

Model equations for groundfish stock assessments in British Columbia using Stock Synthesis 3

Rowan Haigh, Paul J. Starr, Nick Fisch, Andrew M. Edwards

Pacific Biological Station
Fisheries and Oceans Canada,
3190 Hammond Bay Road,
Nanaimo, BC
V9T 6N7

2026

**Canadian Technical Report of
Fisheries and Aquatic Sciences 3806**



Fisheries and Oceans
Canada

Pêches et Océans
Canada

Canada

Canadian Technical Report of Fisheries and Aquatic Sciences

Technical reports contain scientific and technical information that contributes to existing knowledge but which is not normally appropriate for primary literature. Technical reports are directed primarily toward a worldwide audience and have an international distribution. No restriction is placed on subject matter and the series reflects the broad interests and policies of Fisheries and Oceans Canada, namely, fisheries and aquatic sciences.

Technical reports may be cited as full publications. The correct citation appears above the abstract of each report. Each report is abstracted in the data base *Aquatic Sciences and Fisheries Abstracts*.

Technical reports are produced regionally but are numbered nationally. Requests for individual reports will be filled by the issuing establishment listed on the front cover and title page. Out-of-stock reports will be supplied for a fee by commercial agents.

Numbers 1–456 in this series were issued as Technical Reports of the Fisheries Research Board of Canada. Numbers 457–714 were issued as Department of the Environment, Fisheries and Marine Service, Research and Development Directorate Technical Reports. Numbers 715–924 were issued as Department of Fisheries and Environment, Fisheries and Marine Service Technical Reports. The current series name was changed with report number 925.

Rapport technique canadien des sciences halieutiques et aquatiques

Les rapports techniques contiennent des renseignements scientifiques et techniques qui constituent une contribution aux connaissances actuelles, mais qui ne sont pas normalement appropriés pour la publication dans un journal scientifique. Les rapports techniques sont destinés essentiellement à un public international et ils sont distribués à cet échelon. Il n'y a aucune restriction quant au sujet; de fait, la série reflète la vaste gamme des intérêts et des politiques de Pêches et Océans Canada, c'est-à-dire les sciences halieutiques et aquatiques.

Les rapports techniques peuvent être cités comme des publications à part entière. Le titre exact figure au-dessus du résumé de chaque rapport. Les rapports techniques sont résumés dans la base de données *Résumés des sciences aquatiques et halieutiques*.

Les rapports techniques sont produits à l'échelon régional, mais numérotés à l'échelon national. Les demandes de rapports seront satisfaites par l'établissement auteur dont le nom figure sur la couverture et la page du titre. Les rapports épuisés seront fournis contre rétribution par les agents commerciaux.

Les numéros 1 à 456 de cette série ont été publiés à titre de Rapports techniques de l'Office des recherches sur les pêcheries du Canada. Les numéros 457 à 714 sont parus à titre de Rapports techniques de la Direction générale de la recherche et du développement, Service des pêches et de la mer, ministère de l'Environnement. Les numéros 715 à 924 ont été publiés à titre de Rapports techniques du Service des pêches et de la mer, ministère des Pêches et de l'Environnement. Le nom actuel de la série a été établi lors de la parution du numéro 925.

Canadian Technical Report of
Fisheries and Aquatic Sciences 3806

2026

MODEL EQUATIONS FOR GROUND FISH STOCK ASSESSMENTS IN BRITISH COLUMBIA
USING STOCK SYNTHESIS 3

by

Rowan Haigh, Paul J. Starr, Nick Fisch, Andrew M. Edwards

Pacific Biological Station
Fisheries and Oceans Canada,
3190 Hammond Bay Road,
Nanaimo, BC, V9T 6N7

© His Majesty the King in Right of Canada, as represented by the Minister
of the Department of Fisheries and Oceans, 2026
This work is licensed under the [Open Government Licence](#)
Cat. No. Fs97-6/3806E-PDF ISBN 978-0-662-74855-7 ISSN 1488-5379
<https://doi.org/10.60825/r674-jb43>

Correct citation for this publication:

Haigh, R., Starr, P.J., Fisch, N., Edwards, A.M. 2026. Model equations for groundfish stock assessments in British Columbia using Stock Synthesis 3. Can. Tech. Rep. Fish. Aquat. Sci. 3806: vi + 38 p. <https://doi.org/10.60825/r674-jb43>

Contents

ABSTRACT	v
RÉSUMÉ	vi
1. INTRODUCTION	1
1.1. Brief history of Stock Synthesis	5
2. MODEL ASSUMPTIONS	6
3. MODEL NOTATION AND EQUATIONS	8
3.1. Model notation	8
3.2. Model equations	12
3.3. Stochastic components	15
3.4. Prior expectation of parameters	17
4. DESCRIPTION OF MODEL COMPONENTS	18
4.1. Age classes	18
4.2. Years	18
4.3. Commercial Data	18
4.4. Survey Data	19
4.5. Sex	19
4.6. Weights-at-age	19
4.7. Maturity of females	19
4.8. Initial conditions	20
4.9. State dynamics	20
4.10. Selectivities	20
4.11. Derived states	22
4.12. Stock-recruitment function	23
4.13. Fitting to data	23
4.13.1. Abundance indices	23
4.13.2. Age frequencies	23
4.13.3. Ageing error	24
5. DESCRIPTION OF STOCHASTIC COMPONENTS	24
5.1. Parameters	24
5.2. Recruitment deviations	24
5.3. Recruitment allocation	25
5.4. Log-likelihood functions	26
6. BAYESIAN COMPUTATIONS	26
6.1. Phases	27
6.2. Reweighting	27
6.2.1. Abundance	27
6.2.2. Composition	29

