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ATMOSPHERIC ENVIRONMENT SERVICE
DEPARTMENT OF THE ENVIRONMENT – CANADA

Technical Memoranda

EXTREME VALUES OF ANNUAL AND MONTHLY SNOWFALL AMOUNTS AT HALIFAX, N.S.

by

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TEC 778
14 SEPTEMBER, 1972

ENVIRONMENT CANADA - ATMOSPHERIC ENVIRONMENT SERVICE
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ABSTRACT

The largest annual snowfalls and largest monthly snowfalls at Halifax for ninety-eight years of record are examined to determine the expected return periods of annual and monthly snowfalls at that city. Theoretical values of the return period were computed using Gumbel's theory of extreme values and agreement between observed and theoretical values was found to be quite good.

VALEURS ANNUELLES ET MENSUELLES EXTREMES DES HAUTEURS DE CHUTE DE NEIGE A HALIFAX (N.-E.)

par

R.V. Tyner

RESUME

Les plus grosses chutes annuelles et mensuelles de neige à Halifax enregistrées au cours d'une période de quatre-vingt dix ans font l'objet d'une étude pour déterminer la pérodicité annuelle et mensuelle des chutes de beige en cet endroit. Les valeurs théoriques de périodicité ont été calculées en utilisant la théorie de Gumbel des valeurs extrêmes. On a découvert que la concordance entre les valeurs observées et les valeurs théoriques était assez exacte.

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(Manuscript received June 30, 1972)

Halifax, N.S., has a very long record of good quality climatological records dating back more than one hundred years. Although the location of the observing station within the Halifax metropolitan area has changed over the years, the changes in location are considered to have been such as not to affect significantly the amount of annual snowfall recorded. Thus, the long period of observations should provide a very good basis for the determination of extreme values of snowfall amounts for various return periods.

Discussion

If P(x) is the probability that an extreme value will be less than a given quantity x, and y is the value of a dimensionless reduced variable given by

$$Y = a(x - u)$$

where x is a chosen value of the variable,

u is the mode of the observed distribution,

$a = \frac{\pi}{S_x \sqrt{6}}$,

S_x is the standard deviation of the largest values of the groups into which the observations have been partitioned,

then the probabilities of the extreme values of the snowfalls are approximated by

$$P(x) = e^{-e^{-y}},$$

provided that for large values of x, the initial distribution converges rapidly toward zero, a characteristic common to most usual distributions (1, 2).

$1 - P(x)$ is therefore the probability that an extreme value will equal or exceed x, and

$$T(x) = \frac{1}{1 - P(x)},$$

the time when this value of the variable will be observed again.

The magnitude of the variable with a return period T(x) is given by

$$x_T = \bar{x} + S_x K$$

where S_x is as defined above,

$\bar{x}$ is the mean of the extreme values of each of these groups,

$K = (y - \bar{y}_n) \div S_n$,

where $\bar{y}_n$ is the mean of the reduced variable for a particular sample size,

S_n is the standard deviation of the reduced variable for that sample size.

The values of the snowfall amounts for the years 1874 to 1972, inclusive, for the winter months of December, January, February and March were divided arbitrarily into groups of approximately 20, 14, 12, 10 and 5 observations.

The best fit between observed and theoretical values of the return period was obtained from the partition of the observations into ten groups (nine groups of 10 observations, one of 8 observations). In Figure 1, the return period T(x) is plotted against the reduced variable y for the division of the data into ten groups. For large values of x, T(x) is approximated by

$$\ln T(x) = y$$

while for smaller values of x, T(x) is determined from the relations

$$T(x) = \frac{1}{1 - P(x)},$$

where $y = -\ln \ln \frac{1}{P(x)}$.

