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Report on Construction
Hydraulic Model of Alberni Harbour

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Report on Construction

HYDRAULIC MODEL OF ALBERNI HARBOUR

by

J.B. Nuttall

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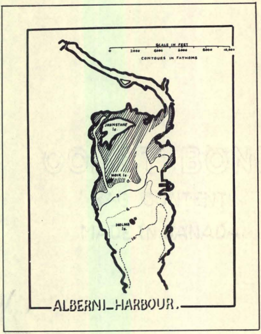
Location of Alberni Inlet

ABSTRACT

A hydraulic model of Alberni Harbour was built to study the mixing of fresh and salt water, the disposal of industrial sewage, and the result of proposed physical changes in the Harbour. The model was built to a scale of 1/1000 horizontally, and 1/84 vertically. A modified form of Lord Kelvin's tide predicting machine is used to compute the tides and thus control a pair of valves which add salt water and remove mixed water. River discharge is manually adjusted. A method of removing water samples from the operating model for chemical analysis was developed as a means of observing salinity distribution. At present the model is ready for verification and experiment.



Alberni Harbour

Figure 1.

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Report on Construction

HYDRAULIC MODEL OF ALBERNI HARBOUR

(File N 7-22 December 1, 1951)

INTRODUCTION

This thesis describes the construction of a hydraulic model of Alberni Harbour. The Model was built to study the mixing of fresh and salt water, disposal of pulp mill sewage, and the effects of proposed physical changes in the harbour. As Alberni Inlet is typical of many British Columbia fiords, the results of this research are expected to be widely applicable.

The oceanographic aspects of Alberni Inlet are described by Tully (7), who used a small model of the harbour in which the scales were,

horizontal . . . . 1/4308.
vertical . . . . . 1/288.

A simple harmonic tide was reproduced and wind effects were simulated. Although it gave much valuable information, its small scale did not allow the precision or detail expected with the present model.

The scales used for the model described in this paper are,

horizontal 1/1000
vertical . . . . . 1/84.

The actual size of the model is therefore 23 feet long by 7 1/2 feet wide, and covers the area shown in Figure 1. The model bed is built of concrete set in a wooden framework.

Since no literature on the control mechanism of density stratified models of the size of the present model is available, an essentially new problem was presented. Tidal control requires an automatic mechanism to compute the tide and apply it to the model. A modified form of Lord Kelvin's tide predicting machine is used as the computer; its output is fed to two valves, one admits salt water, and one removes mixed water. A float mechanism compares the computed tide with the model tide and operates a servo-mechanism which makes minor corrections in the valve settings. The computed tide and prototype time are recorded on a moving tape; the prototype date is recorded by a mechanical counter. River

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discharge is controlled by a manually adjusted vee notch weir.

Current data is to be obtained by observing the movement of injected dyes and neutrally buoyant droplets of carbon tetrachloride and petroleum ether. A water sampler was developed in order to remove small samples from the model for salinity analysis. Several of these samplers may be used simultaneously to produce a synoptic survey of the model.

Model Laws

It is accepted that the following quantities characterize the flow of a homogeneous fluid:

x, y, z	linear dimensions
V	velocity
Δp	pressure difference
ρ	density
g	acceleration due to gravity
μ	dynamic coefficient of viscosity
σ	surface tension
ϵ	elastic modulus

By Buckingham's π Theorem (3), it may be shown that a function describing the fluid motion is:

$$f\left(\frac{V^2\rho}{\epsilon}, \frac{zV^2\rho}{\sigma}, \frac{zV\rho}{\mu}, \frac{V^2}{zg}, \frac{\rho V^2}{\Delta p}, \frac{x}{y}, \frac{y}{z}\right) = 0$$

In order to have complete similarity between model and prototype the seven terms in the above function must be similar for both.

The Cauchy number, $V^2\rho/\epsilon$ may be neglected in water-ways models since the effect of compressibility is minute. Similarily the Weber number, $zV^2\rho/\sigma$ may be omitted provided surface tension has a negligible effect in both prototype and model. Since velocity in the model is fixed by the choice of distance and time scales, the Reynolds number, $zV\rho/\mu$ cannot, in general, be made similar for model and prototype. This is because fluid density and viscosity cannot be arbitrarily chosen. In water-ways models the value of $zV\rho/\mu$ is less than the prototype. The effect of the decreased Reynolds number is to decrease the scale of turbulence in the model. Where the Reynolds number in the model is less than the lower limit for turbulent flow, the model can be expected to bear no relation to the prototype, i. e. near the boundaries. By analogy with Nickuradse's diagram of friction factor f vs $z V\rho/\mu$ for pipes, Figure 2, it appears that decreased Reynold's number in the model will have relatively small effect if the flow is turbulent, provided the model roughness can be made large enough. It has been shown by Allen (1) that it is sometimes difficult to make the boundaries of a distorted scale model sufficiently rough.

