

2.4.2 Amplifier Gains

To calculate the gains, a coefficient function is defined, such that $\mathbf{coeff}\{ax + by\}\{x\} = a$. This allows the system gains then to be determined as

$$K_1 = \mathbf{coeff}\{V_{o1}\}\{V_1\} \qquad (17)$$

$$K_2 = \mathbf{coeff}\{V_{o1}\}\{V_2\} \qquad (18)$$

---

[2] The noise model assumes infinite input impedance, however the amplifiers analyzed could have a finite input impedance.

6

The amplifier gains A_1 and A_2 are calculated from the system gains by removing the effects of the source and load impedances, thus

$$A_1 = \text{limit} \{K_1\} \{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l1} \to \infty\} \tag{19}$$

$$A_2 = \text{limit} \{K_2\} \{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l1} \to \infty\} \tag{20}$$

2.4.3 Equivalent Input Noise

Without loss of generality the input signals to DISO amplifier can be defined as

$$V_1 = \frac{V_d}{2} + V_c \tag{21}$$

$$V_2 = -\frac{V_d}{2} + V_c. \tag{22}$$

Rearranging these expressions gives

$$V_d = V_1 - V_2 \tag{23}$$

$$V_c = \frac{V_1 + V_2}{2}, \tag{24}$$

where V_d is the difference between the inputs (the part of interest) and V_c is the common mode or average of the inputs. Substituting these definitions into Equation 12, and for convenience renaming the output from V_{o1} to V_{od}, the output difference voltage is given by

$$V_{od} = K_d V_d + K_c V_c + K_1 \left(V_{N1} + Z_{s1} I_{N1} + V_{Z_{n1}} \right) + K_2 \left(V_{N2} + Z_{s2} I_{N2} + V_{Z_{n2}} \right) \tag{25}$$

where K_d, the system difference gain, is given as

$$K_d = \frac{K_1 - K_2}{2} \tag{26}$$

and K_c, the system common mode gain, is

$$K_c = K_1 + K_2. \tag{27}$$

Equation 25 can be referred to the input by dividing by the system difference gain creating

$$V_{id} = V_d + \frac{K_c}{K_d} V_c + \frac{K_1}{K_d} \left(V_{N1} + Z_{s1} I_{N1} + V_{Z_{n1}} \right) + \frac{K_2}{K_d} \left(V_{N2} + Z_{s2} I_{N2} + V_{Z_{n2}} \right). \tag{28}$$

All terms in this expression except the input signal V_d are noise and as a result the signal to noise ratio can be easily determined at this point. The equivalent input noise is found by setting the difference and common mode inputs[3] to zero, thus

$$V_{in} = \text{limit} \{V_{id}\} \{V_d \to 0, V_c \to 0\} \tag{29}$$

$$= \frac{K_1}{K_d} \left(V_{N1} + Z_{s1} I_{N1} + V_{Z_{n1}} \right) + \frac{K_2}{K_d} \left(V_{N2} + Z_{s2} I_{N2} + V_{Z_{n2}} \right) \tag{30}$$

The terms of interest are V_{od} from Equation 25, V_{id} from Equation 28 and V_{in} from Equation 30. Their PSDs can be computed using Equation 3.

---

[3]In practise the common mode rejection ratio (that is K_c/K_d) should eliminate the common mode input voltage. However if the common mode rejection ratio is 10^{-6} a common mode signal of 10^{-3} would result in a noise term of 10^{-9} which is of the same order of magnitude of other sources of noise.

2.4.4 Voltage and Current Noise

To compute the voltage and current noise terms start with V_{od} from Equation 25 and create

$$V'_{od} \;=\; \text{limit} \{V_{od}\} \{V_d \to 0, V_c \to 0, Z_{l1} \to \infty, V_{Z_{s1}} \to 0, V_{Z_{s2}} \to 0\} \tag{31}$$

$$=\; K'_1 (V_{N1} + Z_{s1} I_{N1}) + K'_2 (V_{N2} + Z_{s2} I_{N2}) \tag{32}$$

where K'_1 and K'_2 are the system gains with the loading effect of the load impedance removed, that is

$$K'_1 \;=\; \text{limit} \{K_1\} \{Z_{l1} \to \infty\} \tag{33}$$

$$K'_2 \;=\; \text{limit} \{K_2\} \{Z_{l1} \to \infty\} \tag{34}$$

From the single expression in Equation 32 it is not possible to solve for the four terms representing the voltage and current noise. However if conditions are known that set $K'_1 = -K'_2$ (the usual case for a DISO amplifier) and if $Z_{s2} = Z_{s1}$ then

$$V'_{od} \to K'_1 \{V_{N1} - V_{N2} + Z_{s1} (I_{N1} - I_{N2})\}, \tag{35}$$

which when referred to the input gives

$$V'_{id} \;=\; \frac{V'_{od}}{K'_1}$$

$$=\; V_{N1} - V_{N2} + Z_{s1} (I_{N1} - I_{N2}). \tag{36}$$

From this expression the sum of the voltage noise sources known as the combined voltage noise can be determined as

$$V_{N'} \;=\; V_{N1} - V_{N2} \tag{37}$$

$$=\; \text{limit} \{V'_{id}\} \{Z_{s1} \to 0\}, \tag{38}$$

and the combined current noise

$$I_{N'} \;=\; I_{N1} - I_{N2} \tag{39}$$

$$=\; \text{limit} \left\{\frac{V'_{id}}{Z_{s1}}\right\} \{Z_{s1} \to \infty\} \tag{40}$$

Note that the voltage noise is calculated when the source impedance is zero and the current noise is calculated when the source impedance is large. This is analogous to the actual techniques used when making noise measurements on the bench. The PSD for the combined voltage noise from Equation 37 is

$$P_{V_{N'}} = P_{V_{N1}} + P_{V_{N2}}. \tag{41}$$

The quantity usually quoted for a DISO amplifier is the square root of this, $\check{P}_{V_{N'}}$. For the current noise it is usual to assume that the noise sources at each input are equal, that is $P_{I_{N1}} = P_{I_{N2}} = P_{I_N}$. Hence
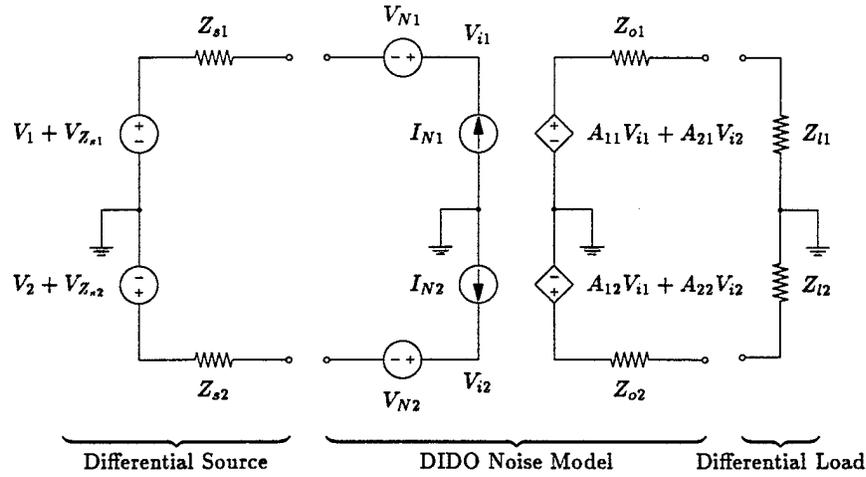
$$P_{I_N} = \frac{1}{2} P_{I_{N'}} \tag{42}$$

8

Figure 4: DIDO noise model with differential source and differential load.

where $\overset{\vee}{P}_{I_N}$ is specified by manufacturers.

The MAPLE subroutine to compute the model parameters of the DISO amplifier is DISOMOD, found in Listing A.4. This routine requires the output voltage of the amplifier with the known sources and load connected, the conditions that equate K_1' and $-K_2'$, and a list of the sources within the amplifier, so that computation of PSDs can be performed.

2.5 DIDO Amplifier Noise Model

The DIDO noise model is similar to that of the DISO amplifier. However it must provide a differential output as shown in Figure 4. The same parameters are required as for the DISO amplifier, except two output impedances and four gains must be determined.

The following circuit equations can be written

$$\frac{V_{i1} - V_{N1} - V_{Z_{s1}} - V_1}{Z_{s1}} = I_{N1} \tag{43}$$

$$\frac{V_{i2} - V_{N2} - V_{Z_{s2}} - V_2}{Z_{s2}} = I_{N2} \tag{44}$$

$$\frac{V_{o1} - A_{11}V_{i1} - A_{21}V_{i2}}{Z_{o1}} + \frac{V_{o1}}{Z_{l1}} = 0 \tag{45}$$

$$\frac{V_{o2} - A_{12}V_{i1} - A_{22}V_{i2}}{Z_{o2}} + \frac{V_{o2}}{Z_{l2}} = 0 \tag{46}$$

which, on solving for the output voltages yields

$$V_{o1} = \frac{Z_{l1}}{Z_{l1} + Z_{o1}} \left\{ A_{11} \left(V_1 + V_{N1} + Z_{s1}I_{N1} + V_{Z_{s1}} \right) + A_{21} \left(V_2 + V_{N2} + Z_{s2}I_{N2} + V_{Z_{s2}} \right) \right\} \tag{47}$$

$$V_{o2} = \frac{Z_{l2}}{Z_{l2} + Z_{o2}} \left\{ A_{12} \left(V_1 + V_{N1} + Z_{s1}I_{N1} + V_{Z_{s1}} \right) + A_{22} \left(V_2 + V_{N2} + Z_{s2}I_{N2} + V_{Z_{s2}} \right) \right\} \tag{48}$$

9

Once again the outputs can be generalized to include the effect of an input impedance by introducing system gains, thus

$$V_{o1} = K_{11}(V_1 + V_{N1} + Z_{s1}I_{N1} + V_{Z_{s1}}) + K_{21}(V_2 + V_{N2} + Z_{s2}I_{N2} + V_{Z_{s2}}) \tag{49}$$

$$V_{o2} = K_{12}(V_1 + V_{N1} + Z_{s1}I_{N1} + V_{Z_{s1}}) + K_{22}(V_2 + V_{N2} + Z_{s2}I_{N2} + V_{Z_{s2}}) \tag{50}$$

The model parameters are determined by manipulating the sources and loads in these expressions similar to the DISO amplifier procedures.