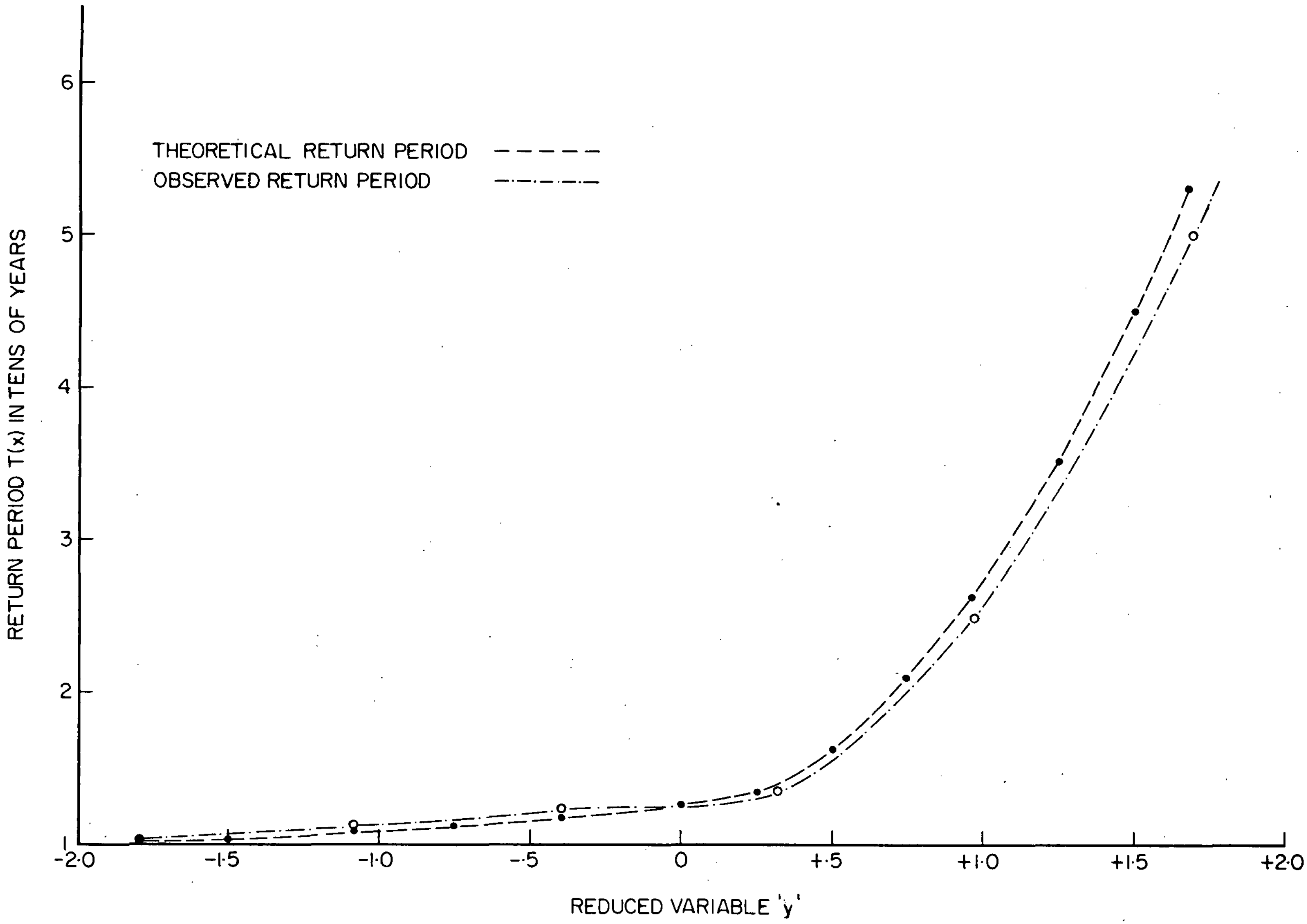

THEORETICAL RETURN PERIOD
OBSERVED RETURN PERIOD
RETURN PERIOD T(x) IN TENS OF YEARS
REDUCED VARIABLE 'y'
6
5
4
3
2
1
-2·0
-1·5
-1·0
-·5
0
+·5
+1·0
+1·5
+2·0

Figure 1.

Values of the reduced variable are displayed in Table 1.

Table 1.