In the following discussion x and y are taken as horizontal dimensions, and z as a vertical dimension. The subscripts m and p refer to the model and the prototype respectively, and the subscript r to the ratio of a quantity in the model to that in the prototype. The letter t represents a time interval.

The Froude number $V^2/z g$ can be satisfied provided suitable model scales are chosen. In order that

$$\left(\frac{V^2}{z g}\right)_m \div \left(\frac{V^2}{z g}\right)_p = 1$$

the equality

$$\frac{V_r^2}{z_r} = 1$$

must be satisfied. Now

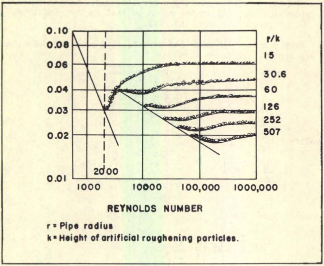
$$V_r = \frac{x_r}{t_r} = \frac{y_r}{t_r}$$

and therefore

$$t_r = x_r z_r^{-\frac{1}{2}}$$

Unless the vertical and horizontal scale ratios are equal, the Euler number, $\rho V^2/\Delta p$, a characteristic of the flow pattern, cannot be made equal in model and prototype. The change in the Euler number is small however. By writing the Bernoulli equation for turbulent flow (4)

$$\frac{V_1^2}{2g} + \frac{u_1^2 + v_1^2 + w_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{V_2^2}{2g} + \frac{u_2^2 + v_2^2 + w_2^2}{2g} + \frac{p_2}{\rho g} + z_2$$



Variation of Resistance Coefficient with Reynolds Number for Pipes

Figure 2.

where u', v', and w' represent instantaneous departures from the mean velocities in the x, y, and z directions respectively. The mean flow is taken to have the velocity V. From the above equation

$$\Delta P = P_2 - P_1 = \rho g \left[\frac{V_1^2 - V_2^2 + u_1'^2 - u_2'^2 + v_1'^2 - v_2'^2 + w_1'^2 - w_2'^2}{2g} + z_1 - z_2 \right]$$

Substituting this value of ΔP in $\rho V^2/\Delta P$, the ratio between model and prototype becomes

$$\frac{\left(\frac{\rho V^2}{\Delta P}\right)_m}{\left(\frac{\rho V^2}{\Delta P}\right)_p} = V_r^2 \left[\frac{(V_1)_p^2 - (V_2)_p^2 + (u_1')_p^2 - (u_2')_p^2 + (v_1')_p^2 - (v_2')_p^2 + (w_1')_p^2 - (w_2')_p^2 + \Delta z_p \cdot 2g \cdot \frac{z_r}{V_r^2}}{V_r^2 \left\{ (V_1)_p^2 - (V_2)_p^2 + (u_1')_p^2 - (u_2')_p^2 + (v_1')_p^2 - (v_2')_p^2 + \left((w_1')_p^2 - (w_2')_p^2\right) \frac{z_r^2}{t_r^2 V_r^2} + \Delta z_p \cdot 2g \cdot \frac{z_r}{V_r^2} \right\}} \right]$$

since $\rho_m = \rho_r$. By the Froude relationship,

$$z_r = V_r^2$$

so the multiplier of $\Delta z \cdot 2g$ in the above equation becomes unity. The multiplier of the $(w')_p$ terms does not since

$$\frac{z_r^2}{t_r^2 V_r^2} = \frac{z_r^2 t_r^2}{t_r^2 x_r^2} = \frac{z_r^2}{x_r^2} \pm 1$$

This means that kinematic similarity cannot be obtained in a distorted scale model and therefore complete mechanical similarity is precluded.

The requirements of x/y and y/z are satisfied only if the model is geometrically similar to the prototype.

A rational basis for the relationship between vertical and horizontal scales is available in Lacy's rule (1) which states

$$z_r = x_r^{\frac{2}{3}}$$

The basis of this rule is the Regime Theory of alluvial channels, of which Lacy was an early contributor. Blench (2) has developed the Regime Theory to its present state in which

$$\frac{V^2}{D} = b$$

where V is the mean velocity of flow in the channel, D is the mean depth, and b is a constant depending upon the material forming the channel bed and upon the supply of bed material from sources other than the bed itself. b is called the "bed factor". The side factor s is defined as

$$\frac{V^3}{W} = s$$

where W is the channel width and s depends upon the material forming the cohesive sides of the channel. The channel slope S is related to the other variable by the relation

$$\frac{V^2}{gDS} = C\left(\frac{VW}{\nu}\right)^{\frac{1}{4}}$$

where C is a non-dimensional constant and ν is the kinematic viscosity of the fluid-sediment complex.