6.3. Prior distributions	31
6.4. MCMC properties	31
7. REFERENCE POINTS, PROJECTIONS, AND ADVICE TO MANAGERS	32
8. SS3 INPUTS	33
CONTACT INFORMATION	33
9. REFERENCES – MODEL EQUATIONS	34

ABSTRACT

Haigh, R., Starr, P.J., Fisch, N., Edwards, A.M. 2026. Model equations for groundfish stock assessments in British Columbia using Stock Synthesis 3. Can. Tech. Rep. Fish. Aquat. Sci. 3806: vi + 38 p. <https://doi.org/10.60825/r674-jb43>

Starting in 2021, the Offshore Rockfish Program in the Groundfish Section of the Stock Assessment and Research Division at the Pacific Biological Station (PBS) in Nanaimo, British Columbia, started using the Stock Synthesis 3 (SS3) population model platform for fisheries stock assessments on Canada's Pacific coast. Prior to SS3, the Program had used Awatea, a model variant of Coleraine. The switch was made because SS3 offered more features and choices than those available in Awatea, and because support for the Coleraine variant had ended. Following the adoption of SS3 by other programs at PBS, it was considered expedient to transfer the model details to a Technical Report that could be cited by all users of SS3 at PBS. This report documents the SS3 platform as a transition from the Awatea platform, and describes the model structure, data inputs, and model fitting procedures in terms familiar to already-established conventions.

RÉSUMÉ

Haigh, R., Starr, P.J., Fisch, N., Edwards, A.M. 2026. Model equations for groundfish stock assessments in British Columbia using Stock Synthesis 3. Can. Tech. Rep. Fish. Aquat. Sci. 3806: vi + 38 p. <https://doi.org/10.60825/r674-jb43>

À compter de 2021, le Programme sur le sébaste hauturier de la Section des poissons de fond de la Division de l'évaluation et de la recherche sur les stocks de la Station biologique du Pacifique (SBP) à Nanaimo, en Colombie-Britannique, a commencé à utiliser la plateforme de modélisation des populations Stock Synthesis 3 (SS3) pour l'évaluation des stocks halieutiques sur la côte pacifique du Canada. Avant SS3, le Programme utilisait Awatea, une variante du modèle Coleraine. Ce changement a été motivé par le fait que SS3 offrait davantage de fonctionnalités et d'options qu'Awatea, et que le support technique de la variante Coleraine avait pris fin. Suite à l'adoption de SS3 par d'autres programmes de la SBP, il a été jugé opportun de consigner les détails du modèle dans un rapport technique pouvant être cité par tous les utilisateurs de SS3 à la SBP. Ce rapport décrit la plateforme SS3 comme une transition depuis Awatea et présente la structure du modèle, les données d'entrée et les procédures d'ajustement du modèle selon des conventions déjà établies.

1. INTRODUCTION

Starting in 2021, the Offshore Rockfish Program (ORP) in the Groundfish Section of the Stock Assessment and Research (StAR) Division at the Pacific Biological Station (PBS) in Nanaimo BC started using the Stock Synthesis 3 (SS3) population model for fisheries stock assessments (FSAs) in British Columbia (BC). Prior to SS3, ORP had used Awatea, a model variant of Coleraine (Hilborn et al. 2003), from 2007 to 2020. Model documentation for Awatea used in Fisheries and Oceans Canada (DFO) stock assessments first appeared in Stanley et al. (2009), and was updated by Edwards et al. (2012b). The switch from Awatea was made because SS3 offered more features and choices than those available in Awatea (e.g., retrospective analysis, log-likelihood analysis, time-varying parameters), and Awatea was no longer being maintained. Since the adoption of SS3 by various DFO Sections (Groundfish, Quantitative Assessment Methods, etc.), it was considered expedient to transfer the model details, which appeared as an appendix in ORP stock assessment documents, to a Technical Report that could be cited by all PBS users of SS3.

The earliest version of Stock Synthesis was developed for the northern anchovy (*Engraulis mordax*) fishery off California (USA) and northern Baja California (Mexico) (Methot 1986, 1989). It provided a framework to incorporate additional data (e.g., environmental effects like ocean temperature) and time-varying selectivity and natural mortality into the analyses. « Anchovy synthesis » (or SS) was extended to a more general framework (SS2) for US West Coast groundfish in 2005 (see Section 1.1.). Stock Synthesis 3 (Methot and Wetzel 2013; Methot et al. 2024), released in 2009, is a statistical age-structured population modelling framework that uses Automatic Differentiation Model Builder (ADMB) software for estimation of population trajectories and their uncertainties (using either maximum likelihood or Bayesian estimation). The primary benefits of the SS3 framework include :

- modelling flexibility – allows customization across diverse scenarios from data-poor to data-rich;
- supports complex functionality – multi-area, multi-gender, and multi-fleet models with variable growth, recruitment, and selectivity;
- supports variety of data sources – length/age frequencies, conditional age-at-lengths [CAAL], tagging data, environmental time series, etc.;
- ubiquitous adoption – understood by many stock assessment scientists;
- streamlines peer-review processes – reviewers are more likely to know SS3 than other, less common FSA models;
- dedicated maintenance – model authors responsive to user questions
- expected transition support – SS3 being phased out in favour of NOAA's Fisheries Integrated Modeling System (FIMS).