Computed Values of y for Selected Return Periods
(period = 10 years, i.e., T(2) = 20-year return period)

T(x)	y	T(x)	y
1.2	-0.582	15	2.695
1.5	-0.094	20	2.978
2.0	0.367	25	3.200
3.0	0.903	30	3.401
4.0	1.247	40	3.689
5.0	1.500	50	3.912
6.0	1.702	100	4.605
7.0	1.852		
8.0	2.030		
9.0	2.139		
10.0	2.252		

For return periods less than or equal to 20, y calculated from

$$\ln \ln \frac{1}{P(x)} = -y.$$

For return periods greater than 20, y calculated from $\ln T(x) = y$.

Observed values of the return period (T(0)) are obtained from $T(0) = \frac{1}{1 - P(0)}$, where $P(0) = \frac{m}{n+1}$;

where m is the number of groups having a snowfall amount less than the observed, and

n is the number of groups.

For the division of the observations of winter snowfall into ten groups, the values of the various quantities occurring in the prediction equations have the following values:

$\bar{x}$ (the mean of the extreme values of the groups) = 103.9

S_x (the standard deviation of these extreme values) = 18.4

$a = \left(\frac{\pi}{S_x \sqrt{6}}\right)$ = .0695

u (the mode of the extreme values) = 95.8.

The values of K for the partition of the observed snowfalls into ten groups are shown in Table 2.

The value of $\overline{y}$, the mean of the reduced variable for sample size n = 10, was obtained from the relation

$$-y = \ln \ln \frac{1}{P(x)}$$

where the values of P are obtained by setting $P(x) = \frac{m}{n+1}$,

where m = 1, 2, ... , n.

For a sample size of 10, $\overline{y} \approx .500$ and S_n, the standard deviation of the reduced variable, approximately .95. For n = 10, K has the following values:

Table 2.

Values of K for n = 10, and Return Periods T

T (periods)	K	T (periods)	K	T (periods)	K
1.2	-1.028	8.0	1.454	50	3.241
1.5	-0.564	9.0	1.533	100	3.900
2.0	-0.126	10.0	1.664		
3.0	0.382	15.0	2.085		
4.0	0.710	20.0	2.354		
5.0	0.950	25.0	2.565		
6.0	1.142	30.0	2.756		
7.0	1.284	40.0	3.030		

Return periods of the four-month winter snowfall x_T obtained from the relation $x_T = \overline{x} + S_x K$ are shown below in Table 3.

Table 3.

T (periods)	T Years	K	Winter Snowfall (inches)
2.0	20	-0.126	101.6
4.0	40	0.710	116.9
5.0	50	0.950	121.4
10.0	100	1.664	134.5
20.0	200	2.354	147.2
50.0	500	3.241	164.5
100.0	1000	3.900	175.7

Return Periods of Largest Monthly Snowfalls

The largest monthly snowfalls for each of the 98 years of record were examined. Here n = 98, and N, the number of events from which the largest event is selected, = 6, comprising the months of November, December, January, February, March and April.

For these data the following values were obtained:

Mean of the largest monthly snowfalls ...... 26.8 in.

Mode of the largest monthly snowfalls ...... 22.55 in.

$a = \frac{1}{.77970\ S_x} = \frac{1}{(.78)(9.45)} =$ .1357

S_x, the standard deviation of the series of largest monthly snowfalls 9.45

Values of the reduced variable y for selected monthly snowfalls x were determined from the expressions

$y = a(x - u)$ (theoretical values of y)

$\ln \ln \frac{1}{P(x)} = -y$ (values of y determined from the observations)

where u is the modal value of the largest monthly snowfalls (22.55 in.),

P(x) the probability that an extreme value will be less than $x = \frac{m}{n+1}$,

n is the number of observations,

m the order of the observations, arranged in ascending order, and are shown in Table 4.

Table 4.