The slope may be expressed as

$$S = \frac{\nu^{\frac{1}{4}}}{gC} \cdot \frac{b^{\frac{5}{6}} s^{\frac{1}{12}}}{Q^{\frac{1}{6}}}$$

where Q is the rate of flow in the channel, i. e. $Q = VWD$

If the same material forms the channel bed and sides in both the model and prototype, the slope equation may be written

$$S_m = \frac{\nu^{\frac{1}{4}}}{gC} \cdot \frac{b^{\frac{5}{6}} s^{\frac{1}{12}}}{Q_m^{\frac{1}{6}}} = \frac{\nu^{\frac{1}{4}}}{gC} \cdot \frac{b^{\frac{5}{6}} s^{\frac{1}{12}}}{(Q_p x_r z_r^{3/2})^{\frac{1}{6}}}$$

for the model, using the Froude scales to relate Q_m and Q_p ; and

$$S_p = \frac{\nu^{\frac{1}{4}}}{gC} \cdot \frac{b^{\frac{5}{6}} s^{\frac{1}{2}}}{Q_p^{\frac{1}{6}}}$$

for the prototype. Taking the ratio of S_m to S_p

$$\frac{S_m}{S_p} = \left(X_r Z_r^{\frac{3}{2}}\right)^{-\frac{1}{6}}$$

but $S = Z/X$, so that

$$\frac{S_m}{S_p} = \frac{Z_m}{X_m} \cdot \frac{X_p}{Z_p} = \frac{Z_r}{X_r} = \left(X_r Z_r^{\frac{3}{2}}\right)^{-\frac{1}{6}}$$

and therefore

$$Z_r = X_r^{\frac{2}{3}}$$

The effect of the Earth's rotation, Coriollis' force, can only be simulated by mounting the entire model on a rotating base. The errors due to the omission of this step are thought to be of the same order of magnitude as the experimental errors in this particular model. Therefore the effect of Coriollis will not be simulated.

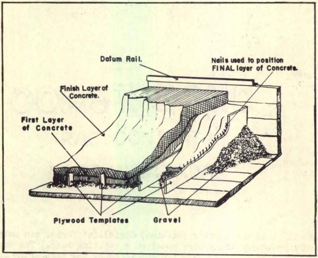
The model scales were chosen as follows: the horizontal scale $X_r = 1/1000$, selected on the basis of laboratory space and water supply, the vertical scale $Z_r = 1/84$, selected with regard to Lacy's rule and the effects of surface tension over the inter-tidal area of the model. Having fixed the horizontal and vertical scales, the time scale becomes

$$t_r = 1/109.1$$

from the Froude law.

Construction

The laboratory in which this model was constructed is a temporary frame building 20 by 40 feet, an office and washroom occupy a space of 8 by 20 feet.



Construction of Model Basin

Figure 3.

1. Model Bed:

The model is set on eight concrete piers in the laboratory floor. Ten inch square beams rest on the piers and support two by eight inch longerons which form the model base. The supporting framework was designed to deflect a maximum of 0.2% of the span between the cross beams when the model is full of water. A wooden box made of two inch planking was built upon the supporting framework to contain the model bed. Female plywood templates cut one-half inch below the required model boundary were set in the box. A layer of gravel was placed on the floor of the box and the first layer of concrete poured. This layer extends to one inch below the top of the templates. Small nails were then driven into the plywood, the heads being placed on the required boundaries of the model. A final layer of concrete was then trowelled to the surface defined by the nail heads. The method of moulding the model bed is shown in Figure 3.

A co-ordinate system was used to place the nails. This consisted of machined rails fixed to either side of the model box and a cross rail complete with vertical measuring rod. The assembly is accurately made and set so that horizontal measurements can be made to within 1/16 inch on the model, or five feet in the prototype; vertical measurements can be made to within 1/32 inch on the model or three inches in the prototype. These are equal to the accuracy of the available charts of the area and are considered adequate.

Photographs of the model in operation are shown in Figure 13.

In order to accommodate bed movement studies in the tide flat area, this region was left at a scaled depth of seven fathoms when the concrete was poured. The tide flats were later built up with sharp sand to fit male templates. A spray of portland cement and water was used to "fix" the area for the testing of control gear. The crust will be removed and adjustment made before the final tests are begun. This area is shown in Figure 14.