The Stock Synthesis Development Team at the National Oceanic and Atmospheric Administration (NOAA, U.S. Dept. Commerce) provides executables and documentation on how to run SS3. The SS3 source code is available on GitHub, as is an SS3 Graphical User Interface (GUI) called « Stock Synthesis Interface ».

SS3 has been used in age-structured assessments at PBS for (BC groundfish stocks) from 2021 to the present:

- 2026 – Dover Sole (DOL: BC) (Fisch et al. 2026)
- 2025 – Silvergray Rockfish (SGR: BC) (Starr and Haigh 2026a)
- 2025 – Lingcod (LIN: BC Outside) (Haggarty et al. 2025)
- 2024 – Yellowtail Rockfish (YTR: BC) (Starr and Haigh 2026b)
- 2024 – Petrale Sole (PEL: BC) (Mazur et al. 2026)
- 2023 – Pacific Ocean Perch (POP: 5ABC, 3CD, & 5DE) (Starr and Haigh 2025)
- 2022 – Canary Rockfish (CAR: BC) (Starr and Haigh 2023)
- 2021 – Yellowmouth Rockfish (YMR: BC) (Starr and Haigh 2022c)

Offshore Pacific Hake (*Merluccius productus*) was first assessed using Stock Synthesis in 1997 (Dorn and Saunders 1997). Stock assessments for Pacific Hake continue to use Stock Synthesis and are conducted annually by the « Joint Technical Committee of the Pacific Hake/Whiting Agreement Between the Governments of the United States and Canada », up to the latest one by Johnson et al. (2025).

Awatea has been used in age-structured assessments for (BC stocks) from 2007 to 2024:

- 2024 – Bocaccio (BOR: BC) update of 2019 assessment in 2024 (DFO 2024);
- 2021 – Bocaccio (BOR: BC) update of 2019 assessment in 2021 (DFO 2022);
- 2020 – Rougheye/Blackspotted Rockfish (REBS: 5DE & 3CD5AB) (Starr and Haigh 2022b);
- 2019 – Bocaccio (BOR: BC) (Starr and Haigh 2022a);
- 2019 – Widow Rockfish (WWR: BC) (Starr and Haigh 2021a);
- 2018 – Redstripe Rockfish (RSR: 5DE & 3CD5ABC) (Starr and Haigh 2021b);
- 2017 – Pacific Ocean Perch (POP: 5ABC) (Haigh et al. 2018);
- 2014 – Yellowtail Rockfish (YTR: BC) (DFO 2015);
- 2013 – Silvergray Rockfish (SGR: BC) (Starr et al. 2016);
- 2013 – Rock Sole (ROL: 5AB & 5CD) (Holt et al. 2016);
- 2012 – Pacific Ocean Perch (POP: 3CD) (Edwards et al. 2014b);
- 2012 – Pacific Ocean Perch (POP: 5DE) (Edwards et al. 2014a);
- 2011 – Yellowmouth Rockfish (YMR: BC) (Edwards et al. 2012a),
- 2010 – Pacific Ocean Perch (POP: 5ABC) (Edwards et al. 2012b);
- 2009 – Canary Rockfish (CAR: BC) update of 2007 assessment (DFO 2009);
- 2007 – Canary Rockfish (CAR: BC) (Stanley et al. 2009).

Offshore (slope and shelf) rockfish were assessed for 14 years using the simpler age-structured model Awatea, a version of Coleraine (Hilborn et al. 2003) modified and maintained by Allan Hicks, then at the University of Washington, now at the International Pacific Halibut Commission (IPHC). Both Awatea and SS3 are platforms for implementing ADMB software (ADMB Project 2009), which provides (i) maximum posterior density (MPD) estimates using a function minimiser and automatic differentiation, and (ii) an approximation of the posterior distribution of the parameters using the Markov Chain Monte Carlo (MCMC) method, specifically using the Metropolis algorithm (Gelman et al. 2004, although see Section 6.4.).

During the Awatea years, the nomenclature of some parameter names from that platform became de facto descriptors of parameter action:

- μ denotes the age at full selectivity;
- v_L denotes the variability of the left-hand (ascending) side of the selectivity curve approaching μ ;
- v_R denotes the variability of the right-hand (descending) side of the selectivity curve leaving μ ; and
- Δ denotes the male offset of μ

The chief strength of Coleraine|Awatea is the use of a robust likelihood formulation proposed by Fournier et al. (1998) for composition data by sex and age (or length). The robust normal model was used over the more traditional multinomial error model because it reduced the influence of observations with standardised residuals > 3 standard deviations (Fournier et al. 1990). Fournier et al. (1990) identified two types of deviations:

- type I – occasional occurrence of an event of very low probability; and
- type II – probability of observing an event with higher frequency than normal in the population (e.g., school of young fish).

Their robustified likelihood function reduces both types of deviations.

SS3 offers two error models: the multinomial and a compound Dirichlet-multinomial. The latter can estimate effective sample sizes that are similar to iterative reweighting methods, but without requiring multiple iterations of running the assessment model (Thorson et al. 2017).