Values of the Reduced Variable y for Selected Values of x

x (Maximum Monthly Snow)	$P(x) = \frac{m}{n+1}$	$y = a(x - u)$	$y = -\ln \ln \frac{1}{P(x)}$
55 in.	.99	4.403	4.605
50 in.	.99	3.725	3.912
45 in.	.96	3.046	2.996
40 in.	.90	2.368	2.169
35 in.	.84	1.689	1.749
30 in.	.70	1.011	1.022
25 in.	.49	0.332	0.352
20 in.	.22	-0.347	-0.408
15 in.	.10	-1.025	-0.829
10 in.	.03	-1.704	-1.252
7.5 in.	.01	-2.042	-1.525

Values of the theoretical and observed values of the reduced variable y plotted against these values of maximum monthly snowfall are shown in Figure 2. The agreement between the theoretical values of y and the values of y determined from the observations is quite good. Differences between theoretical and observed values of the reduced variable at both extremes of values of the observed maximum monthly snowfalls are due most probably to the small number of such large (or small) values of x.

Values of the theoretical and observed maximum monthly precipitation y, along with the theoretical and observed return periods for specific values of y, are shown below in Table 5.

Observed values of the return period are calculated from the expression $T(x) = \frac{n}{n - m + 1}$,

where $m = 1, 2, \ldots\ldots, n$

with the data arranged in ascending order; theoretical values of the return period from $T(x) \frac{1}{1 - P(x)}$

where $\ln \ln \frac{1}{P(x)} = -y$.

Theoretical values of x are obtained from the expression $y = a(x - u)$.

