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EXTREME TEMPERATURE RECURRENCE

by

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ABSTRACT

A technique is devised, following a study of Extreme Temperatures (1), to show how the probability of the occurrence, or recurrence of extreme temperatures during any time interval L, may be estimated. All that is required is a short formula and hand-calculator, or, a set of tables.

OCCURRENCES PÉRIODIQUES DES TEMPÉRATURES EXTRÊMES

par

John R. Hendricks

RÉSUMÉ

A la suite d'une étude des températures extrêmes, une technique a été développée pour montrer de quelle façon la probabilité de occurrence - periodique ou non-des températures extrêmes pendant un intervalle de temps L, peut être évaluée. Cela nécessite, tout simplement, une formule courte et des calculs manuels, ou, une série de tables.

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(Manuscript received September 21, 1973)

Introduction

It is the purpose of this paper to show how Kendall's (2) probability of the occurrence of an event E, within its return period R, for large R, given by:

$$\text{Prob (E)} = 1 - 1/e = .6321206 \qquad (1)$$

may be extended to give the probability of an event E occurring, or recurring during any time-period desired.

The limiting process, which Kendall outlined before, is displayed in Table I.

Theory

The probability of the non-occurrence of an event E within its return period R, for large R, is simply:

$$\text{Prob (non-E)} = 1/e = .3678794 \qquad (2)$$

The probability of non-occurrence within two successive return periods, using either the arguments as outlined before; or, using simply the product of probabilities on two successive events, yields either way:

$$1/e^2 = 0.135335$$

and, for three successive return periods:

$$1/e^3 = 0.049787$$

and, in general, for k successive return periods:

$$1/e^k$$

Further generalization may be considered, where k is non-integral, and is defined as a continuous function by:

$$L = kR \tag{3}$$

where L is any desired time-length. Whence the probability of the occurrence of an event E within any period of time L, for large R, becomes:

$$\text{Prob(E within L)} = 1 - 1/e^k \tag{4}$$

Application

Equation (4) may be applied directly, given the return period R, and the time-length L desired, to finding the probability of the occurrence of an event E within the time-length L.

However, for application to the long-in-time return periods for rare temperature occurrences, in a quick-answer situation in a forecast office, Tables II, III and IV have been prepared. The recurrences or occurrences which have less than a 5-year return period for all practical purposes, for rare temperature situations, may be considered to occur or recur at almost any time, and to be of no special significance for this type of study.

As is shown for a time period of 150 days, displayed in Table I, the difference between $(1 - 1/e)$ and $(1 - 1/150)^{150}$ is about 1 part in 1,000; and for extreme temperature studied on a monthly basis, for longer than 5-year intervals (longer than about 150 days) the difference becomes negligible.

The theory invoked refers not only to extreme values but to any statistical event with a constant probability of occurrence in a given time unit. The application intended is for extremes of temperature but it is also applicable to the occurrence of a temperature above a given threshold value, for instance, provided that the probability or return period of such occurrence is known.

Example

For example, at Regina, 110°F occurred on the 5th day of July, 1937, and set the all-time high temperature for Regina. The Return Period, as calculated, is given as 41.134914 years, or simply 41 years for practical purposes.

Following the event, one person might ask, "What is the probability of this event occurring again next year?" Using Table II, and interpolating between the 40- and 45-year return periods, 1 year is found at 2 and 3 percent. Therefore, a quick-answer is: "There is about a $2\frac{1}{2}$ percent chance of having this occur again next year."

Similarly, anyone asking, "What is the probability of ever experiencing a $110^{o}F$ temperature during a normal lifetime (70 years) at Regina?", a search of Table IV, upon interpolation yields the answer: "About an 83 percent chance that one will experience this event!"

The 50% column, Table III, may be used to show the length of time required for a 50/50 chance. In this example, there is a 50/50 chance of having an occurrence of $110^{o}F$ in July at Regina once every 28 years (or, generation, 30 years).