Water supply to the laboratory is:

	Maximum Supply	Maximum Head
Fresh Water	0.03 c.f.s.	43 feet
Salt Water	0.11 c.f.s.	31 feet

2. Tide Computer:

In predicting the tides at Port Alberni the Hydrographic Office uses 61 terms of the series (6)

$$H = z_0 + \sum_1^m f_n z_n \cos(a_n t + \alpha_n)$$

where H is predicated tide at any time

TABLE I

Harmonic Constants of the Tide at
Port Alberni B.C.
Latitude 49°14'N, Longitude 124°49'W
February 1, 1948.

	H	g		H	g
Z_0	6.451		T_2	.052	4.4
Sa	0.437	281.6	S_2	.876	24.6
Ssa	0.107	209.1	R_2	.031	151.0
Mm	0.031	153.4	K_2	.247	17.2
MSt	0.075	163.8	MSN_2	.015	183.8
Mf	0.016	137.6	KV_2	.015	250.4
			$2SM_2$	.009	333.5
$2Q_1$	.016	75.9			
σ_1	.011	135.0	MO_3	.002	333.5
Q_1	.140	114.7	M_3	.009	328.0
ρ_1	.012	70.0	SO_3	.012	228.4
O_1	.809	118.8	MK_3	.017	183.3
MP_1	.032	325.3	SK_3	.008	14.0
M_1	.050	123.7			
X_1	.028	139.5	MN_4	.004	116.5
π_1	.009	116.5	M_4	.019	71.5
P_1	.400	121.4	SN_4	.006	108.4
S_1	.007	81.8	MS_4	.019	168.1
K_1	1.300	125.7	MK_4	.010	78.7
ψ_1	.040	197.5	S_4	.009	234.5
ϕ_1	.013	73.3	SK_4	.003	270.0
J_1	.076	129.1			
SO_1	.013	138.0	$2MN_6$	.003	161.7
OO_1	.042	136.9	M_6	.008	209.7
			MSN_6	.005	90.0
OQ_2	.016	137.5	$2MS_6$	.005	291.8
MNS_2	.007	188.2	$2MK_6$	.010	251.7
μ_2	.065	329.5	$2SM_6$	.006	218.6
N_2	.665	343.8	MSK_6	.003	198.4
V_2	.119	349.4			
OP_2	.016	251.6			
M_2	3.128	4.3	θ_1	.027	158.2
MKS_2	.007	26.5	$2N_2$	.086	309.8
λ_2	.017	353.3			
L_2	.083	0.7			

Revisions for 1952 predictions
Sa 0.236 307.6
Ssa 0.119 239.1

Figure 4.

a , is the angular speed of the constituent which is constant for any one component and depends on astronomical data only,

f , factor for reducing mean amplitude to year of prediction and depends on astronomical data only,

z , mean amplitude of constituent determined by analysis of tidal records,

α , phase angle of constituent determined from tidal records and may be adjusted to begin the series at any time.

The tidal height may be predicated with good accuracy by using only four of the components listed in Figure 4 and taking the value of f equal to unity. These components are M_2 due to the moon, S_2 due to the sun, and K_1 and O_1 due to the moon's declination. The series then becomes

$$H = z_0 + M_2 \cos(a_{M_2} t + \alpha_{M_2}) + S_2 \cos(a_{S_2} t + \alpha_{S_2})$$
$$+ K_1 \cos(a_{K_1} t + \alpha_{K_1}) + O_1 \cos(a_{O_1} t + \alpha_{O_1})$$

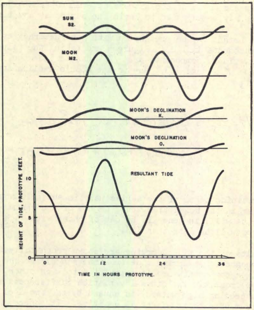
If all the remaining terms of the series were in phase at some time, the resultant error in the model tide would be 14% of the maximum range. Since these components will, in general, be out of phase with each other, the resultant error will be less than 14%.

The results of the tide computer are shown graphically in Figure 5 where the sequence for a spring tide (maximum range) are shown. When the M_2 and S_2 components are in opposition to each other, neap tides (minimum range) result. The diurnal variation (differences in the height of the two succeeding high waters) is caused by the two declination terms and is a maximum when their effect is additive. It can be shown that the tidal cycle will never repeat itself. "Near repetition" however, occurs frequently, the most obvious case being the monthly repetition of the spring and neap tides.