2.5.1 Output Impedances

For the output impedances the procedure is repeated for both outputs. For example at the second output the effect of the load impedance can be removed to produce

$$V'_{o2} = \mathbf{limit}\{V_{o2}\}\{Z_{l2} \to \infty\} \tag{51}$$

and then

$$Z_{o2} = Z_{l12}\frac{V'_{o2} - V_{o2}}{V_{o2}} \tag{52}$$

2.5.2 Amplifier Gains

First the system gains are determined as

$$K_{11} = \mathbf{coeff}\{V_{o1}\}\{V_1\} \tag{53}$$

$$K_{21} = \mathbf{coeff}\{V_{o1}\}\{V_2\} \tag{54}$$

$$K_{12} = \mathbf{coeff}\{V_{o2}\}\{V_1\} \tag{55}$$

$$K_{22} = \mathbf{coeff}\{V_{o2}\}\{V_2\}. \tag{56}$$

Removing the effects of the source and load impedances yields the amplifier gains as

$$A_{11} = \mathbf{limit}\{K_{11}\}\{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l1} \to \infty\} \tag{57}$$

$$A_{21} = \mathbf{limit}\{K_{21}\}\{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l1} \to \infty\} \tag{58}$$

$$A_{12} = \mathbf{limit}\{K_{12}\}\{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l2} \to \infty\} \tag{59}$$

$$A_{22} = \mathbf{limit}\{K_{22}\}\{Z_{s1} \to 0, Z_{s2} \to 0, Z_{l2} \to \infty\} \tag{60}$$

2.5.3 Output Difference Voltage

The signal of interest at the output of the DIDO amplifier is the difference of the two output voltages, which is

$$V_{od} = V_{o1} - V_{o2} \tag{61}$$

$$= K_1(V_1 + V_{N1} + Z_{s1}I_{N1} + V_{Z_{s1}}) + K_2(V_2 + V_{N2} + Z_{s2}I_{N2} + V_{Z_{s2}}), \tag{62}$$

10

where the system gains K_1 and K_2 are defined as

$$K_1 = K_{11} - K_{12} \tag{63}$$

$$K_2 = K_{21} - K_{22}. \tag{64}$$

If the input voltages are again represented in terms of the difference and common mode voltages of the inputs, the output difference voltage becomes

$$V_{od} = K_d V_d + K_c V_c + K_1 \left(V_{N1} + Z_{s1} I_{N1} + V_{Z_{s1}} \right) + K_2 \left(V_{N2} + Z_{s2} I_{N2} + V_{Z_{s2}} \right) \tag{65}$$

where K_d and K_c are defined in terms of K_1 and K_2 as before. The expression in Equation 65 is the same as Equation 25 and thus the procedures previously used to determine the DISO amplifier noise model parameters can be repeated here.

The MAPLE subroutine DIDOMOD found in Listing A.4, computes the DIDO noise parameters when passed the output voltages, the conditions that satisfy $K_1' = -K_2'$, and a list of the sources within the amplifier.
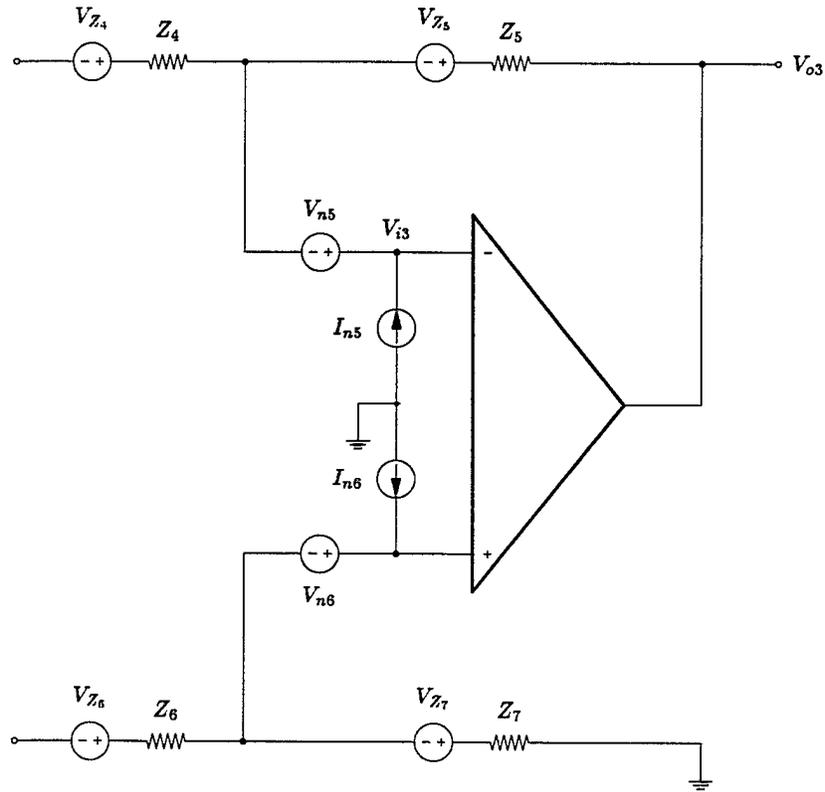
Figure 5: Second Stage of the Instrumentation Amplifier with Noise Sources

3   Noise Analysis of the Instrumentation Amplifier

The noise analysis of the three opamp instrumentation amplifier is divided into three parts, the DISO second stage, the DIDO first stage and then the entire instrumentation amplifier, which is the cascade connection of these two stages.

3.1   Second Stage of the Instrumentation Amplifier

The second stage of the instrumentation amplifier, shown in Figure 5, is known as a four resistor differential amplifier. With the same sources and load as were used for the DISO model, the defining circuit equations are

$$\frac{V_{i3} - V_{n5} - V_{Z_4} - V_1 - V_{Z_{n1}}}{Z_{s1} + Z_4} + \frac{V_{i3} - V_{n5} + V_{Z_5} - V_{o3}}{Z_5} = I_{n5} \tag{66}$$

$$\frac{V_{i3} - V_{n6} - V_{Z_6} - V_2 - V_{Z_{n2}}}{Z_{s2} + Z_6} + \frac{V_{i3} - V_{n6} + V_{Z_7}}{Z_7} = I_{n6}. \tag{67}$$

Solving for the output voltage yields

$$V_{o3} = -\frac{Z_5}{Z_{s1} + Z_4}\left(V_1 + V_{Z_{n1}} + V_{Z_4}\right) + \frac{Z_7\left(Z_{s1} + Z_4 + Z_5\right)}{\left(Z_{s2} + Z_6 + Z_7\right)\left(Z_{s1} + Z_4\right)}\left(V_2 + V_{Z_{n2}} + V_{Z_6}\right)$$

12

$$+\frac{Z_{s1}+Z_4+Z_5}{Z_{s1}+Z_4}\left(V_{n6}-V_{n5}\right)+\frac{\left(Z_{s2}+Z_6\right)\left(Z_{s1}+Z_4+Z_5\right)}{\left(Z_{s2}+Z_6+Z_7\right)\left(Z_{s1}+Z_4\right)}\left(I_{n6}Z_7-V_{Z_7}\right)$$

$$-I_{n5}Z_5+V_{Z_5}. \tag{68}$$

This output voltage is of the same form as Equation 25 and the procedures for determining the DISO amplifier noise parameters can be applied. The MAPLE procedure DISOMOD requires the above output voltage, the conditions that set $K_1'=-K_2'$ (which are $Z_6=Z_4$ and $Z_7=Z_5$) and a list of the noise sources within the amplifier.

The results of the analysis can be simplified if the normal operating conditions of the amplifier are substituted into the results. These conditions are

$$Z_6=Z_4, \qquad Z_7=Z_5, \qquad Z_{s2}=Z_{s1},$$

$$P_{V_{Z_6}}=P_{V_{Z_4}}, \qquad P_{V_{Z_7}}=P_{V_{Z_5}}, \qquad P_{V_{Z_{n2}}}=P_{V_{Z_{n1}}},$$

$$P_{V_{n5}}=P_{V_{n6}}=\frac{1}{2}P_{V_{n5}'}, \qquad P_{I_{n6}}=P_{I_{n5}}.$$

Now from the MAPLE output in Listing A.1 the voltage and current noise for the second stage are

$$P_{I_N} = P_{I_{n5}}+\frac{P_{V_{n5}'}}{2\left|Z_5\right|^2}+\frac{P_{V_{Z_5}}}{\left|Z_5\right|^2} \tag{69}$$

$$P_{V_N'} = \left|\frac{Z_4+Z_5}{Z_5}\right|^2 P_{V_{n5}'}+2\left|Z_4\right|^2 P_{I_{n5}}+2P_{V_{Z_4}}+2\left|\frac{Z_4}{Z_5}\right|^2 P_{V_{Z_5}}. \tag{70}$$

When the differential gain of this stage is large, $Z_5\gg Z_4$ and the voltage noise can be approximated as

$$P_{V_N'}\approx P_{V_{n5}'}+2\left|Z_4\right|^2 P_{I_{n5}}+2P_{V_{Z_4}}. \tag{71}$$

A similar analysis of this amplifier stage is available in [2].

3.2   First Stage of the Instrumentation Amplifier

The first stage of the instrumentation amplifier, complete with noise sources, is shown in Figure 6. With the sources and loads connected to this stage the four defining equations for the circuit are

$$\frac{V_{i1}-V_{n1}-V_1-V_{Z_{n1}}}{Z_{s1}}=I_{n1} \tag{72}$$

$$\frac{V_{i1}-V_{n2}+V_{Z_2}-V_{o1}}{Z_2}+\frac{V_{i1}-V_{n2}-V_{Z_1}+V_{n4}-V_{i2}}{Z_l}=I_{n2} \tag{73}$$

$$\frac{V_{i2}-V_{n3}-V_2-V_{Z_{n2}}}{Z_{s2}}=I_{n3} \tag{74}$$

$$\frac{V_{i2}-V_{n4}+V_{Z_3}-V_{o2}}{Z_3}+\frac{V_{i2}-V_{n4}+V_{Z_1}+V_{n2}-V_{i1}}{Z_l}=I_{n4}. \tag{75}$$

Solving for the output voltages, from these four equations, gives

$$V_{o1}=\frac{Z_1+Z_2}{Z_1}\left(V_1+V_{n1}-V_{n2}+I_{n1}Z_{s1}+V_{Z_{n1}}\right)$$

13
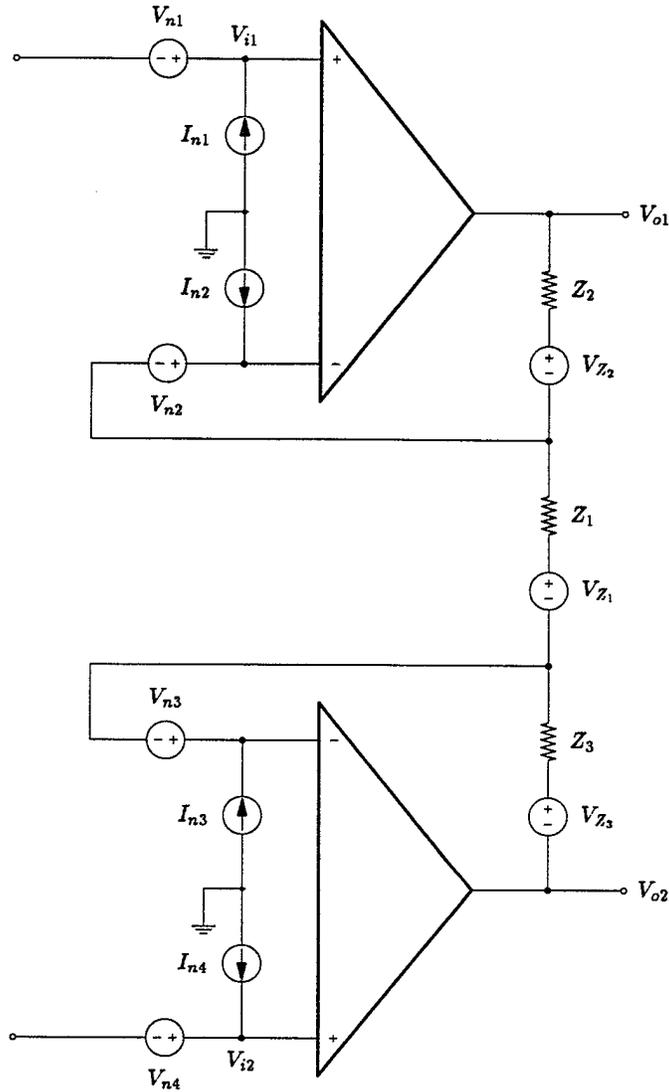
Figure 6: First Stage of the Instrumentation Amplifier with Noise Sources

$$-\frac{Z_2}{Z_1}\left(V_2 + V_{n3} - V_{n4} + I_{n3}Z_{s2} + V_{Z_{n2}}\right) - I_{n2}Z_2 + V_{Z_2} \tag{76}$$

$$V_{o2} = \frac{Z_1 + Z_3}{Z_1}\left(V_2 + V_{n3} - V_{n4} + I_{n3}Z_{s2} + V_{Z_{n2}}\right)$$

$$-\frac{Z_3}{Z_1}\left(V_1 + V_{n1} - V_{n2} + I_{n1}Z_{s1} + V_{Z_{n1}}\right) - I_{n4}Z_3 + V_{Z_3}. \tag{77}$$