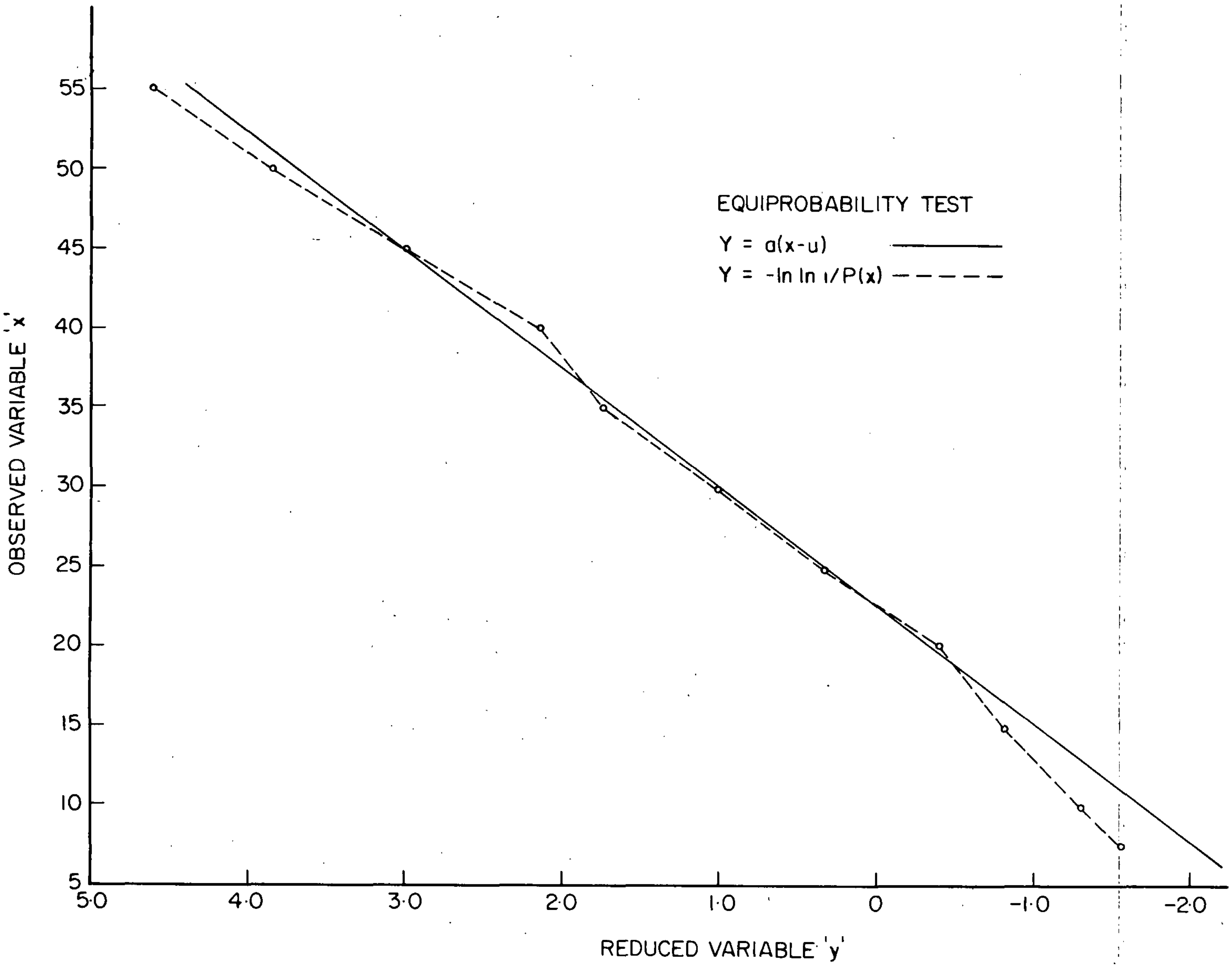

EQUIPROBABILITY TEST
Y = a(x-u)
Y = -ln ln 1/P(x)
OBSERVED VARIABLE 'x'
REDUCED VARIABLE 'y'
55
50
45
40
35
30
25
20
15
10
5
5·0
4·0
3·0
2·0
1·0
0
-1·0
-2·0

Figure 2.

Table 5.

Values of Observed and Theoretical Monthly Maximum Snowfalls and Return Periods for Specific Values of the Reduced Variable y

	Return Period (yrs.)		Maximum Monthly Snowfall	
y	Observed	Theoretical	Observed	Theoretical
4.0	98.0	84.3	55.9 in.	52.0 in.
3.75	49.0	44.0	53.0	50.1
3.5	32.7	33.3	46.7	47.4
3.0	19.6	21.0	43.7	43.8
2.5	12.2	12.6	43.0	40.3
2.0	7.5	7.8	38.3	36.8
1.5	4.9	5.0	34.0	33.3
1.0	3.2	3.2	29.7	29.8
.5	2.1	2.2	26.1	26.2
0	1.6	1.6	23.1	22.7
-0.5	1.22	1.24	18.9	19.2
-1.0	1.06	1.07	12.6	15.7
-1.5	1.0	1.0	7.6	11.4