The data inputs to SS3 comprise four files – « starter.ss », « data.ss », « control.ss », and « forecast.ss » – instead of a single file used by Awatea. Parameter control and priors appear in the control.ss file, and data appear in the data.ss file; these two files can be named anything the user wishes because the starter.ss file specifies their names. The names for the starter.ss and forecast.ss files must remain invariant. Unlike Awatea, which requires specifying an input file from the command line (e.g. « awatea -ind filename.txt »), calling SS3 is done by typing only « ss »¹ on the command line (assuming « ss.exe » occurs on the system's PATH) because the software assumes the presence of the four files listed above. Typically, stock assessments use the safe version of SS3 (e.g., « ss3.30.23_win.exe »), which performs bound checks, rather than the « optimised » version, which is reportedly *fast and optimised for speedy execution*. (The safe version can be renamed or copied to « ss.exe » for convenience.) The options in SS3 for fitting the data are more complex than those for Awatea and offer a greater degree of flexibility; however, this flexibility requires a steep learning curve and increases opportunities for inadvertent errors.

Reweighting abundance and composition data is desirable because it allows the model to give greater weight to the data source that is considered more reliable. Reweighting abundance data is accomplished by adding process error to index observation error (Francis 2011), essentially downweighting its importance.

¹or call « ss3 » where « ss3.exe » occurs on the PATH

There are three methods for reweighting composition data:

- mean-age weighting by Francis (2011), specifically method TA1.8 in Table A1 of his Appendix A;
- harmonic mean ratio based on McAllister and Ianelli (1997);
- weighting factor based on an estimable parameter (θ) from the Dirichlet-multinomial (D-M) distribution (Thorson et al. 2017).

All reweighting methods other than the D-M are performed on the data inputs to SS3, either by changing precision values in `data.ss` (e.g., adding process error to index observation error) or adding weight factors to the `control.ss` file. The D-M method for composition is built into SS3 as a likelihood component of the objective function. See Section 6.2. for more details.

The running of SS3 and the visualisation of output can be streamlined using custom code on the R statistical platform (R Core Team 2025). The best software for interacting with SS3 is provided by the R package « `r4ss` ». According to the authors:

“It is based on the original work of Ian Stewart [NOAA] begun around 2005 and released as an open source R package in 2009. The package has a long list of authors and has benefited from a large community of users making suggestions and reporting issues.”

– Ian Taylor et al., `r4ss` vignette, 2021

The ORP uses a collection of repositories called « PBS Software » (online : github.com/pbs-software), which was started by Jon T. Schnute with the creation of « `PBSmapping` » in 2003 to provide a mapping framework in S-Plus (and later in R). The repository « `PBSsynth` » adopts the « `r4ss` » package, modifying a few of their functions and creating new ones to facilitate the running and evaluation of SS3 model runs. `PBSsynth` relies on a set of data manipulation and plotting functions housed in « `PBStools` ».

An additional R package that remains essential for running MCMC simulations is provided by Cole Monnahan : « `adnuts` » (Monnahan 2018). The package makes use of ADMB’s « no U-turn sampling » (NUTS) algorithm (Monnahan and Kristensen 2018; Monnahan et al. 2019) and parses the workload into multiple, parallel chains. Both operations reduce the run time for MCMCs from days to hours (see Section 6.4. for more details); however, if the SS3 model is complex or if the likelihood surface is flat, run time can extend into days.

Using the above R-code products, figures and tables of output can be automatically produced from the R console. Additionally, the R function « `Sweave` » (Leisch 2002) collates, via $\text{\LaTeX}$, the large amount of figures and tables into PDF files for model runs, facilitating quick views of run results². Once model exploration is finished, separate functions build the results appendix from existing figures, newly-generated figures, and decision tables of probabilities with respect to reference points. `Sweave` has been somewhat superseded by « `knitr` » (Xie 2025) and « R Markdown » (Allaire et al. 2025),

²Alternatively, one can run the « `r4ss` » function « `SS_plots` » to generate multiple plots from the MLE fit.

with the R package « csasdown » (Anderson et al. 2026) custom built to format DFO documents.

Methot and Wetzel (2013) provide mathematical notation of equations used in the SS3 model in their Appendix A. Presented herein are select elements of SS3 that have been used to implement age-structured assessment models for ORP stocks in recent years, including model notation and equations for a statistical catch-at-age model formulation, specifics of the Bayesian estimation procedure and reweighting scheme, and methods for calculating reference points and performing projections. Similarities and differences between previously applied DFO Awatea models and more recent DFO SS3 models are presented throughout to help document the transition from Awatea to SS3. Note that some equation terms that appear in Appendix A of Methot and Wetzel (2013) have been dropped in the subset below, e.g., δ_s (duration of the season) was assumed to equal unity.

1.1. BRIEF HISTORY OF STOCK SYNTHESIS

An historical summary of SS3 is provided by Methot and Wetzel (2013). Additionally, Punt et al. (2020) provide a discussion on the development and future of integrated analysis (IA) for fisheries stock assessments.

Stock Synthesis (SS) evolved through several key stages, starting with initial development for northern anchovy in the 1980s, then re-development as a generalized model for west coast groundfish, and finally a third stage using ADMB for computation. The current version, SS3, builds upon these earlier models and continues to evolve with ongoing updates and refinements³. A general timeline is summarised below.

1980s : Initial development

- SS began as an IA approach, taking a more inclusive approach to modelling population dynamics and utilizing various fishery and survey data.
- The foundational work was done by Fournier and Archibald (1982).
- SS was initially developed for northern anchovy in California, with the core concepts established therein (Methot 1986, 1989).