The necessary condition to equate the system gains is $Z_3 = Z_2$. The results from DIDOMOD can be greatly simplified if the following normal operating conditions are substituted into the results.

$$Z_3 = Z_2, \qquad Z_{s2} = Z_{s1},$$

$$P_{V_{Z_3}} = P_{V_{Z_2}}, \qquad P_{V_{Z_{n2}}} = P_{V_{Z_{n1}}},$$

14

$$P_{V_{n1}} = P_{V_{n2}} = P_{V_{n3}} = P_{V_{n4}} = \frac{1}{2}P_{V'_{n1}}, \qquad P_{I_{n2}} = P_{I_{n3}} = P_{I_{n4}} = P_{I_{n1}}.$$

With these substitutions the voltage and current noise from Listing A.2 are

$$P_{I_N} = P_{I_{n1}} \qquad \qquad (78)$$

and $\qquad \qquad (79)$

$$P_{V'_N} = 2P_{V'_{n1}} + 2\left|\frac{Z_1 Z_2}{Z_1 + 2Z_2}\right|^2 P_{I_{n1}} + 4\left|\frac{Z_2}{Z_1 + 2Z_2}\right|^2 P_{V_{Z_1}} + 2\left|\frac{Z_1}{Z_1 + 2Z_2}\right|^2 P_{V_{Z_2}} \qquad (80)$$

If the differential gain is large, $Z_2 \gg Z_1$ and the voltage noise can be approximated as

$$P_{V'_N} \approx 2P_{V'_{n1}} + \frac{1}{2}|Z_1|^2 P_{I_{n1}} + P_{V_{Z_1}}. \qquad (81)$$

Note that with a large gain the equivalent thermal noise of the feed back impedance Z_2 can be neglected.

3.3   Full Instrumentation Amplifier

The instrumentation amplifier as seen in Figure 7 is a DISO amplifier and is analyzed as such in Listing A.3. From the analysis the combined voltage and current noise of the instrumentation amplifier are

$$P_{I_N} = P_{I_{n1}} \qquad \qquad (82)$$

$$P_{V'_N} = 2P_{V'_{n1}} + 2\left|\frac{Z_1 Z_2}{Z_1 + 2Z_2}\right|^2 P_{I_{n1}} + 4\left|\frac{Z_2}{Z_1 + 2Z_2}\right|^2 P_{V_{Z_1}} + 2\left|\frac{Z_1}{Z_1 + 2Z_2}\right|^2 P_{V_{Z_2}}$$

$$+ \left|\frac{Z_1}{Z_1 + 2Z_2}\right|^2 \left(\left|\frac{Z_4 + Z_5}{Z_5}\right|^2 P_{V'_{n5}} + 2|Z_4|^2 P_{I_{n5}} + 2P_{V_{Z_4}} + 2\left|\frac{Z_4}{Z_5}\right|^2 P_{V_{Z_5}}\right) \qquad (83)$$

Inspection of Equation 83 reveals the well known result [2] that the total combined voltage noise of the two cascaded stages is the noise of the first stage plus the noise of the second stage divided by the difference gain of the first stage squared.

Assuming that the gain of the first stage is large means the noise of the second stage can be ignored. Thus, the voltage noise can be approximated as

$$P_{V'_N} \approx 2P_{V'_{n1}} + \frac{1}{2}|Z_1|^2 P_{I_{n1}} + P_{V_{Z_1}} \quad . \qquad (84)$$

Equation 84 is the primary result of this report. Care must be taken when applying this approximation to all frequencies because the impedances and gains of the instrumentation amplifier are frequency dependent. The accuracy of this approximation is considered in the next section.
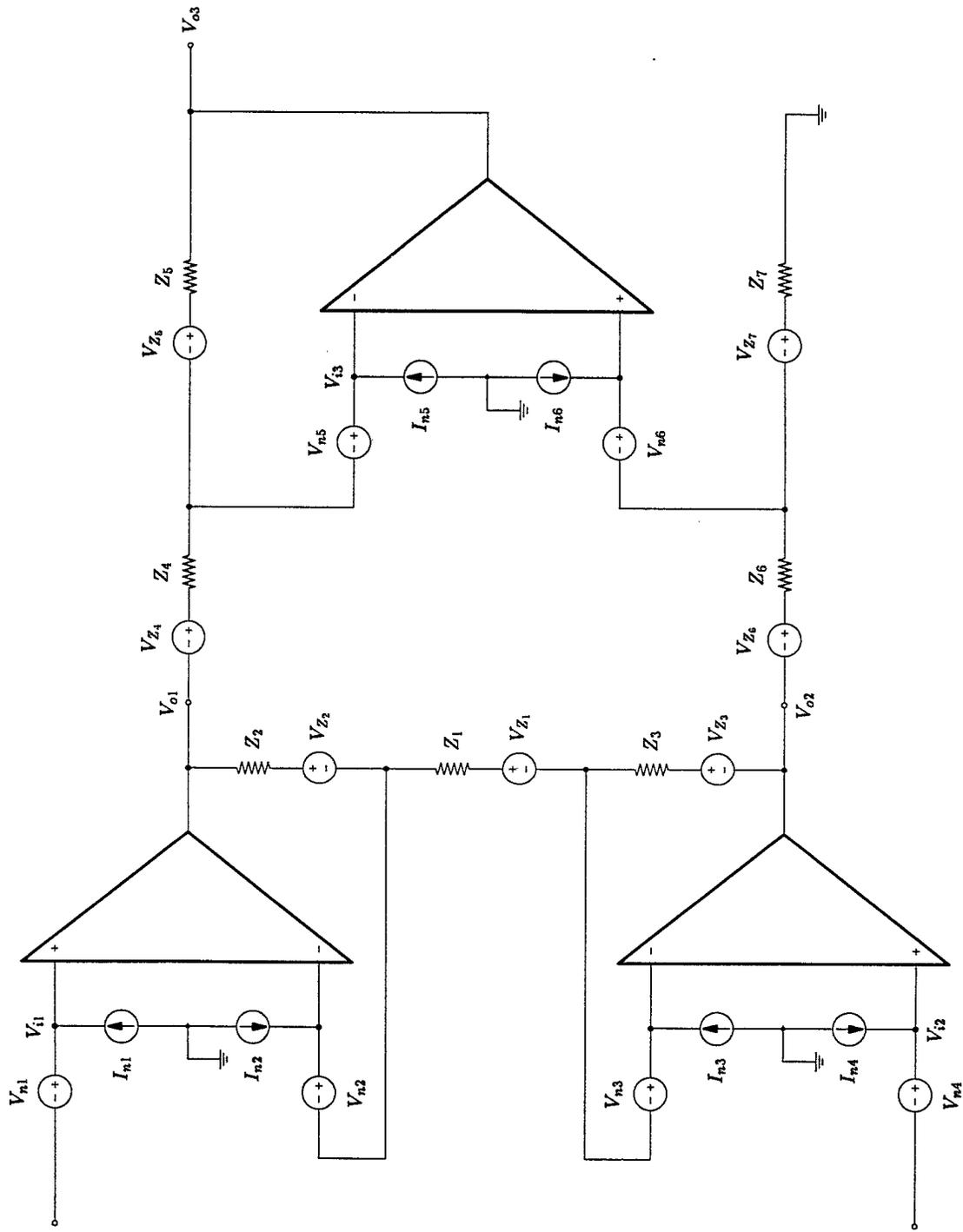
15

Figure 7: The Instrumentation Amplifier with Noise Sources

16

4   Measurements

To verify the noise analyses an instrumentation amplifier was constructed using the following values for the impedances:

$$Z_1 \rightarrow \text{series combination of } R_1 = 10k\Omega \text{ and } C_1 = 20\mu F$$
$$Z_2 = Z_3 \rightarrow \text{parallel combination of } R_2 = 500k\Omega \text{ and } C_2 = 390pF$$
$$Z_4 = Z_6 \rightarrow R_4 = 2K\Omega$$
$$Z_5 = Z_7 \rightarrow \text{parallel combination of } R_5 = 200k\Omega \text{ and } C_5 = 2.2nF.$$

With these selections the difference gain of both stages is 100. The Z_1 combination provides ac coupling at $0.8Hz$, the Z_2 combination provides roll off at $800Hz$, and the Z_4 combination rolls off at $400Hz$.

To determine the noise performance of the OP10 and the OP227 each was used as a dual opamp in the first stage of the instrumentation amplifier. For the second stage an OP05 opamp (which has the same noise specifications as an OP10) was employed. To reveal the current noise of the two opamps source resistances of $0k\Omega, 1k\Omega, 10k\Omega$ and $100k\Omega$ were connected to the inputs of the instrumentation amplifier. For each source resistance and opamp the output signals V_{o1}, V_{o2} and V_{o3} were digitally recorded. Further amplification of the noise signals was provided by custom built DREP post amplifiers, which include antialiasing filters at $390Hz$. During the recording of these results proper grounding practices for low noise measurements must be followed [3] and the system must be adequately shielded from external electromagnetic radiation. Working at a remote site where the background radiation is small and a solid electrical ground exists is advantageous. For each measurement 20 minutes of data were collected at a sampling rate of 1000 samples per second. To compute the PSD of the signals, the data were split into 100 second intervals (providing $10^{-2}Hz$ frequency resolution) with 50% overlap. For each interval the data were padded to $2^{19} = 524288$ points and the PSD computed and averaged. The results were displayed using a logarithmic frequency axis. To limit the number of data points the PSD was logarithmically subsampled at high frequencies (see Figures 8 and 9). For each graph are plotted

$$\check{P}_{V_{o3}} - \text{measured and modelled root spectral density of the output } V_{o3},$$
$$\check{P}_{V_{od}} - \text{measured and modelled root spectral density of the difference of } V_{o1} \text{ and } V_{o2},$$
$$\text{and } \check{P}_{V_{in}} - \text{modelled root spectral density of the noise referred to the input.}$$

The modelled results are plotted as shaded regions representing the typical to maximum expected values of the noise. Note that it is not possible to measure the voltage and current noise directly.

From these results observe that for small source impedances and frequencies above $10Hz$, the OP227 is quieter. However, when the source resistances are increased and the frequencies of interest are below $10Hz$, the OP10 is quieter. In addition, from the graphs with the OP227 the measured results at low frequencies tend to suggest that the voltage and current noise of the OP227 may be better modelled using a $1/f^\alpha$ noise model with $\alpha > 1$.