The rate of change of the tidal height is a first approximation of the valve opening required to add or remove water from the model. Thus if

$$H = z \cos(at + \alpha)$$

$$\frac{dH}{dt} = az \cos\left(at + \alpha + \frac{\pi}{2}\right)$$



Method of Summing
Tidal Components

Figure 5.

but

$$\frac{dH}{dt} = \frac{Q}{A}$$

where A is surface area of the model, so that

$$Q = A a z \cos(at + \alpha + \frac{\pi}{2})$$

Therefore the valves can be controlled by a series similar to that used for the tidal height summation except the components must be $\pi/2$ radians in advance of the height components and of a length proportion to az .

A pully and cable system, Figures 6 and 15, is used to integrate these series and the result is fed directly to the control valves in the case of the derivative, and to the control float and the tide recorder, in the case of the height summation. Power for the tide computer is supplied by a one-quarter horsepower electric induction motor.

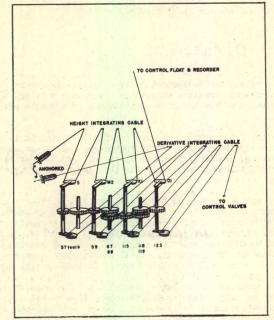
By using cables to sum the tidal components, a further maximum error 0.6% is introduced, which is considered negligible. Spur gears are employed to produce the angular velocity ratio between the component cranks. Owing to the expense of precisely reproducing angular velocities, maximum allowable error of 0.0033% was selected arbitrarily and the gearing designed within this limit. The long period tidal components can be set on the computer manually and will reduce the errors in tidal height somewhat.

The tide computer may be set up as follows: the angular position of the components is calculated for the prototype time at which the experiment is to begin, the component is locked in a known position, the remaining cranks adjusted to their proper relative positions, and the driving gears are then locked to their shafts.

3. Control Valves:

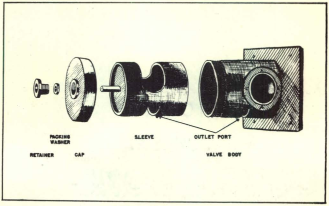
Since the motion available to operate the control valves was proportional to the discharge, valves having a nearly linear characteristic were required. Two brass sleeve valves with rectangular ports were specially made (Figure 7) because no suitable type is available commercially. The equation of discharge for the valves is

$$Q = \beta C_d \sqrt{2g(h - KQ^2)}$$



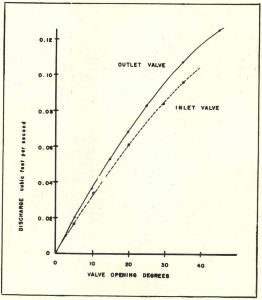
Tide Computer

Figure 6.



Control Valve Design

Figure 7.



Characteristics of Control Valves

Figure 8.

where β . . . . port area.

C_d . . . . the discharge coefficient.

K_1 . . . . factor proportional to the head losses in the approach piping.

h . . . . maximum available head.

solving for Q and expanding by the binomial theorem,

$$Q = \beta C_d \sqrt{2gh}\left[1 + \beta^2 C_d^2 g K_1 + \beta^4 C_d^4 g^2 K_1^2 + \cdots\right]$$

Thus, if K_1 is small, the relation between Q and β is nearly linear. The observed characteristic curves taken "in situ", together with the drawings, are shown in Figure 8. The method of connecting the control valves to the derivative cable and servo motor is shown schematically in Figure 9.

4. Control Float and Servomechanism.

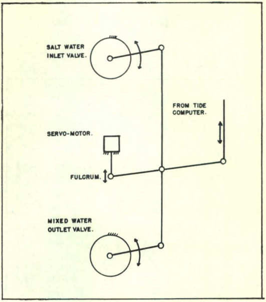
The control float is a metal box which floats in a stilling basin built into the downstream end of the model, Figure 16. The height of the tide in the model is compared with the computed tide by a probe arm connected to the tide computer output. Three platinum wires forming the probing element dip into the mercury cup on the control float. The servo-mechanism is arranged to give stepped proportional control about the centre probe as shown in Figure 10. When the tide is in error such that either the upper probe is touching the mercury surface or the lower probe is free of the mercury surface, the voltage applied to the servo-motor is increased above that applied when the surface is hunting about the end of the centre probe and a more rapid correction results.

When the river discharge is varied, the servo-motor will move the fulcrum, Figure 9, to a new mean position and further oscillation about this position is due to variation in the model surface area with tidal height, Figure 11, and to normal hunting.