Generations and lifetimes are sometimes regarded as more meaningful to the general public. This paper enables one to convert Return Periods to these other time intervals.

Short Time-Period Estimation

In a forecast office situation, a short time-period estimation of probability, may suddenly be required. Since it is known that:

$$1 - e^{-k} = k \qquad \text{for small k,}$$

such as k less than $\frac{1}{4}$, and especially k less than 1/10; whenever L turns out to be less than R/4, or, especially less than R/10, the probability of an event E within L, simply reduces to "k" for all practical purposes.

For example, knowing that the Return Period of a killing frost, ($28^{o}F$, or less,) at Regina in July is 472 years, one may estimate the likelihood of a killing frost during the next five years at Regina in July, by solving:

$$5 = k(472)$$

mentally. This yields k = .01, which shows this possibility is at about a 1% chance of occurrence.

Conclusions

The probability of occurrence or recurrence, of any temperature, or range of temperatures, has wide application in a forecast office situation. Using the Probability Study of Extreme Temperatures (1) as a history base and Tables II, III and IV, gives a straightforward and simple approach to the answering of questions on the rate of return of rare temperature occurrences.

APPROVED,

J.R.H. Noble,
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References

1. Hendricks, John R. and Lee Rea.,: A Probability Study of Extreme Temperatures, Parts I, II and III, ...

2. Kendall, G.R.,: Statistical Analysis of Extreme Values, National Research Council Sub-Committee on Hydrology, July 3, 1959.

Return Period in days m	Probability of not occurring (1 - 1/m)^m	Probability of occurring 1 - (1 - 1/m)^m
2	.25	.75
3	.2962962	.7037038
4	.3164062	.6835938
5	.3276800	.6723200
6	.3348980	.6651020
7	.3399166	.6600833
8	.3436088	.6563912
9	.3464392	.6535608
10	.3486783	.6513217
11	.3504940	.6495060
12	.3519954	.6480046
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28	.3612104	.6387896
29	.3614437	.6385563
30	.3616610	.6383390
31	.3618645	.6381355
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150	.3666490	.6333510

TABLE I

This table demonstrates, as Kendall (2) has shown, how the probability of the occurrence of an event within its return period diminishes as the return period becomes larger; and, how it approaches the limit $1 - 1/e = .6321206$.

TIME INTERVALS (in years) FROM KNOWN PROBABILITIES AND RETURN PERIODS

Short time intervals

Return Period in years	Probabilities 1%	2%	3%	4%	5%	10%	20%
5	nil	nil	nil	nil	nil	nil	1
10	nil	nil	nil	nil	1	1	2
15	nil	nil	nil	1	1	2	3
20	nil	nil	1	1	1	2	4
25	nil	1	1	1	1	3	6
30	nil	1	1	1	2	3	7
35	nil	1	1	1	2	4	8
40	nil	1	1	2	2	4	9
45	nil	1	1	2	2	5	10
50	1	1	2	2	3	5	11
55	1	1	2	2	3	6	12
60	1	1	2	2	3	6	13
65	1	1	2	3	3	7	14
70	1	1	2	3	4	7	16
75	1	2	2	3	4	8	17
80	1	2	2	3	4	8	18
85	1	2	3	3	4	9	19
90	1	2	3	4	5	9	20
95	1	2	3	4	5	10	21
100	1	2	3	4	5	11	22
110	1	2	3	5	6	12	25
120	1	2	4	5	6	13	27
130	1	3	4	5	7	14	29
140	1	3	4	6	7	15	31
150	2	3	5	6	8	16	33
160	2	3	5	7	8	17	36
170	2	3	5	7	9	18	38
180	2	4	5	7	9	19	40
190	2	4	6	8	10	20	42
200	2	4	6	8	10	21	45
k	.010	.020	.030	.041	.051	.105	.223

TABLE II

Knowing the return period and the probability of return, one can estimate the length of time required to support the probability. Knowing the return period and the time interval, one can inversely find the probability of returning during the time interval. The value k, for equation (4) is given on the last line for use in extending the tables, if required.