In keeping with the method of quoting noise used by opamp manufacturers, Table 2 lists the modelled combined voltage noise of the opamps (based on the manufacturer's specifications) and the combined voltage noise of the instrumentation amplifier using the two opamp types. As a single opamp the OP227 is about a factor of three quieter than the OP10. However when used in the instrumentation amplifier configuration this improvement is not realized. To shed light on why, and to further understand the results, a more detailed look at Equation 83 is required. The equation is composed of eight terms that are dependent on $P_{V'_{n1}}, P_{I_{n1}}, P_{V_{Z1}}, P_{V_{Z2}}, P_{V'_{n5}}, P_{I_{n5}}, P_{V_{Z4}}$ and $P_{V_{Z5}}$. In Figure 10 these components are numbered from 1–8 respectively. The first four components

17

Figure 8: Noise Measurements – modelled and measured noise at the first and second stage outputs along with the modelled equivalent input noise for an OP10 and OP227 front end with source impedances of 0Ω and $1k\Omega$.
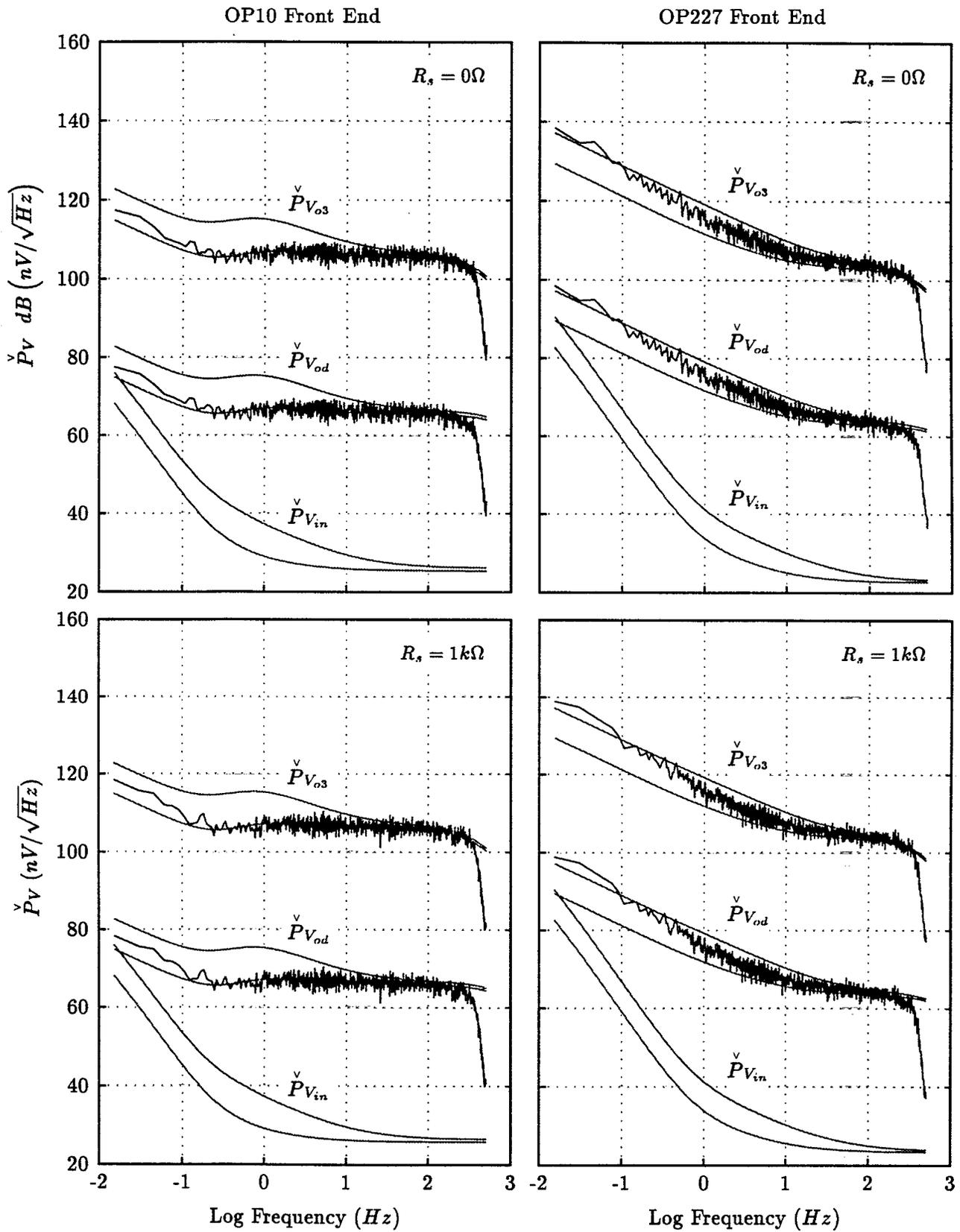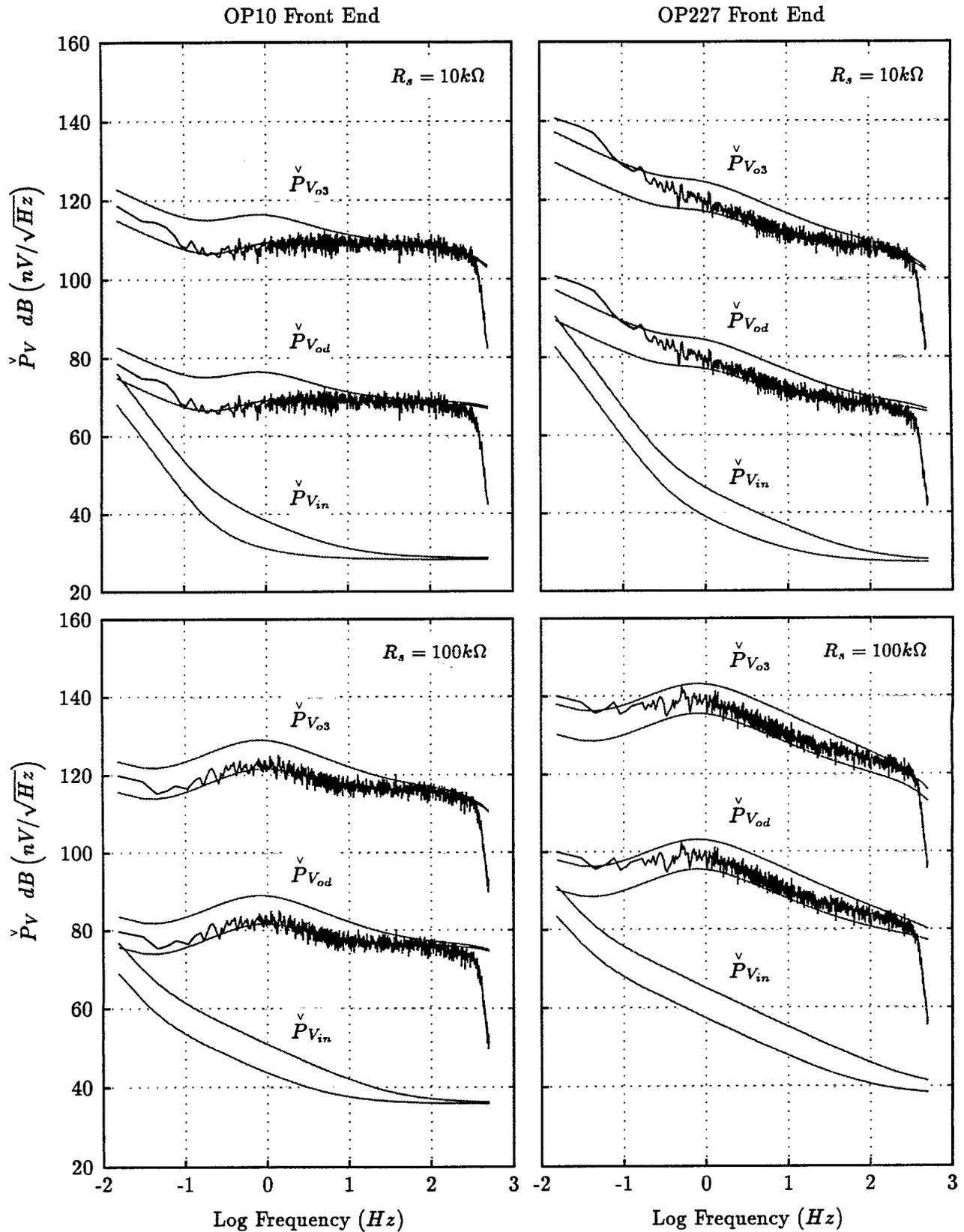
18

Figure 9: Noise Measurements – modelled and measured noise at the first and second stage outputs along with the modelled equivalent input noise for an OP10 and OP227 front end with source impedances of $10k\Omega$ and $100k\Omega$.

19

Table 2: Combined voltage noise comparisons of a single OP10 and OP227 opamp with the instrumentation amplifier with an OP10 and OP227 front end.

Combined Voltage Noise $\overset{\vee}{P}_{V_N'}$ $\left(nV/\sqrt{Hz}\right)$				
	Single Opamp		Instrumentation Amplifier	
$f\,(Hz)$	OP10	OP227	OP10	OP227
0.1	44.3 (151)	184 (474)	17.4 (40.7)	915 (2230)
1	16.7 (48.7)	28.2 (73.3)	6.2 (13.4)	48.8 (115)
10	10.5 (18.6)	19.6 (29.7)	3.5 (5.5)	17.9 (31.8)
100	9.7 (11.9)	18.7 (21.2)	3.0 (4.0)	14.0 (16.7)
1000	9.6 (11.1)	18.5 (20.2)	3.0 (3.8)	13.6 (14.3)

are from the first stage and are drawn with dash-dot lines, the last four components are from the second stage and are plotted with dotted lines. The noise from the second stage is the same for both graphs. Also plotted are the overall combined voltage using a solid line and the approximation of Equation 84 with a dash line. First note that the accuracy of the approximation is frequency dependent. At low frequencies ($< 10^{-2} Hz$) and high frequencies ($> 10^3 Hz$), Z_2 is no longer much greater than Z_1. Also the terms due to $P_{V_{n1}'}$, $P_{I_{n1}}$ and $P_{V_{Z1}}$, that is lines 1,2 and 3, are the major contributors to the total result, consistent with the approximation. It can also be seen that $P_{I_{n1}}$ dominates at low frequencies ($< 1 Hz$ for the OP10 and $< 10 Hz$ for the OP227). And in this region the voltage noise is proportional to $1/f^3$ because of the ac coupling provided by Z_1. This effect is removed at the outputs by the transfer function of the first stage. Lastly these plots reveal that the OP227 does not achieve the expected noise performance because the thermal noise of Z_1 dominates in the region from $10^2 - 10^4 Hz$.

To achieve the minimum noise, that is

$$P_{V_N'} = 2P_{V_{n1}'}, \tag{85}$$

the impedance of Z_1 must be lowered so that both its equivalent thermal noise and product with the opamp current noise are negligible in comparison to the opamp voltage noise. This could be achieved by decreasing all the resistors by a common factor. However to keep the same bandwidth the capacitors would have to be increased by the same factor. Naturally there are limitations to both these changes because the resistor values will become too small and the physical size of capacitors becomes a problem as their value is increased. For example, for the OP227 configuration a factor of 100 would be required to reduce the current noise term at $10^{-2} Hz$ to the same level as the voltage noise. This would shrink R_1 to 100Ω and increase C_1 to $2000\mu F$, an exceptionally large capacitance. Compromising by using a factor of 10 would still require C_1 to be $200\mu F$.