The servo-motor used is a one-fifteenth horse-power shunt wound motor. This motor has a built-in speed reducer to give an output speed of nine revolutions per minute. This is connected by a sprocket and chain to a sprocket with a threaded hub which engages a screw attached to the fulcrum. Limit switches are attached to the fulcrum to give a maximum limit to its movement.

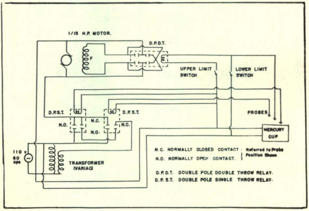
5. Tide Recorder:

This device records the computed tide, actual tide, and prototype time on a strip of adding machine tape. The recorder is operated by a selsyn



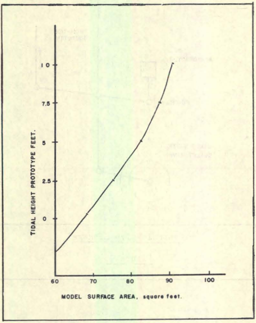
Control Valve Linkage

Figure 9.



Servo-Motor Control Circuit

Figure 10.



Height of Tide vs. Model Surface Area

Figure 11.

motor-generator system from the tide computer. During experiments the computed tide only is recorded, the actual tide being recorded when a check on the accuracy of the control mechanism is required. A photograph is shown in Figure 17.

6. River Headworks:

River flow is manually controlled in the model, using a 30 degree vee notch weir discharging into a basin and overflowing into the Somass River channel, Figure 14. Records of daily observations of the discharge of the Sproat and Stamp Rivers, which join to form the Somass, are available from the Water Resources and Drainage Board, and indicate that adjustment at intervals of 1 1/2 hours (weekly in the prototype) is sufficient, except in times of freshet. Very accurate reproduction of river flow can be obtained in this manner with relatively little expense.

Instrumentation

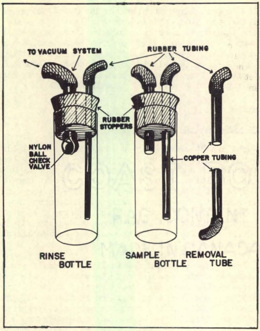
1. Methods of Observation:

Current data can be obtained by a commercially available instrument, the "Midget Current Meter", or by photographing neutrally buoyant droplets of carbon tetrachloride and petroleum ether. The current meter will give good results at velocities above 0.1 feet per second. The photographic method gives excellent accuracy and may be used with very low velocities in confined areas since the dye has a negligible effect on the flow pattern. It requires considerable time and equipment however. Flow paths are to be observed by the introduction of a dye with its density adjusted to correspond to a specific salinity, or, in the case of sewage disposal studies, the density of the sewage.

Salinity is to be determined chemically from small samples of water taken from the model during operation. The water sampler shown in Figure 12 is used to remove the samples. The sample is taken by immersing the withdrawal tube to a predetermined position in the model, and applying a vacuum to the rinse bottle at the time the sample is required. The water sample is thus drawn through the withdrawal tube and sample bottle, and into the rinse bottle until the water level reaches the nylon ball check valve. The forces of the water on the nylon ball cause it to close the end of the copper tube thus preventing further withdrawal of water. The rinse bottle is employed to make certain that no residual water from the previous sample is present. The rinse bottle and sample bottle are then removed from their rubber stoppers, the rinse water thrown away and the sample kept for analysis. In practice the water samplers are to be used in banks of five or more to correspond to the method of taking water samples in nature where several water "bottles" are lowered on a cable.

2. Verification:

The model will be verified by comparing the distribution of salinity with that observed in nature (7) under comparable conditions of tide and river discharge. It will be necessary to adjust the roughness of the fixed bed portion of the model in order to give a realistic reproduction of turbulence.



Water Sampler

Figure 12.

The validity of the results pertaining to bed movement will be checked by simulating natural conditions existing at some time in the past and operating the model so as to reproduce some phenomena which has been observed to occur. A comparison of results obtained in the model with the natural phenomena will provide an index of accuracy of bed movement.

Discussion

It is proposed to study the harbour oceanographically as an example of a tidal estuary, and to investigate the consequences of proposed physical changes in the river bed, and its approach to the harbour. This requires that the oceanographic structure and its variations in relation to tide and river discharge be investigated by observing the distribution of salinity, which corresponds to the density distribution since there will be no temperature gradients. Corresponding studies of the currents are required for the expression of the mechanics of the system. Bed movement study is required to determine the consequences of the proposed engineering works. At present the model is ready for verification and experiment.