1990s : Generalization and west coast groundfish focus

- SS was re-developed as a more generalized model, focusing on US West Coast groundfish.
- The FORTRAN-coded models for sablefish and hake were merged into a fully generalized framework, which was then termed « SS2 ».
- SS2 was used for US West Coast groundfish assessments starting in 2005.

2005–2009 : ADMB and SS3

- The transition to ADMB (Automatic Differentiation Model Builder) provided a major upgrade, enabling rapid model convergence and variance estimation.

³Official support for SS3 has not entirely ended at the time of writing; however, transition to a replacement platform called « Fisheries Integrated Modeling System » (FIMS) is underway.

- This version, still called SS2, continued to be used for US west coast groundfish assessments.
- A major re-work of internal structures in 2009 led to the renaming as SS3. These changes included:
 - conversion to a two-sex model, allowing for sex-specific growth and population dynamics;
 - inclusion of sex-specific conditional age-at-length data;
 - overhaul of size data handling (stronger filtering methods, ability to downweight unreliable data);
 - shift from age-based to length-based selectivity; introduction of more robust fitting algorithms (logistic and double-normal vs. spline smoothing);
 - separation of fisheries into distinct fleets operating within model subareas.

2010s–2025 : Ongoing development and refinements

- SS3 continues to evolve, with regular updates and new features being added (see NOAA's SS3 Virtual Lab).
- The model is coded in C++ and utilizes ADMB for parameter estimation.
- The current version, SS3, is publicly available on GitHub, which also provides archived versions.
- A Shiny app⁴ graphical user interface (SS3 helper) provides additional support.
- Stock Synthesis Interface (SSI) provides a guided user interface for configuring, running, and viewing output from SS3 models.
- Stock Assessment Continuum Tool (SACT, previously called « Stock Synthesis Data-limited Tool ») uses SS3 to implement a variety of assessment configurations. This tool was initially built to implement several standard data-limited assessment methods within the SS3 modeling framework. Under the new moniker, it can also implement more complex and data-rich configurations.
- An output processor package, « r4ss », provides a collection of R functions for interacting with Stock Synthesis.

2. MODEL ASSUMPTIONS

Model assumptions will vary from one stock assessment to the next. Sometimes, parameters like natural mortality are not estimable. Sometimes, priors need to be tightened to constrain parameter estimates within reasonable ranges. Regardless, the following model assumptions (usually year-age-sex structured) provide a typical starting point:

1. The assessed BC population of Requested Rockfish (RQR) comprises one offshore stock for the entire BC coast, which encompasses PMFC⁵ areas 3CD5ABCDE. Three subareas are specified within the coastwide stock using the multi-area option

⁴a web application built using the R programming language, primarily via the R package « shiny »

⁵Pacific Marine Fisheries Commission. PMFC areas are similar to but differ slightly from [Groundfish Management Areas](#).

within SS3 that allocates coastwide recruitment among subareas. This multi-area functionality (not typical but included for illustrative purposes) has been used for recent POP and SGR assessments in BC (Starr and Haigh 2025, 2026a).

2. The RQR fishery is dominated by trawl gear (> 95% by catch in the last five years). Annual catches used in the model are taken by 'Trawl' fisheries (bottom + midwater gear) coastwide. Negligible catch by other gears (halibut longline, sablefish trap, lingcod & salmon troll, and rockfish hook & line) is added to the Trawl catch to account for total annual removals. The annual catch is known without error and occurs at the end of each year.
3. The Beverton-Holt stock-recruitment relationship is time-invariant.
4. Selectivity is different among fleets (fisheries and surveys), and is invariant over time. Selectivity parameters for fleets are estimated when ageing data are available. Selectivity for fleets without ageing data can either be fixed or linked to a fleet with estimated selectivity.
5. Spawning stock biomass (B , often referred to as « SSB ») comprises mature females.
6. Mortality, maturity, and growth parameters are specific to a coastwide population.
7. Natural mortality M is estimated using a normal prior, and is invariant over time. This parameter differs between the two sexes.
8. Maturity-at-age for females is fixed and invariant over time, usually estimated outside the model using a logistic fit.
9. Growth parameters by sex are estimated outside the model (via von Bertalanffy MLE) and remain fixed and invariant over time within the model. These parameters differ between the two sexes most of the time.
10. Female fecundity-at-age is directly proportional to female mean weights-at-age.
11. Recruitment at age 0 is 50% females and 50% males.
12. Recruitment standard deviation (σ_R) is fixed at 0.9.
13. An ageing error (AE) vector is based on estimates from NOAA's R package « AgeingError » (Richards et al. 1992; Punt et al. 2008).
14. Relative abundance indices are proportional to the vulnerable biomass part way through a year after mortality (fishing and natural) has occurred up until that point.
15. The « log » function in equations and computations refers to natural logarithms.

Additional to model assumptions, data assumptions include:

1. Only fish ages determined using the preferred otolith break-and-burn methodology (MacLellan 1997) are used because ages determined by surface ageing methods (chiefly before 1978) are likely biased (Beamish 1979). Surface ageing is deemed suitable for very young rockfish (ages 1-3).
2. Commercial samples of catch-at-age are representative of the fishery in a given year if there are ≥ 2 sampling events (called « samples ») and ≥ 50 (or 75, depending on the data richness) aged otoliths in that year.