20
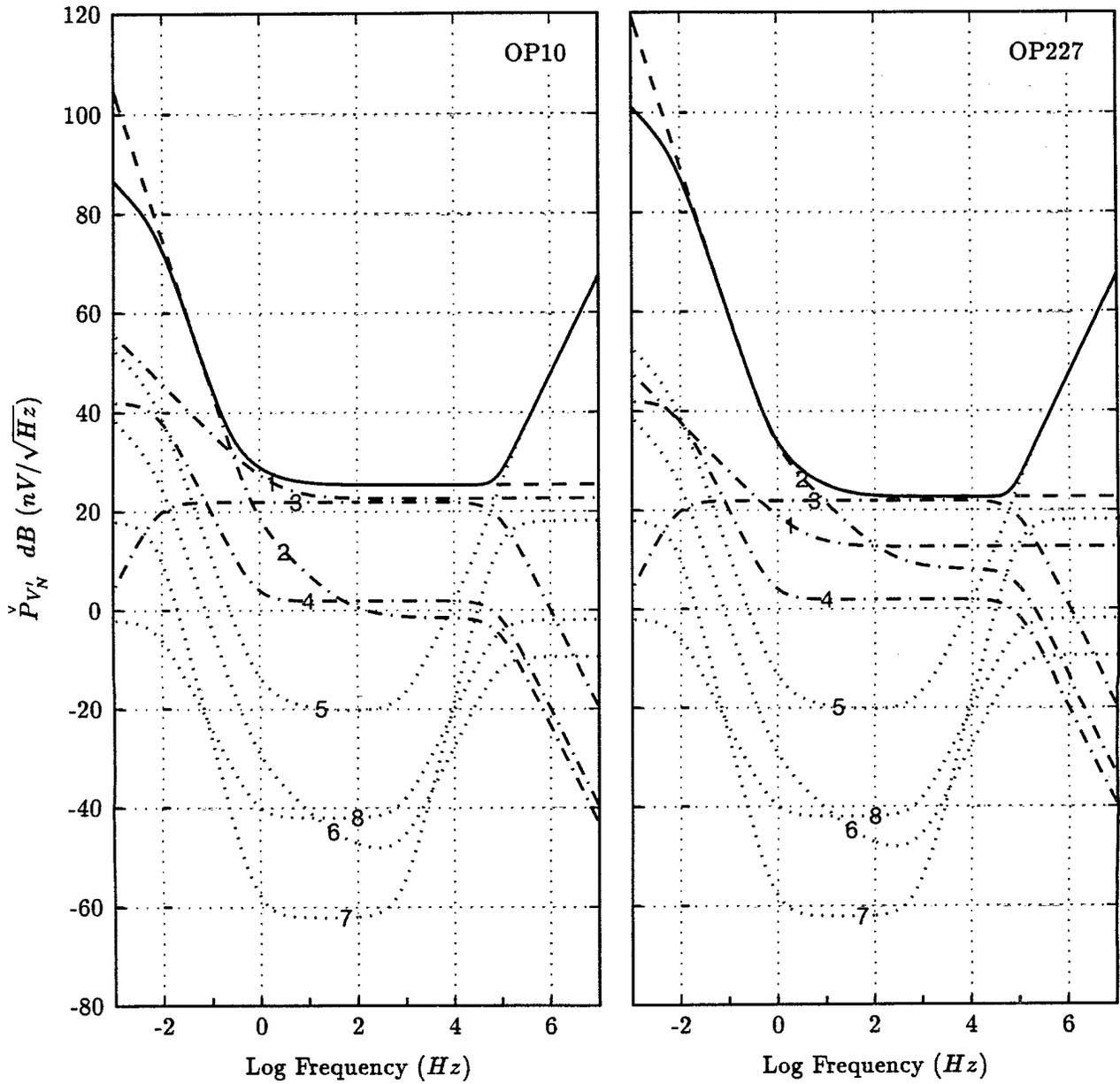
Figure 10: Breakdown of the combined voltage noise of the instrumentation amplifier with an OP10 and OP227 used in the first stage. The solid line is the combined voltage, the dashed line is the approximation, the dash-dot lines numbered 1–4 are the noise from the first stage and the dotted lines numbered 5–8 are the noise from the second stage.

21

5   Summary

A noise analysis of an instrumentation amplifier has been presented and measurements to verify the analysis given. MAPLE routines that determine the noise parameters of DISO and DIDO amplifiers have also been developed. From the analysis a simple and concise expression for the voltage noise of the instrumentation amplifier has been found which can be used when designing a low noise instrumentation amplifier. Trade-offs between satisfying the gain and bandwidth using practical components must be balanced against noise performance.

References

[1] C.D. Motchenbacher and F.C. Fitchen. *Low-Noise Electronic Design*. John Wiley and Sons Inc., Toronto, Ont., 1973.

[2] C.D. Motchenbacher and J.A. Connelly. *Low-Noise Electronic System Design*. John Wiley and Sons Inc., Toronto, Ont., 1993.

[3] H.W. Ott. *Noise Reduction Techniques in Electronic Systems*. John Wiley and Sons Inc., Toronto, Ont., 1988.

[4] J.G. Graeme, G.E. Tobey, and L.P. Huelsman, editors. *Operational Amplifers Design and Applications*. The Burr-Brown Electronics Series. McGraw-Hill Book Company, Toronto, Ont., 1971.

[5] J.G. Graeme. *Applications of Operational Amplifers Third Generation Techniques*. The Burr-Brown Electronics Series. McGraw-Hill Book Company, Toronto, Ont., 1971.

[6] P. Horowitz and W. Hill. *The Art of Electronics*. Cambridge University Press, New York, N.Y., 1989.

[7] B.M. Oliver and J.M. Cage. *Electronic Measurements and Instrumentation*. McGraw-Hill Book Company, Toronto, Ont., 1971.

[8] N. Netzer. The design of low-noise amplifiers. *Proceedings of the IEEE*, 69(6):728–741, June 1981.

[9] J.S. Bendat and A.G. Piersol. *Random Data: Analysis and Measurement Procedures*. John Wiley and Sons Inc., Toronto, Ont., 1971.

[10] Precision Monolithics Inc., Santa Clara,CA. *Analog Integrated Circuits Data Book*, 1990.

[11] L.P. Dennis. Differential amplifier input impedance. *Proceedings of the IEEE*, 58:1288–1289, August 1970.

A   Maple Listings

This appendix provides Maple listings for the noise analysis of the instrumentation amplifier. Maple provides the means to perform difficult algebraic manipulations, with computer efficiency. The worksheets shown are for the first and second stage of the instrumentation amplifier and the cascade connection of the two stages. The last listing is of the macros that were developed to compute the noise models' parameters as described in Section 2.

Listing A.1: STAGE2.MS

Second Stage of Instrumentation Amplifier

```
> restart; read '/noise/noiselib.mpl';
```

```
> eq1 := (Vi3 - Vn5 - VZ4 - V1 - VZs1)/(Zs1 + Z4) + (Vi3 - Vn5 + VZ5 - Vo3)/Z5 = In5;
```

$$eq1 := \frac{Vi3 - Vn5 - VZ4 - V1 - VZs1}{Zs1 + Z4} + \frac{Vi3 - Vn5 + VZ5 - Vo3}{Z5} = In5$$

```
> eq2 := (Vi3 - Vn6 - VZ6 - V2 - VZs2)/(Zs2 + Z6) + (Vi3 - Vn6 + VZ7)/Z7 = In6;
```

$$eq2 := \frac{Vi3 - Vn6 - VZ6 - V2 - VZs2}{Zs2 + Z6} + \frac{Vi3 - Vn6 + VZ7}{Z7} = In6$$

```
> soln := solve({eq1,eq2},{Vo3,Vi3}): assign(soln):
```

```
> vars := [V1,V2,Vd,Vc,Vn5,Vn6,In5,In6,VZs1,VZs2,VZ4,VZ5,VZ6,VZ7]:
```

```
> Pvars := PPvars(vars):
```

```
> Vo3 := ff(Vo3,vars);
```

$$Vo3 := -\frac{Z5\,V1}{Zs1 + Z4} + \frac{Z7\,(Z5 + Zs1 + Z4)\,V2}{(Z7 + Zs2 + Z6)(Zs1 + Z4)} - \frac{(Z5 + Zs1 + Z4)\,Vn5}{Zs1 + Z4}$$

$$+ \frac{(Z5 + Zs1 + Z4)\,Vn6}{Zs1 + Z4} - In5\,Z5 + \frac{Z7\,(Zs2 + Z6)(Z5 + Zs1 + Z4)\,In6}{(Z7 + Zs2 + Z6)(Zs1 + Z4)}$$

$$- \frac{Z5\,VZs1}{Zs1 + Z4} + \frac{Z7\,(Z5 + Zs1 + Z4)\,VZs2}{(Z7 + Zs2 + Z6)(Zs1 + Z4)} - \frac{Z5\,VZ4}{Zs1 + Z4} + VZ5$$

$$+ \frac{Z7\,(Z5 + Zs1 + Z4)\,VZ6}{(Z7 + Zs2 + Z6)(Zs1 + Z4)} - \frac{(Zs2 + Z6)(Z5 + Zs1 + Z4)\,VZ7}{(Z7 + Zs2 + Z6)(Zs1 + Z4)}$$

```
> conditions := [Z6=Z4,Z7=Z5]:
```

```
> DIFFERTAB := disomod('Vo3',vars,conditions):
```

```
> conditions := [conditions[],PVn5=PVn5_/2,PVn6=PVn5_/2,PIn6=PIn5,PVZ6=PVZ4,PVZ7=PVZ5,Z
> s2=Zs1,PVZs2=PVZs1]:
```

```
> DIFFERTAB := ggtab(subs(conditions,eval(DIFFERTAB)),Pvars);
```

$$DIFFERTAB := table([$$

$$Kd = -\frac{Z5}{Zs1 + Z4}$$

$$PIN = \frac{1}{2}\frac{PVn5_-}{|Z5|^2} + PIn5 + \frac{PVZ5}{|Z5|^2}$$

$$PVod = \left|\frac{Z5}{Zs1 + Z4}\right|^2 PVd + \left|\frac{Z5 + Zs1 + Z4}{Zs1 + Z4}\right|^2 PVn5_- + 2\,|Z5|^2\,PIn5$$

$$+ 2\left|\frac{Z5}{Zs1 + Z4}\right|^2 PVZs1 + 2\left|\frac{Z5}{Zs1 + Z4}\right|^2 PVZ4 + 2\,PVZ5$$

$$PVid = PVd + \left|\frac{Z5 + Zs1 + Z4}{Z5}\right|^2 PVn5_- + 8\left|\frac{1}{2}Zs1 + \frac{1}{2}Z4\right|^2 PIn5$$

$$+ 2\,PVZs1 + 2\,PVZ4 + 2\left|\frac{Zs1 + Z4}{Z5}\right|^2 PVZ5$$

$$PVN_- = \left|\frac{Z5 + Z4}{Z5}\right|^2 PVn5_- + 2\,|Z4|^2\,PIn5 + 2\,PVZ4 + 2\left|\frac{Z4}{Z5}\right|^2 PVZ5$$

$$A1 = -\frac{Z5}{Z4}$$

25

$$A2 = \frac{Z5}{Z4}$$

$$Zo1 = 0$$

$$])$$

```
> DIFFERTAB := ggtab(subs(Z4+Z5=Z5,Zs1+Z4+Z5=Z5,Zs1+Z4=Z4,eval(DIFFERTAB)),Pvars);
```