A criterion for the scaling of density difference between salt and fresh water may be established as follows: In Keulegan's presentation of the Richardson-Prandtl number (5)

$$R_i = \frac{\rho\left(\frac{dV}{dz}\right)^2}{g\,\frac{d\rho}{dz}} > \frac{\nu_s}{\nu_e}$$

for turbulent exchange of density attribute where ν_s is the coefficient of diffusion of salt, and ν_e is the coefficient of eddy viscosity, the rate of doing work per unit volume by shear stresses is shown to be

$$\frac{dW}{dt} = \rho\,\nu_e\left(\frac{dV}{dz}\right)^2$$

and the rate of doing work per unit volume against gravity is shown to be

$$\frac{dE}{dt} = g\,\nu_s\,\frac{d\rho}{dz}$$

In order to have the same relative amount of vertical transfer of salt in both model and prototype, the vertical velocity of salt transfer, and hence the rate of doing work against gravity on a per unit volume basis in the model, must be increased in the ratio of the vertical to the horizontal scales, Z_r/X_r as

$$\frac{dE}{dt} = g(\nu_s)_m \frac{d\rho_m}{dZ_m} \cdot \frac{Z_r}{X_r}$$

Assuming that the ratio of the work done against gravity to that dissipated by the viscous action is the same for similar flow conditions in both model and prototype, so that

$$\frac{dW}{dt} = K \frac{dE}{dt}$$

then

$$Kg(\nu_e)_m \frac{d\rho_m}{dZ_m} \cdot \frac{Z_r}{X_r} = \rho(\nu_e)_m \left(\frac{dV_m}{dZ_m}\right)^2$$

or

$$Kg\left(\frac{\nu_s}{\nu_e}\right) \frac{d\rho_m}{dZ_p} \cdot \frac{Z_r}{Z_r X_r} = \rho \left(\frac{dV_p}{dZ_p}\right)^2 \left(\frac{V_r}{Z_r}\right)^2$$

from which

$$Kg\left(\frac{\nu_s}{\nu_e}\right)_m \frac{d\rho_m}{dZ_p} \cdot \frac{1}{X_r} = \rho \left(\frac{dV_p}{dZ_p}\right)^2 \cdot \frac{1}{Z_r}$$

since the Froudian model scales are used in which $V_r^2 = Z_r$. Written for the prototype, this equation becomes

$$Kg\left(\frac{\nu_s}{\nu_e}\right)_p \left(\frac{d\rho_p}{dZ_p}\right) = \rho \left(\frac{dV_p}{dZ_p}\right)^2$$

taking ratios

$$\left(\frac{\nu_s}{\nu_e}\right)_m \cdot \left(\frac{\nu_e}{\nu_s}\right)_p \cdot \frac{1}{X_r} \cdot \frac{d\rho_m}{d\rho_p} = \frac{1}{Z_r}$$

Now if ν_s/ν_e is the same for both model and prototype, then

$$\frac{\delta\rho_m}{\delta\rho_p} = \frac{x_r}{z_r}$$

Therefore the density difference should be diminished in the ratio of the scale distortion in order to give comparable density vs. depth curves for both model and prototype.

The validity of the above result depends on two assumptions which are not "a priori" correct. These are

1. The ratio of the work done against gravity to that dissipated by viscous action is the same for similar flow conditions in both model and prototype, and
2. The ratio ν_s/ν_e is the same for both model and prototype.

The assumption 1. is probably in error because faster relative movement is required in the model. Therefore a greater proportion of the work done by shear stresses would be converted into heat energy. If such is the case a further small reduction in the density scale ratio is required.

The validity of assumption 2. can only be proven by experiment. The above result is in agreement with von Arx (8), who suggests the same result on the basis of experiments with density stratified models.

The use of a pulley and cable system to integrate the tide has the following disadvantages:

1. Requires a great deal of space for the cables.
2. Is difficult to set to a specific instant of time when beginning an experiment. The operator cannot be certain of an initial crank setting within about three degrees in angle.
3. Elasticity and clearances in the mechanism results in a slightly distorted output curve.

While the effects of the above points are expected to have negligible effect on the experimental results, they are not desirable and should not be present in future designs of tide computers.

The control valves have functioned well but do leak at the packing gland. The original design called for an "O ring" seal in place of the packing washer, however, the valve shaft was pitted during the brazing of the valve sleeve component and a graphite impregnated cotton washer had to be used because of rough surfaces. The valve shaft should have been made extra large, a fine surfaced turned after brazing, and an "O ring" fitted. Original clearance between the 5.187 inch diameter valve sleeve and the valve body was about 0.003 inches. This had to be increased by lapping before satisfactory operation was obtained. The valve characteristic, Figure 8, could be made linear by properly shaping the valve ports.