3. MODEL NOTATION AND EQUATIONS

Model notation in Table 1 uses a hypothetical Requested Rockfish (RQR) that is assessed coastwide using three subareas⁶ for illustrative purposes. The table format is developed from that first used by Schnute et al. (2001), and follows the notation guidelines of Edwards and Auger-Méthé (2019). Model equations appear in Tables 2 and 3, and example prior distributions appear in Table 4. The model description is divided into the model equations, stochastic components and Bayesian priors. Full details of notation and equations are given after the tables. Model inputs, housed on GitHub, are described in Section 8.

The model equations in Table 2 iteratively calculate numbers of fish in each age class (and of each sex) through time, using the commercial catch data, weights-at-age, maturity-at-age, and known fixed parameter values. In practice, known fixed values are not available for all parameters; therefore, many of them need to be estimated. For recruitment deviations, stochasticity is added (Table 3).

Incorporation of the prior distributions for estimated parameters is necessary for a full Bayesian implementation, the goal of which is to minimise the objective function $\mathcal{F}(\Theta)$ given by (49). This function is derived as the sum of the negative log likelihoods from the the model equations, stochastic components, and priors of the model.

3.1. MODEL NOTATION

Table 1. Notation for the SS3 catch-at-age model (continued overleaf). The assessment model uses only ‘cohorts’ (age-classes by year) even though SS3 recognises finer subdivisions of time called ‘morphs’ (seasons), which can be further characterised by ‘platoons’ (rates of growth).

Symbol	Description and units
Indices: subscripts, sometimes variables	
a	► age class, where $a = 0, 1, 2, 3, \dots, A$, corresponds to actual ages 0, ..., 60 ▷ a' = reference age near youngest age well-represented in data ▷ a'' = reference age near oldest age well-represented in data
t	► model year, where $t = 1, 2, 3, \dots, T$, corresponds to actual years: 1935, ..., 2026, and $t = 0$ represents unfished equilibrium conditions
g	► index for series (abundance composition) data: 1 – 5ABC Trawl Fishery (commercial data) 2 – 5DE Trawl Fishery (commercial data) 3 – 3CD Trawl Fishery (commercial data) 4 – QCS Synoptic survey series 5 – WCHG Synoptic survey series 6 – WCVI Synoptic survey series 7 – HS Synoptic survey series 8 – GIG Historical survey series 9 – NMFS Triennial survey series

⁶to track catch and CPUE by area-based fleets

Symbol	Description and units
s	► sex, 1=females, 2=males
l	► length bin, where $l = 1, 2, 3, \dots, \Lambda$, and Λ is bin index of largest length
α	► productivity parameter related to survival rates at low stock densities
β	► density-dependence parameter controlling how quickly recruitment rate reaches its asymptote
$\hat{a}$	► subarea, where 1 = 5ABC, 2 = 5DE, 3 = 3CD
$\dot{a}$	► allometric scaling constant/intercept (for body weight, fecundity, etc.)
$\dot{b}$	► allometric scaling exponent/slope (for body weight, fecundity, etc.)
Index ranges	
A	► accumulator age-class, $A \in \{60\}$
G	► number of fleets (fisheries and surveys)
Λ	► number of length bins
T	► number of model years, $T = 92$ (1935-2026); end year for data : 2025
T_g	► sets of model years for survey abundance indices from series g , listed here for clarity as actual years (subtract 1934 to give model year t): $T_1 = \{1996, \dots, 2024\}$ $T_2 = \{1996, \dots, 2024\}$ $T_3 = \{1996, \dots, 2024\}$ $T_4 = \{2003:2005, 2007, 2009, 2011, 2013, 2015, 2017, 2019, 2021, 2023, 2025\}$ $T_5 = \{1997, 2006:2008, 2010, 2012, 2016, 2018, 2020, 2022, 2024\}$ $T_6 = \{2004, 2006, 2008, 2010, 2012, 2014, 2016, 2018, 2021:2022, 2024\}$ $T_7 = \{2005, 2007, 2009, 2011, 2013, 2015, 2017, 2019, 2021, 2023\}$ $T_8 = \{1967, 1969, 1971, 1973, 1976:1977, 1984, 1994\}$ $T_9 = \{1980, 1983, 1989, 1992, 1995, 1998, 2001\}$
U_g	► sets of model years with proportion-at-age data for series g : $U_1 = \{1978:1981, 1986, 1988, 1990:2007, 2009:2015, 2017, 2023\}$ $U_2 = \{1990, 1994:2004, 2009:2011, 2013, 2024\}$ $U_3 = \{1998:2006, 2013:2014, 2023:2024\}$ $U_4 = \{2004, 2009, 2011, 2019, 2023\}$ $U_5 = \{1997, 2010, 2020, 2022, 2024\}$ $U_6 = \{2018, 2021:2022\}$
Data and fixed parameters	
$\tilde{a}_a$	► age a incremented to mid-year values by adding 0.5; used to calculate proportion expected age Ψ_a (without ageing error)
ϑ	► degree of nonlinearity in ageing error function ξ_a (Punt et al. 2008)
p_{atgs}	► observed weighted proportion of fish from series g in each year $t \in U_g$ that are age-class a and sex s ; so $\sum_{a=1}^A \sum_{s=1}^2 p_{atgs} = 1$ for each $t \in U_g$