$$DIFFERTAB := \text{table}([$$

$$Kd = -\frac{Z5}{Z4}$$

$$PIN = \frac{1}{2}\frac{PVn5_-}{|Z5|^2} + PIn5 + \frac{PVZ5}{|Z5|^2}$$

$$PVod = \left|\frac{Z5}{Z4}\right|^2 PVd + \left|\frac{Z5}{Z4}\right|^2 PVn5_- + 2\,|Z5|^2\,PIn5 + 2\left|\frac{Z5}{Z4}\right|^2 PVZs1$$

$$+ 2\left|\frac{Z5}{Z4}\right|^2 PVZ4 + 2\,PVZ5$$

$$PVid = PVd + PVn5_- + 8\left|\frac{1}{2}Zs1 + \frac{1}{2}Z4\right|^2 PIn5 + 2\,PVZs1 + 2\,PVZ4$$

$$+ 2\left|\frac{Z4}{Z5}\right|^2 PVZ5$$

$$PVN_- = PVn5_- + 2\,|Z4|^2\,PIn5 + 2\,PVZ4 + 2\left|\frac{Z4}{Z5}\right|^2 PVZ5$$

$$A1 = -\frac{Z5}{Z4}$$

$$A2 = \frac{Z5}{Z4}$$

$$Zo1 = 0$$

$$])$$

```
>
```

26

Listing A.2: STAGE1.MS

Front End of Instrumentation Amplifer

```
> restart; read '/noise/noiselib.mpl';
```

```
> eq1 := (Vi1 - Vn1 - V1 - VZs1)/Zs1 = In1;
```
$$eq1 := \frac{Vi1 - Vn1 - V1 - VZs1}{Zs1} = In1$$

```
> eq2 := (Vi1 - Vn2 + VZ2 - Vo1)/Z2 + (Vi1 - Vn2 - VZ1 + Vn4 - Vi2)/Z1 = In2;
```
$$eq2 := \frac{Vi1 - Vn2 + VZ2 - Vo1}{Z2} + \frac{Vi1 - Vn2 - VZ1 + Vn4 - Vi2}{Z1} = In2$$

```
> eq3 := (Vi2 - Vn3 - V2 - VZs2)/Zs2 = In3;
```
$$eq3 := \frac{Vi2 - Vn3 - V2 - VZs2}{Zs2} = In3$$

```
> eq4 := (Vi2 - Vn4 - VZ3 - Vo2)/Z3 + (Vi2 - Vn4 + VZ1 + Vn2 - Vi1)/Z1 = In4;
```
$$eq4 := \frac{Vi2 - Vn4 - VZ3 - Vo2}{Z3} + \frac{Vi2 - Vn4 + VZ1 + Vn2 - Vi1}{Z1} = In4$$

```
> soln := solve({eq1,eq2,eq3,eq4},{Vo1,Vo2,Vi1,Vi2}): assign(soln):
```
```
> vars := [V1,V2,Vd,Vc,Vn1,Vn2,Vn3,Vn4,In1,In2,In3,In4,VZs1,VZs2,VZ1,VZ2,VZ3]:
```
```
> Pvars := PPvars(vars):
```
```
> Vo1 := ff(Vo1,vars);
```
$$Vo1 := \frac{(Z1 + Z2)\,V1}{Z1} - \frac{Z2\,V2}{Z1} + \frac{(Z1 + Z2)\,Vn1}{Z1} - \frac{(Z1 + Z2)\,Vn2}{Z1} - \frac{Z2\,Vn3}{Z1}$$
$$+ \frac{Z2\,Vn4}{Z1} + \frac{Zs1\,(Z1 + Z2)\,In1}{Z1} - Z2\,In2 - \frac{Z2\,Zs2\,In3}{Z1} + \frac{(Z1 + Z2)\,VZs1}{Z1}$$
$$- \frac{Z2\,VZs2}{Z1} - \frac{Z2\,VZ1}{Z1} + VZ2$$

```
> Vo2 := ff(Vo2,vars);
```
$$Vo2 := -\frac{Z3\,V1}{Z1} + \frac{(Z3 + Z1)\,V2}{Z1} - \frac{Z3\,Vn1}{Z1} + \frac{Z3\,Vn2}{Z1} + \frac{(Z3 + Z1)\,Vn3}{Z1}$$
$$- \frac{(Z3 + Z1)\,Vn4}{Z1} - \frac{Z3\,Zs1\,In1}{Z1} + \frac{Zs2\,(Z3 + Z1)\,In3}{Z1} - Z3\,In4 - \frac{Z3\,VZs1}{Z1}$$
$$+ \frac{(Z3 + Z1)\,VZs2}{Z1} + \frac{Z3\,VZ1}{Z1} - VZ3$$

```
> conditions:=[Z3=Z2]:
```
```
> INAMPFTAB := didomod('Vo1','Vo2',vars,conditions):
```
```
> conditions:=[conditions[],PVn1=PVn1_/2,PVn2=PVn1_/2,PVn3=PVn1_/2,PVn4=PVn1_/2,PIn2=PI
> n1,PIn3=PIn1,PIn4=PIn1,PVZ3=PVZ2,Z3=Z2,PVZs2=PVZs1,Zs2=Zs1]:
```
```
> INAMPFTAB := ggtab(subs(conditions,eval(INAMPFTAB)),Pvars);
```
$$INAMPFTAB := table([$$

$$Zo1 = 0$$

$$PIN = PIn1$$

$$PVod = 2\left|\frac{Z1 + 2\,Z2}{Z1}\right|^2 PVn1_- + 2\,PVZ2 + 2\left|\frac{Z1 + 2\,Z2}{Z1}\right|^2 PVZs1$$

$$+ \left(2\,|Z2|^2 + 2\left|\frac{Zs1\,(Z1 + 2\,Z2)}{Z1}\right|^2\right)PIn1 + 4\left|\frac{Z2}{Z1}\right|^2 PVZ1$$

27

$$+ \left| \frac{Z1 + 2\,Z2}{Z1} \right|^2 PVd$$

$$PVid = 2\,PVn1_- + 2 \left| \frac{Z1}{Z1 + 2\,Z2} \right|^2 PVZ2 + 2\,PVZs1$$

$$+ \left(2 \left| \frac{Z2\,Z1}{Z1 + 2\,Z2} \right|^2 + 2\,|Zs1|^2 \right) PIn1 + 4 \left| \frac{Z2}{Z1 + 2\,Z2} \right|^2 PVZ1 + PVd$$

$$PVN_- = 2\,PVn1_- + 2 \left| \frac{Z2\,Z1}{Z1 + 2\,Z2} \right|^2 PIn1 + 4 \left| \frac{Z2}{Z1 + 2\,Z2} \right|^2 PVZ1$$

$$+ 2 \left| \frac{Z1}{Z1 + 2\,Z2} \right|^2 PVZ2$$

$$Kdd = \frac{Z1 + 2\,Z2}{Z1}$$

$$Zo2 = 0$$

$$A11 = \frac{Z1 + Z2}{Z1}$$

$$A21 = -\frac{Z2}{Z1}$$

$$A12 = -\frac{Z2}{Z1}$$

$$A22 = \frac{Z1 + Z2}{Z1}$$

$$])$$

```
> INAMPFTAB := ggtab(subs(Z1+2*Z2=2*Z2,Z1+Z2=Z2,eval(INAMPFTAB)),Pvars);
```

$$INAMPFTAB := \text{table}([$$

$$Zo1 = 0$$

$$PIN = PIn1$$

$$PVod = 8 \left| \frac{Z2}{Z1} \right|^2 PVn1_- + 2\,PVZ2 + 8 \left| \frac{Z2}{Z1} \right|^2 PVZs1$$

$$+ \left(2\,|Z2|^2 + 8 \left| \frac{Zs1\,Z2}{Z1} \right|^2 \right) PIn1 + 4 \left| \frac{Z2}{Z1} \right|^2 PVZ1 + 4 \left| \frac{Z2}{Z1} \right|^2 PVd$$

$$PVid = 2\,PVn1_- + \frac{1}{2} \left| \frac{Z1}{Z2} \right|^2 PVZ2 + 2\,PVZs1 + \left(\frac{1}{2}\,|Z1|^2 + 2\,|Zs1|^2 \right) PIn1$$

$$+ PVZ1 + PVd$$

$$PVN_- = 2\,PVn1_- + \frac{1}{2}\,PIn1\,|Z1|^2 + PVZ1 + \frac{1}{2} \left| \frac{Z1}{Z2} \right|^2 PVZ2$$

$$Kdd = 2\,\frac{Z2}{Z1}$$

$$Zo2 = 0$$

$$A11 = \frac{Z2}{Z1}$$

$$A21 = -\frac{Z2}{Z1}$$

$$A12 = -\frac{Z2}{Z1}$$

$$A22 = \frac{Z2}{Z1}$$

])

Listing A.3: INAMP.MS

Instrumentation Amplifer

```
> restart; read '/noise/noiselib.mpl';
```

```
> eq1 := (Vi1 - Vn1 - V1 - VZs1)/Zs1 = In1;
```

$$eq1 := \frac{Vi1 - Vn1 - V1 - VZs1}{Zs1} = In1$$

```
> eq2 := (Vi1 - Vn2 + VZ2 - Vo1)/Z2 + (Vi1 - Vn2 - VZ1 + Vn4 - Vi2)/Z1 = In2;
```

$$eq2 := \frac{Vi1 - Vn2 + VZ2 - Vo1}{Z2} + \frac{Vi1 - Vn2 - VZ1 + Vn4 - Vi2}{Z1} = In2$$

```
> eq3 := (Vi2 - Vn3 - V2 - VZs2)/Zs2 = In3;
```

$$eq3 := \frac{Vi2 - Vn3 - V2 - VZs2}{Zs2} = In3$$

```
> eq4 := (Vi2 - Vn4 + VZ3 - Vo2)/Z3 + (Vi2 - Vn4 + VZ1 + Vn2 - Vi1)/Z1 = In4;
```

$$eq4 := \frac{Vi2 - Vn4 + VZ3 - Vo2}{Z3} + \frac{Vi2 - Vn4 + VZ1 + Vn2 - Vi1}{Z1} = In4$$

```
> eq5 := (Vi3 - Vn5 - VZ6 - Vo2)/Z6 + (Vi3 - Vn5 + VZ7)/Z7 = In5;
```

$$eq5 := \frac{Vi3 - Vn5 - VZ6 - Vo2}{Z6} + \frac{Vi3 - Vn5 + VZ7}{Z7} = In5$$

```
> eq6 := (Vi3 - Vn6 - VZ4 - Vo1)/Z4 + (Vi3 - Vn6 + VZ5 - Vo3)/Z5 = In6;
```

$$eq6 := \frac{Vi3 - Vn6 - VZ4 - Vo1}{Z4} + \frac{Vi3 - Vn6 + VZ5 - Vo3}{Z5} = In6$$

```
> vars:=[V1,V2,Vd,Vc,Vn1,Vn2,Vn3,Vn4,Vn5,Vn6,In1,In2,In3,In4,In5,In6,VZs1,VZs2,VZ1,VZ2,VZ3,V
> Z4,VZ5,VZ6,VZ7]:
> Pvars := PPvars(vars):
> soln := solve({eq1,eq2,eq3,eq4,eq5,eq6},{Vo1,Vo2,Vo3,Vi1,Vi2,Vi3}): assign(soln):
> Vo3 := ff(Vo3,vars);
```