The use of electronic devices has many advantages in hydraulic model studies and could have been used here to advantage had the author's knowledge and the funds available, been sufficient.

A tide computer adaptable to any hydraulic model could be made using the design of Lord Kelvin's tide predicting machine as a basis, but having the output taken off in the form of a voltage which could be used to operate the control valves. The cables could be replaced by a scotch yoke on each component crank connected to a variable resistance. These resistances could be summed by a series circuit and compared with tidal height.

The control float could be replaced with a probe in which a 0.002 inch wire is the only material in contact with the model water surface. A probe of this type is currently being developed at the National Research Council, Fraser River Model, and shows promise. In operation the end of the probe wire hunts continuously about the water surface. An accuracy of 0.003 inches is thought to be obtainable in this way.

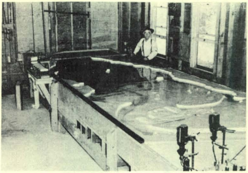
The probe could be connected to a potentiometer, which compared with the tide computer output, would operate the control valves through a servo-mechanism.

The method described above has none of the disadvantages of the present system and is completely adaptable.

Acknowledgment

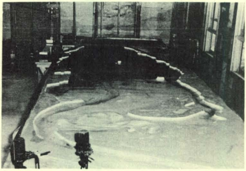
The model was constructed on the premises of the Pacific Biological Station of the Fisheries Research Board, at Nanaimo, B.C., and the machine work done in the shop of the Department of Mechanical Engineering at the University of British Columbia.

The author wishes to express his sincere gratitude to Mr. Prentice Bloedel of Bloedel, Stewart and Welch, Ltd., who provided valuable field data and the Bloedel, Stewart and Welch Fellowship in Oceanography which made this research possible; to Dr. J. P. Tully, Oceanographer in Charge of the Pacific Oceanographic Group, Fisheries Research Board, under whose direction this work was carried out; and to Professor W. O. Richmond of the University of British Columbia, for the many suggestions regarding the mechanical aspects of the model and for invaluable criticism of the manuscript.



The Model in Operation

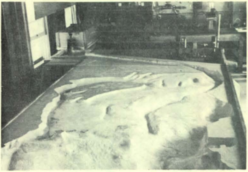
Tide recorder is on the left, control valves behind the man. Dimensions of the model are 23 feet by 7½ feet.



The Model

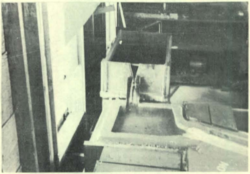
The bottles in the foreground are part of the dye injectors used for current observations.

Figure 13.



Tide Flat Area

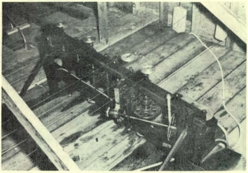
This area is built up in sand and 'fixed' with a spray of cement and water.



River Headworks

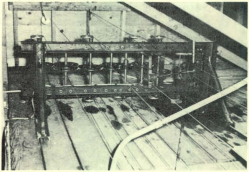
River flow is regulated by varying the head behind the weir.

Figure 14.



Tide Computer

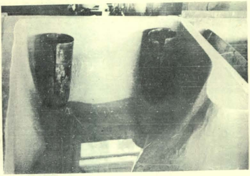
The induction motor is visible on the right. This drives the computer by a leather belt.



Tide Computer

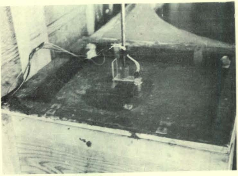
The curve of tidal height is taken from the upper cranks, the derivative from the lower. No derivative crank was placed on the left hand crank (S_2) because length was short.

Figure 15.



Standpipes

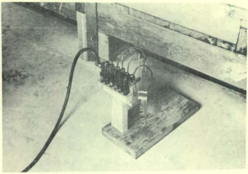
Vertical velocity distribution can be partly controlled by the shape of the openings



Control Float

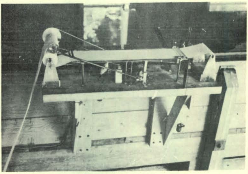
0.005 inch copper wires connect moving parts to remainder of electrical circuit.
The position of the probe arm is controlled by the Tide Computer.

Figure 16.



Water Sampler

The wooden base is used for storage only. When in use the copper tubes are immersed in the model to a predetermined depth.



Tide Recorder.

This is used to record the computed tide during an experiment. Prototype hours are marked on the tape and a counter records the date. Actual model tide can be traced on the under side of the tape for comparison. The tide recorder is driven by a Selsyn motor-generator from the tide computer.

Figure 17.

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