Symbol	Description and units
c	► small constant (e.g., 0.0001) added to observed and expected proportions-at-length and age to make log likelihood calculations more robust
n_{tg}	► specified sample size that yields corresponding p_{atgs}
$\tilde{n}_{tg}$	► effective sample size based on $\hat{p}_{atgs}$
φ_{las}	► age-length transition matrix for female fish ($s=1$) by age a and length l
m_a	► proportion of age-class a for females that are mature, fixed from data
o_a	► egg production (eggs/kg) of age-class a females
f_a	► fecundity of age-class a females
w_{as}	► average weight (kg) of individual of age-class a and sex s (using fixed parameters)
σ_R	► standard deviation parameter for recruitment process error, $\sigma_R = 0.9$
ϵ_t	► recruitment deviations arising from process error, where $\epsilon_t \sim \mathcal{N}(0, \sigma_R^2)$
b_t	► recruitment bias adjustment fraction applied during year t ► modulates recruitment variance over time: $b_t \sigma_R^2$ ► ranges from 0 (data-poor years) to 1 (data-rich years)
$\hat{x}$	► estimated values of observed data x (generalised)
$\hat{m}_g$	► month when data are collected for fleet g indices
$L_{\infty, s}$	► theoretical maximum length that an average fish of sex s in a given population would reach given an infinitely long period of time to grow; ► k = von Bertalanffy growth coefficient; ► t_0 = theoretical age when size is zero.

Estimated parameters

Θ	► set of estimated parameters:
R_0	► unfished equilibrium recruitment of age-0 fish (numbers of fish, 1000s)
$(\hat{p}_{\hat{a}}, \nu_{t, \hat{a}}, \zeta_{\hat{a}})$	► log proportion recruitment allocated to subarea $\hat{a}$; SD added to $\hat{p}_{\hat{a}}$; SE of $\nu_{t, \hat{a}}$
M_s	► natural mortality rate for sex $s = 1, 2$
h	► steepness parameter for Beverton-Holt recruitment
q_g	► catchability for fleets ($g = 1, \dots, 9$); can also be determined analytically
β_{itg}	► double-normal parameters for female ($s=1$) selectivity $S_{ag, s=1}$ where index $i = 1, \dots, 6$ for the six β parameters by year t and abundance index series $g = 1, \dots, 6$, using : ► $j_{1:2, atg}$ = joiner functions for ascending descending components ► $\pi_{1:2, atg}$ = ascending descending limbs when estimating minimum and maximum age at selectivity ► $\gamma_{0:3, tg}$ = exponential terms in $\pi_{1:2, atg}$ <u>Awatea equivalents :</u> ► $\mu_{tg} = \beta_{1tg}$ ► $\log v_{Ltg} = \beta_{3tg}$ ► $\log v_{Rtg} = \beta_{4tg}$

Symbol	Description and units
Δ_{itg}	▶ shift in vulnerability for male ($s=2$) selectivity $S_{ag,s=2}$ where index $i = 1, \dots, 5$ for the five Δ parameters and subscripts tg are the same as those for β <u>Awatea equivalents :</u> ▷ $\Delta_{tg} = \Delta_{1tg}$
Derived states	
L_{ats}	▶ mean size at age a of sex s at start of year t ▷ L' = reference length for a'; ▷ L'' = reference length for a''; ▷ $\check{L}_l, \dot{L}_l$ = minimum and middle length of length bin l, respectively; ▷ $\check{L}_l, \dot{L}_l$ = minimum and middle length of length bin l, respectively.
N_{ats}	▶ number of age-class a fish (1000s) of sex s at start of year t
B_t	▶ spawning biomass (tonnes mature females) at start of year t
B_0	▶ unfished spawning biomass (tonnes mature females) at start of year 0
R_t	▶ recruitment of age-0 fish (numbers of fish, 1000s) in year t
ρ_t	▶ recruitment deviations (log thousands age-0 fish) in year t
V_{tg}	▶ vulnerable biomass (tonnes, females + males) in the middle of year t
D_{tg}	▶ mid-season retained dead biomass (tonnes, females + males)
F_{tg}	▶ instantaneous fishing mortality rate for time period t by fishery g ▷ hybrid method uses Pope's approximation and Baranov's equation ▷ calculations facilitated by temporary variables $\mathcal{T}_{tg}$ and joiners $\mathcal{J}_{tg}$
$F_{\max}$	▶ maximum allowable fishing mortality rate (fixed at $F=2.9$)
Z_{ats}	▶ total mortality rate (natural & fishing) for time period t and sex s
Log-likelihood components	
$\mathcal{L}_{1g}(\Theta \{\widehat{I}_{tg}\})$	▶ CPUE or abundance index series, $\widehat{I}_{tg}$, by year (t) and fleet (g) ▷ κ_{tg} = standard deviation of $\widehat{I}_{tg}$ ▷ κ'_{tg} = SD offset specified by user (extra variance added to CV of $\widehat{I}_{tg}$)
$\mathcal{L}_{2g}(\Theta \{d_{tg}\})$	▶ discard biomass, d_{tg}, by year (t) and fleet (g) ▷ δ_{tg} = standard deviation of d_{tg} ▷ δ'_{tg} = SD offset specified by user (extra variance added to CV of d_{tg})
$\mathcal{L}_{3g}(\Theta \{\bar{w}_{tg}\})$	▶ mean body weight, $\bar{w}_{tg}$, by year (t) and fleet (g) ▷ ψ_{tg} = standard deviation of $\bar{w}_{tg}$ ▷ ψ'_{tg} = SD offset specified by user (extra variance added to CV of $\bar{w}_{tg}$)
$\mathcal{L}_{4g}(\Theta \{l_{tg}\})$	▶ length composition
$\mathcal{L}_{5g}(\Theta \{a_{tg}\})$	▶ age composition
$\mathcal{L}_{6g}(\Theta \{z_{tg}\})$	▶ generalised size composition
$\mathcal{L}_{7g}(\Theta \{C_{tg}\})$	▶ catch, C_{tg}, by year (t) and fleet (g) ▷ τ_{tg} = standard deviation of C_{tg}
$\mathcal{L}_R(\Theta \{R_{tg}\})$	▶ recruitment deviations ▷ σ_R = standard deviation parameter for recruitment process error