TIME INTERVALS (in years) FROM KNOWN PROBABILITIES AND RETURN PERIODS

Medium time intervals

Return Period in years	25%	30%	40%	50%	60%	63%	70%
5	1	2	3	3	5	5	6
10	3	4	5	7	9	10	12
15	4	5	8	10	14	15	18
20	6	7	10	14	18	20	24
25	7	9	13	17	23	25	30
30	9	11	15	21	27	30	36
35	10	12	18	24	32	35	42
40	12	14	20	28	37	40	48
45	13	16	23	31	41	45	54
50	14	18	26	35	46	50	60
55	16	20	28	38	50	55	66
60	17	21	31	42	55	60	72
65	19	23	33	45	60	65	78
70	20	25	36	49	64	70	84
75	22	27	38	52	69	75	90
80	23	29	41	55	73	80	96
85	24	30	43	59	78	85	102
90	26	32	46	62	82	90	108
95	27	34	49	66	87	95	114
100	29	36	51	69	92	100	120
110	32	39	56	76	101	110	132
120	35	43	61	83	110	120	144
130	37	46	66	90	119	130	157
140	40	50	72	97	128	140	169
150	43	54	77	104	137	150	181
160	46	57	82	111	147	160	193
170	49	61	89	118	156	170	205
180	52	64	92	125	165	180	217
190	55	68	97	132	174	190	229
200	58	71	102	139	183	200	241
K	.288	.357	.511	.693	.916	1.0	1.204

TABLE III

Knowing the return period and the probability of return, one can estimate the length of time required to support the probability. Knowing the return period and the time interval, one can inversely find the probability of returning during the time interval. The value k, for equation (4) is given on the last line for use in extending the tables, if required.

TIME INTERVALS (in years) FROM KNOWN

PROBABILITIES AND RETURN PERIODS

Long Time Intervals

Return Period in years	Probabilities 75%	80%	90%	95%	96%	97%	98%	99%
5	7	8	12	15	16	18	20	23
10	14	16	23	30	32	35	39	46
15	21	24	35	45	48	53	57	69
20	28	32	45	60	64	70	78	92
25	35	40	58	75	81	88	98	115
30	42	48	69	90	97	105	117	138
35	49	56	81	105	113	123	137	161
40	55	64	92	120	129	140	156	184
45	62	72	103	135	145	158	176	207
50	69	81	115	150	161	176	196	230
55	76	89	127	165	177	193	215	253
60	83	97	138	180	193	211	235	276
65	90	105	150	195	209	228	254	299
70	97	113	161	210	225	246	274	322
75	104	121	173	225	242	263	293	345
80	111	129	184	240	258	281	313	368
85	118	139	196	255	274	298	332	391
90	125	145	207	270	290	316	352	414
95	132	153	219	285	306	333	371	437
100	139	161	230	300	322	351	391	460
110	152	177	253	329	354	386	430	506
120	166	193	276	359	386	421	469	552
130	180	209	299	389	419	456	508	599
140	194	225	322	419	451	491	547	645
150	208	242	345	449	483	527	587	691
160	222	258	368	479	515	562	626	737
170	236	274	391	509	547	597	665	783
180	249	290	414	539	580	632	704	829
190	263	306	437	569	612	667	743	875
200	277	322	460	599	644	702	782	921
K	1.386	1.610	2.300	2.995	3.22	3.51	3.91	4.604

TABLE IV

Knowing the return period and the probability of return, one can estimate the length of time required to support the probability. Knowing the return period and the time interval, one can inversely find the probability of returning during the time interval. The value k, for equation (4) is given on the last line for use in extending the tables, if required.

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4 pps. 2 refs. 4 tables.

Subject Reference: 1. Temperature
2. Probability Forecasting

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