$$Vo3 := -\frac{\%2\,V1}{Z1\,Z4\,(Z7+Z6)} + \frac{\%1\,V2}{Z1\,Z4\,(Z7+Z6)} - \frac{\%2\,Vn1}{Z1\,Z4\,(Z7+Z6)}$$

$$+ \frac{\%2\,Vn2}{Z1\,Z4\,(Z7+Z6)} + \frac{\%1\,Vn3}{Z1\,Z4\,(Z7+Z6)} - \frac{\%1\,Vn4}{Z1\,Z4\,(Z7+Z6)}$$

$$+ \frac{(Z5+Z4)\,Vn5}{Z4} - \frac{(Z5+Z4)\,Vn6}{Z4} - \frac{Zs1\,\%2\,In1}{Z1\,Z4\,(Z7+Z6)} + \frac{Z5\,Z2\,In2}{Z4}$$

$$+ \frac{Zs2\,\%1\,In3}{Z1\,Z4\,(Z7+Z6)} - \frac{Z7\,Z3\,(Z5+Z4)\,In4}{Z4\,(Z7+Z6)} + \frac{Z6\,Z7\,(Z5+Z4)\,In5}{Z4\,(Z7+Z6)}$$

$$- Z5\,In6 - \frac{\%2\,VZs1}{Z1\,Z4\,(Z7+Z6)} + \frac{\%1\,VZs2}{Z1\,Z4\,(Z7+Z6)}$$

$$+ \frac{(Z4\,Z7\,Z3 + Z5\,Z7\,Z3 + Z6\,Z5\,Z2 + Z7\,Z5\,Z2)\,VZ1}{Z1\,Z4\,(Z7+Z6)} - \frac{Z5\,VZ2}{Z4}$$

$$+ \frac{Z7\,(Z5+Z4)\,VZ3}{Z4\,(Z7+Z6)} - \frac{Z5\,VZ4}{Z4} + VZ5 + \frac{Z7\,(Z5+Z4)\,VZ6}{Z4\,(Z7+Z6)}$$

$$- \frac{Z6\,(Z5+Z4)\,VZ7}{Z4\,(Z7+Z6)}$$

$$\%1 := Z7\,Z1\,Z4 + Z4\,Z7\,Z3 + Z5\,Z7\,Z1 + Z5\,Z7\,Z3 + Z6\,Z5\,Z2 + Z7\,Z5\,Z2$$

$$\%2 := Z4\,Z7\,Z3 + Z5\,Z7\,Z3 + Z6\,Z5\,Z1 + Z6\,Z5\,Z2 + Z7\,Z5\,Z2 + Z5\,Z7\,Z1$$

30

```
> conditions := [Z3=Z2,Z6=Z4,Z7=Z5]:
> INAMPTAB := disomod('Vo3',vars,conditions):
> conditions := [conditions[],Zs2=Zs1,PVZs2=PVZs1,PVn1=PVn1_/2,PVn2=PVn1_/2,PVn3=PVn1_
> /2,PVn4=PVn1_/2,PVn5=PVn5_/2,PVn6=PVn5_/2,PIn2=PIn1,PIn3=PIn1,PIn4=PIn1,PIn6=PIn5,PV
> Z3=PVZ2,PVZ6=PVZ4,PVZ7=PVZ5]:
> INAMPTAB := ggtab(subs(conditions,eval(INAMPTAB)),Pvars);
```

$INAMPTAB := \text{table}([$

$$PVid = 2\,PVn1_- + \left|\frac{(Z5+Z4)\,Z1}{(2\,Z2+Z1)\,Z5}\right|^2 PVn5_- + 4\left|\frac{Z2}{2\,Z2+Z1}\right|^2 PVZ1$$

$$+ 2\left|\frac{Z1\,Z4}{Z5\,(2\,Z2+Z1)}\right|^2 PVZ5 + 2\left|\frac{Z1}{2\,Z2+Z1}\right|^2 PVZ2$$

$$+ \left(2\left|\frac{Z2\,Z1}{2\,Z2+Z1}\right|^2 + 2\,|Zs1|^2\right) PIn1 + 2\left|\frac{Z1\,Z4}{2\,Z2+Z1}\right|^2 PIn5 + 2\,PVZs1$$

$$+ 2\left|\frac{Z1}{2\,Z2+Z1}\right|^2 PVZ4 + PVd$$

$$A1 = -\frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}$$

$$A2 = \frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}$$

$$Kd = -\frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}$$

$$PVod = 2\left|\frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}\right|^2 PVn1_- + \left|\frac{Z5+Z4}{Z4}\right|^2 PVn5_-$$

$$+ 4\left|\frac{Z5\,Z2}{Z4\,Z1}\right|^2 PVZ1 + 2\,PVZ5 + 2\left|\frac{Z5}{Z4}\right|^2 PVZ2$$

$$+ \left(2\left|\frac{Z5\,Z2}{Z4}\right|^2 + 2\left|\frac{(2\,Z2+Z1)\,Z5\,Zs1}{Z4\,Z1}\right|^2\right) PIn1 + 2\,|Z5|^2\,PIn5$$

$$+ 2\left|\frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}\right|^2 PVZs1 + 2\left|\frac{Z5}{Z4}\right|^2 PVZ4 + \left|\frac{(2\,Z2+Z1)\,Z5}{Z4\,Z1}\right|^2 PVd$$

$$PIN = PIn1$$

$$Zo1 = 0$$

$$PVN_- = 2\,PVn1_- + \left|\frac{(Z5+Z4)\,Z1}{(2\,Z2+Z1)\,Z5}\right|^2 PVn5_- + 2\left|\frac{Z2\,Z1}{2\,Z2+Z1}\right|^2 PIn1$$

$$+ 2\left|\frac{Z1\,Z4}{2\,Z2+Z1}\right|^2 PIn5 + 4\left|\frac{Z2}{2\,Z2+Z1}\right|^2 PVZ1 + 2\left|\frac{Z1}{2\,Z2+Z1}\right|^2 PVZ2$$

$$+ 2\left|\frac{Z1}{2\,Z2+Z1}\right|^2 PVZ4 + 2\left|\frac{Z1\,Z4}{Z5\,(2\,Z2+Z1)}\right|^2 PVZ5$$

$])$

31

```
> INAMPTAB := ggtab(subs(Z1+2*Z2=2*Z2,Z4+Z5=Z5,eval(INAMPTAB)),Pvars);
```

$INAMPTAB := \text{table}([$

$$PVid = 2\,PVn1_{-} + \frac{1}{4}\left|\frac{Z1}{Z2}\right|^2 PVn5_{-} + PVZ1 + \frac{1}{2}\left|\frac{Z1\,Z4}{Z5\,Z2}\right|^2 PVZ5$$

$$+ \frac{1}{2}\left|\frac{Z1}{Z2}\right|^2 PVZ2 + \left(\frac{1}{2}\,|Z1|^2 + 2\,|Zs1|^2\right) PIn1 + \frac{1}{2}\left|\frac{Z1\,Z4}{Z2}\right|^2 PIn5 + 2\,PVZs1$$

$$+ \frac{1}{2}\left|\frac{Z1}{Z2}\right|^2 PVZ4 + PVd$$

$$A1 = -2\,\frac{Z5\,Z2}{Z4\,Z1}$$

$$A2 = 2\,\frac{Z5\,Z2}{Z4\,Z1}$$

$$Kd = -2\,\frac{Z5\,Z2}{Z4\,Z1}$$

$$PVod = 8\left|\frac{Z5\,Z2}{Z4\,Z1}\right|^2 PVn1_{-} + \left|\frac{Z5}{Z4}\right|^2 PVn5_{-} + 4\left|\frac{Z5\,Z2}{Z4\,Z1}\right|^2 PVZ1 + 2\,PVZ5$$

$$+ 2\left|\frac{Z5}{Z4}\right|^2 PVZ2 + \left(2\left|\frac{Z5\,Z2}{Z4}\right|^2 + 8\left|\frac{Z2\,Z5\,Zs1}{Z4\,Z1}\right|^2\right) PIn1 + 2\,|Z5|^2\,PIn5$$

$$+ 8\left|\frac{Z5\,Z2}{Z4\,Z1}\right|^2 PVZs1 + 2\left|\frac{Z5}{Z4}\right|^2 PVZ4 + 4\left|\frac{Z5\,Z2}{Z4\,Z1}\right|^2 PVd$$

$$PIN = PIn1$$

$$Zo1 = 0$$

$$PVN_{-} = 2\,PVn1_{-} + \frac{1}{4}\left|\frac{Z1}{Z2}\right|^2 PVn5_{-} + \frac{1}{2}\,|Z1|^2\,PIn1 + \frac{1}{2}\left|\frac{Z1\,Z4}{Z2}\right|^2 PIn5$$

$$+ PVZ1 + \frac{1}{2}\left|\frac{Z1}{Z2}\right|^2 PVZ2 + \frac{1}{2}\left|\frac{Z1}{Z2}\right|^2 PVZ4 + \frac{1}{2}\left|\frac{Z1\,Z4}{Z5\,Z2}\right|^2 PVZ5$$

$])$

```
>
```

32

Listing A.4: NOISELIB.MPL

```
disomod := proc(Vo1,vars,conditions)
  local reserved,required,unknowns,limits,Vod,Vid,Pvars,K1,K2,temp,conds,IN,VN_;
  global V1,V2,VZs1,VZs2,Vc,Vd,ZL1,Zs1,Zs2,disotab,Zo1,A1,A2,Kd,PVod,PVid,PVN_,PIN;
# Check Inputs
if nargs=0 then ERROR('No arguments ') fi;
if nargs<>3 then ERROR('Three arguments required ') fi;
unknowns := indets(eval(Vo1));
unassign('Zo1','A1','A2','Kd','PVod','PVid','PVN_','PIN');
reserved := {Zo1,A1,A2,Kd,PVod,PVid,PVN_,PIN};
if evalb(nops(reserved intersect unknowns)<>0) then
  print(reserved intersect unknowns);
  ERROR('Vo1 is a function of reserved variables see ?disomodel ');
fi;
required := {V1,V2,Zs1,Zs2,VZs1,VZs2};
if evalb(nops(required intersect unknowns)<>nops(required)) then
  print(required intersect unknowns);
  ERROR('Vo1 must be function of required variables see ?disomodel ')
fi;
# Output Impedance
limits := {ZL1=infinity};
temp := simplify(limit(eval(Vo1),limits));
Zo1 := simplify(ZL1*(temp-eval(Vo1))/eval(Vo1));
# Amplifier Gains
K1 := simplify(coeff(collect(eval(Vo1),V1,recursive,simplify),V1));
K2 := simplify(coeff(collect(eval(Vo1),V2,recursive,simplify),V2));
limits := {Zs1=0,Zs2=0,ZL1=infinity,ZL2=infinity};
A1 := simplify(limit(K1,limits));
A2 := simplify(limit(K2,limits));
#
Kd := simplify((K1-K2)/2);
Vod := ff(subs(V1=Vd/2+Vc,V2=-Vd/2+Vc,eval(Vo1)),vars);
Vid := ff(Vod/Kd,vars);
Pvars := PPvars(vars);  # create list of PSD vars used by gg
PVod := gg(noisesum(Vod,vars),Pvars);
PVid := gg(noisesum(Vid,vars),Pvars);
#
# Voltage and Current Noise
limits := {ZL1=infinity};
conds := [conditions[],Zs2=Zs1];
K1 := limit(K1,limits);
K2 := limit(K2,limits);
K1 := simplify(subs(conds,K1));
K2 := simplify(subs(conds,K2));
# check that conds equate K1 = -K2
if evalb(K1<>-K2) then
  print('K1 = ',K1);
  print('K2 = ',K2);
  ERROR('Conditions given do not set K1 = -K2 see ?didomod ');
fi;
limits := {Vd=0,Vc=0,VZs1=0,VZs2=0,ZL1=infinity,ZL2=infinity};
Vod := ff(limit(Vod,limits),vars);
Vid := ff(subs(conds,Vod)/K1,vars);
VN_ := ff(limit(Vid,Zs1=0),vars);
PVN_ := gg(noisesum(VN_,vars),Pvars);
IN := ff(limit(Vid/Zs1,Zs1=infinity),vars);
PIN := gg(noisesum(IN,vars)/2,Pvars);
# Create table for output
disotab := table();
```