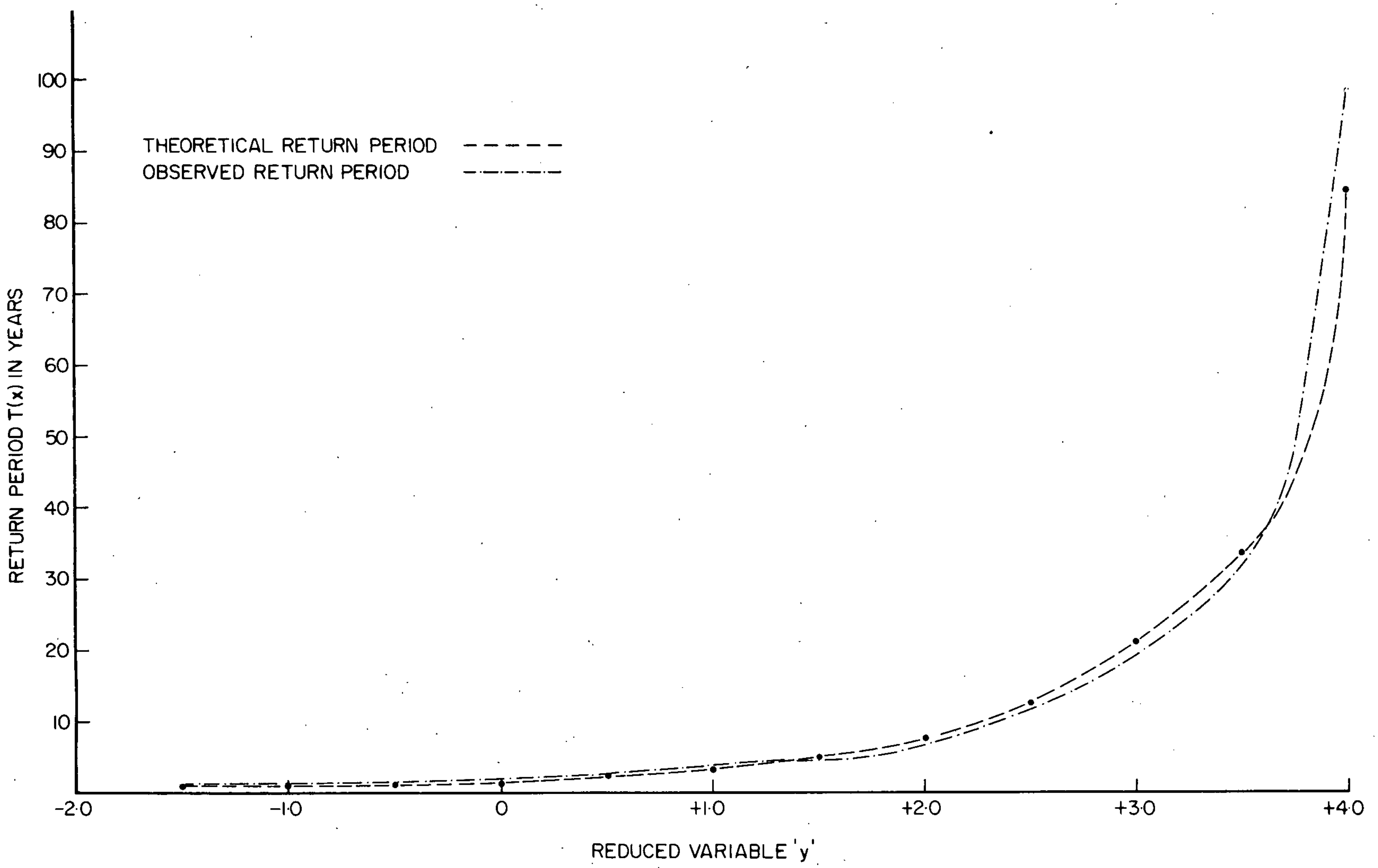
Values of the theoretical and return period plotted against these values of the reduced variable appear in Figure 3.

For this sample of 98 observations, the mean value of the reduced variable $\overline{y}$, was .5640 with standard deviation S_n of 1.2050. For n = 98, $K = (y - \overline{y}) \div S_n$ has the values set out in Table 6 for selected return periods T(x). The magnitude of the expected return period snowfall X_T is determined using these values of K, the standard deviation of the extreme values of monthly snowfalls $S_x = 9.45$, and the mean of the maximum monthly snowfalls 26.8 in.

Table 6.

Computed Values of K and X_T for Selected Return Periods T(x) for n = 98

T(x)	K	X_T
2	-0.0840	26.0
5	0.7775	34.2
10	1.4091	40.1
15	1.7618	43.4
20	2.0356	46.0
25	2.2215	47.8
50	2.7900	53.2
100	3.3535	58.5
150	3.5510	60.4
200	3.9289	63.9
500	4.6889	71.1



THEORETICAL RETURN PERIOD
OBSERVED RETURN PERIOD
RETURN PERIOD T(x) IN YEARS
100
90
80
70
60
50
40
30
20
10
-2·0
-1·0
0
+1·0
+2·0
+3·0
+4·0
REDUCED VARIABLE 'y'

Figure 3.

For $T(x) \leq 100$, y is computed from the relationships

$$T(x) = \frac{1}{1 - P(x)}$$

$$-y = \ln \ln \frac{1}{P(x)}$$

while, for values of T(x) larger than 100, the approximate relationship

$$y = \ln T(x)$$

is the more convenient method of calculation.

The author wishes to thank Mr. Graham Powell, of the Halifax Atlantic Weather Central, for his assistance in abstracting the data used for this analysis.

APPROVED,

J.R.H. Noble,
Assistant Deputy Minister,
Atmospheric Environment Service.

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Subject Reference: 1. Statistical Forecasting
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