33

```
        disotab['Zo1'] := Zo1;
        disotab['A1'] := A1;
        disotab['A2'] := A2;
        disotab['Kd'] := Kd;
        disotab['PIN'] := PIN;
        disotab['PVN_'] := PVN_;
        disotab['PVid'] := PVid;
        disotab['PVod'] := PVod;
        unassign('Zo1','A1','A2','Kd','PVod','PVid','PVN_','PIN');
        eval(disotab);
        end:
        'help/text/disomod' := TEXT(
        'HELP FOR: disomod ',
        'CALLING SEQUENCE:  disotab := disomod('Vo1',vars,conditions) ',
        '  Determines the noise parameters of a diff input - single output amplifier',
        '  ERROR if Vo is a function of Zo1,A1,A2,PVN_,PIN',
        '  ERROR if Vo is not a of V1,V2,Zs1,Zs2,VZs1,VZs2',
        '  returns Zo1,A1,A2,Kd,PVod,PVid,PVN_,PIN',
        '  conditions - conditions for which A1 = -A2'
        ):
        #
        didomod := proc(Vo1,Vo2,vars,conditions)
          local reserved,required,unknowns,limits,Vod,Vid,Pvars,
                K1,K2,K11,K21,K12,K22,temp,conds,A1,A2,IN,VN_;
          global V1,V2,VZs1,VZs2,Vc,Vd,ZL1,ZL2,Zs1,Zs2,
                didotab,Zo1,Zo2,A11,A21,A12,A22,Kdd,PVod,PVid,PVN_,PIN;
        # Check Inputs
        if nargs=0 then ERROR('No arguments ') fi;
        if nargs<>4 then ERROR('Four arguments required ') fi;
        unknowns := indets(eval(Vo1)) union indets(eval(Vo2));
        unassign('Zo1','Zo2','A11','A21','A12','A22','Kdd','PVod','PVid','PVN_','PIN');
        reserved := {Zo1,Zo2,A11,A21,A12,A22,Kdd,PVod,PVid,PVN_,PIN};
        if evalb(nops(reserved intersect unknowns)<>0) then
          print(reserved intersect unknowns);
          ERROR('Vo1 or Vo2 are a function of reserved variables see ?didomodel ');
        fi;
        required := {V1,V2,Zs1,Zs2,VZs1,VZs2};
        if evalb(nops(required intersect unknowns)<>nops(required)) then
          print(required intersect unknowns);
          ERROR('Vo1 and Vo2 must be function of required variables see ?didomodel ')
        fi;
        # Output Impedances
        limits := {ZL1=infinity};
        temp := simplify(limit(eval(Vo1),limits));
        Zo1 := simplify(ZL1*(temp-eval(Vo1))/eval(Vo1));
        limits := {ZL2=infinity};
        temp := simplify(limit(eval(Vo2),limits));
        Zo2 := simplify(ZL2*(temp-eval(Vo2))/eval(Vo2));
        # Amplifier Gains
        K11 := simplify(coeff(collect(eval(Vo1),V1,recursive,simplify),V1));
        K21 := simplify(coeff(collect(eval(Vo1),V2,recursive,simplify),V2));
        K12 := simplify(coeff(collect(eval(Vo2),V1,recursive,simplify),V1));
        K22 := simplify(coeff(collect(eval(Vo2),V2,recursive,simplify),V2));
        limits := {Zs1=0,Zs2=0,ZL1=infinity,ZL2=infinity};
        A11 := simplify(limit(K11,limits));
        A21 := simplify(limit(K21,limits));
        A12 := simplify(limit(K21,limits));
        A22 := simplify(limit(K22,limits));
        #
```

34

```
K1 := simplify(K11-K12);
K2 := simplify(K21-K22);
Kdd := simplify((K1-K2)/2);
Vod := ff(subs(V1=Vd/2+Vc,V2=-Vd/2+Vc,eval(Vo1)-eval(Vo2)),vars);
Vid := ff(Vod/Kdd,vars);
Pvars := PPvars(vars);  # create list of PSD vars used by gg
PVod := gg(noisesum(Vod,vars),Pvars);
PVid := gg(noisesum(Vid,vars),Pvars);
#
# Voltage and Current Noise
limits := {ZL1=infinity,ZL2=infinity};
conds := [conditions[],Zs2=Zs1,ZL2=ZL1];
K1 := limit(K1,limits);
K2 := limit(K2,limits);
K1 := simplify(subs(conds,K1));
K2 := simplify(subs(conds,K2));
# check that conds equate K1 = -K2
if evalb(K1<>-K2) then
  print('K1 = ',K1);
  print('K2 = ',K2);
  ERROR('Conditions given do not set K1 = -K2 see ?didomod ');
fi;
limits := {Vd=0,Vc=0,VZs1=0,VZs2=0,ZL1=infinity,ZL2=infinity};
Vod := ff(limit(Vod,limits),vars);
Vid := ff(subs(conds,Vod)/K1,vars);
VN_ := ff(limit(Vid,Zs1=0),vars);
PVN_ := gg(noisesum(VN_,vars),Pvars);
IN := ff(limit(Vid/Zs1,Zs1=infinity),vars);
PIN := gg(noisesum(IN,vars)/2,Pvars);
# Create table for output
didotab := table();
didotab['Zo1'] := Zo1;
didotab['Zo2'] := Zo2;
didotab['A11'] := A11;
didotab['A21'] := A21;
didotab['A12'] := A12;
didotab['A22'] := A22;
didotab['Kdd'] := Kdd;
didotab['PIN'] := PIN;
didotab['PVN_'] := PVN_;
didotab['PVid'] := PVid;
didotab['PVod'] := PVod;
unassign('Zo1','Zo2','A11','A21','A12','A22','Kdd','PVod','PVid','PVN_','PIN');
eval(didotab);
end:
'help/text/didomod' := TEXT(
'HELP FOR: didomod ',
'CALLING SEQUENCE:  didotab := didomod('Vo1','Vo2',vars) ',
'  Determines the noise parameters of a diff in - diff output amplifier',
'  ERROR if Vo1 and Vo2 are functions of Zo1,Zo2,A11,A12,A21,A22,PIN,PVN_,PVid,PVod',
'  ERROR if Vo1 and Vo2 are not a function of V1,V2,Zs1,Zs2,VZs1,VZs2',
'  returns Zo1,Zo2,A11,A12,A21,A22,PIN,PVN_,PVid,PVod'
):
#
ff := (x,vars) -> collect(simplify(x),vars,distributed,factor):
#
gg := (x,Pvars) -> collect(simplify(x),Pvars,distributed,factor):
#
ggtab := proc(tabl,Pvars)
```

35

```
    local ind,ent,i,ggtable;    # apply gg to a table
ind := indices(tabl);
ent := entries(tabl);
ggtable := table({seq(op(1,ind[i])=gg(op(1,ent[i]),Pvars),i=1..nops({ind}))});
end:
#
noisesum := proc(V,vars) local dV,nvars,PP,i;    # Compute PSD of V
nvars := nops(vars);
PP := 0;
for i to nvars do
  dV := diff(V,vars[i]);
  dV := simplify(dV/sign(dV));
  PP := PP + abs(dV)^2*cat('P',vars[i]);
od;
end:
#
PPvars := proc(vars)
  local i,Pvars;  # Compute PSD variables from input variables
Pvars := [];
for i to nops(vars) do
  Pvars := [Pvars[],cat('P',op(i,vars))];
od;
end:
```

36

Distribution

Report Number:	DREP Technical Memorandum 94-153
Title:	Noise Analysis of an Instrumentation Amplifier
Author:	T.C. Richards
Date:	March 1995
Security Grading:	UNCLASSIFIED

Initial Distribution

5 — DSIS
1 — DREP/Mr. T.C. Richards
1 — DREP/HEM
1 — DREP/Dr. P. Holtham
1 — DREP/Dr. H. Wilson
1 — DREP/Mr. G. Schattschneider
1 — DREP/Mr. J. Perkins
1 — DREP/Mr. B. Dunstan
1 — DREP/Mr. D. Schneider

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when the overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Establishment sponsoring a contractor's report, or tasking agency, are entered in section 8.) DREP	2. SECURITY CLASSIFICATION (overall security classification of the document, including special warning terms if applicable) UNCLASSIFIED

3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S,C or U) in parentheses after the title.)

Noise Analysis of an Instrumentation Amplifier

4. AUTHORS (Last name, first name, middle initial)

Richards, Troy, C

5. DATE OF PUBLICATION (month and year of publication of document) March 1995	6a. NO. OF PAGES (total containing information. Include Annexes, Appendices, etc.) 41	6b. NO. OF REFS (total cited in document) 11

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

Technical Memorandum

8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.)

Defence Research Establishment Pacific, FMO Victoria, BC V0S 1B0

9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant)	9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)

10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.) Technical Memorandum 94-153	10b. OTHER DOCUMENT NOS. (Any other numbers which may be assigned this document either by the originator or by the sponsor)

11. DOCUMENT AVAILABILITY (any limitations on further dissemination of the document, other than those imposed by security classification)

(X) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; further distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; further distribution only as approved
( ) Distribution limited to government departments and agencies; further distribution only as approved
( ) Distribution limited to defence departments; further distribution only as approved
( ) Other (please specify):

12. DOCUMENT ANNOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availabilty (11). However, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)

SECURITY CLASSIFICATION OF FORM

13. ABSTRACT (a brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both offical languages unless the text is bilingual).

Same as document

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus-identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)
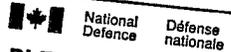
Instrumentation Amplifier
Noise
Maple

SECURITY CLASSIFICATION OF FORM

#151078

NO. OF COPIES NOMBRE DE COPIES	COPY NO. COPIE Nº	INFORMATION SCIENTIST'S INITIALS INITIALES DE L'AGENT D'INFORMATION SCIENTIFIQUE
1	7	JL
AQUISITION ROUTE FOURNI PAR ▶ DRDP		
DATE ▶ 7 Apr 95		
DSIS ACCESSION NO. NUMÉRO DSIS ▶		

DND 1158 (6-87)

▌✶▐ National Défense
Defence nationale

**PLEASE RETURN THIS DOCUMENT
TO THE FOLLOWING ADDRESS:**
DIRECTOR
SCIENTIFIC INFORMATION SERVICES
NATIONAL DEFENCE
HEADQUARTERS
OTTAWA, ONT. - CANADA K1A 0K2

**PRIÈRE DE RETOURNER CE DOCUMENT
À L'ADRESSE SUIVANTE:**
DIRECTEUR
SERVICES D'INFORMATION SCIENTIFIQUES
QUARTIER GÉNÉRAL
DE LA DÉFENSE NATIONALE
OTTAWA, ONT. - CANADA K1A 0K2