Symbol	Description and units
$\mathcal{L}_{\phi_j}(\Theta \{\phi_j\})$	► priors for each parameter ϕ_j ▷ μ_{ϕ_j} = prior mean value of parameter ϕ_j ▷ σ_{ϕ_j} = standard deviation for prior of parameter ϕ_j
$\mathcal{L}_{P_t}(\Theta \{P_t\})$	► random parameter deviations (if time-varying)
$\mathcal{L}(\Theta)$	► total log-likelihood
Prior distributions and objective function	
$\phi_j(\Theta)$	► prior distribution for parameter j
$\phi(\Theta)$	► joint prior distribution for all estimated parameters
$\mathcal{F}(\Theta)$	► objective function to be minimised

3.2. MODEL EQUATIONS

Table 2. Model equations. Using the catch, weight-at-age and maturity data, with fixed values for all parameters, the initial conditions are calculated from (1)-(6), and then state dynamics are iteratively calculated through time using the main equations (7), selectivity functions (8)-(14), and the derived states (16)-(34). Estimated observations for survey biomass indices and proportions-at-age can then be calculated using (38) and (39). In Table 3, the estimated observations of these are compared to data. Equation numbers in blue refer to source equations, where SS = Stock Synthesis Appendix A (Methot and Wetzel 2013), COL = Coleraine (Hilborn et al. 2003), and AE92 = Ageing Error Model 1992 (Richards et al. 1992).

Initial conditions ($t = 0$; $s = 1, 2$)

$$N_{a0s} = 0.5R_0e^{-aM_s} ; \quad 0 \leq a \leq 3A-1 \quad \text{SS (A.1.1) (1)}$$

$$N_{A0s} = \sum_{a=A}^{3A-1} N_{a0s} + (N_{3A-1,0s} e^{-M_{As}}) / (1 - e^{-M_{As}}) \quad \text{SS (A.1.2) (2)}$$

$$B_0 = B_1 = \sum_{a=0}^A f_{as} N_{a0s} ; \quad \text{where } f_{as} = w_{as} m_{as} o_{as} \quad \langle s=1 \text{ (female)} \rangle \quad \text{SS (A.1.19) (3)}$$

$$L_{a0s} = \begin{cases} \check{L}_1 + (a/a') (L'_s - \check{L}_1) & ; \quad a \leq a' \\ L_{\infty s} + (L'_s - L_{\infty s}) e^{-k_s(a-a')} & ; \quad a' < a \leq A-1 \end{cases} \quad \text{SS (A.1.3) (4)}$$

$$L_{\infty s} = L'_s + (L''_s - L'_s) / (1 - e^{-k_s(a''-a')}) \quad \text{SS (A.1.5) (5)}$$

$$L_{A0s} = \frac{\sum_{a=A}^{2A} [e^{-0.2(a-A)}] [L_{As} + ((a-A)/A)(L_{\infty s} - L'_{0s})]}{\sum_{a=A}^{2A} e^{-0.2(a-A)}} \quad \text{SS (A.1.6) (6)}$$

State dynamics ($2 \leq t \leq T$; $s = 1, 2$)

$$N_{a,t+1,s} = \begin{cases} 0.5R_{a=0,t+1,s} & ; a = 0 \\ N_{a-1,t,s} e^{-Z_{a,t,s}} & ; 1 \leq a \leq A-1 \\ N_{A-1,t,s} e^{-Z_{A-1,t,s}} + N_{A,t,s} e^{-Z_{A,t,s}} & ; a = A \end{cases} \quad \text{SS (A.1.20) (7)}$$

Selectivity pattern 20 ($g = 1, \dots, 6$)

$$S_{ags} = \pi_{1ags}(1 - j_{1ags}) + j_{1ags} \left[(1 - j_{2ags}) + j_{2ags} \pi_{2ags} \right] \quad \text{SS (A.1.30) (8)}$$

$$j_{1ags} = 1 / \left[1 + e^{-20(a-\beta_{1gs})/(1+|a-\beta_{1gs}|)} \right] \quad \langle \text{first joiner function: ascending} \rangle \quad \text{SS (A.1.31) (9)}$$

$$j_{2ags} = 1 / \left[1 + e^{-20(a-a_{gs}^*)/(1+|a-a_{gs}^*|)} \right] \quad \langle \text{second joiner function: descending} \rangle \quad \text{SS (A.1.31) (10)}$$

$$a_{gs}^* = \beta_{1gs} + (0.99A - \beta_{1gs})/(1 + \beta_{2gs}) \quad \langle \text{assuming age bin}=1y \rangle \quad \text{SS (A.1.34) (11)}$$

$$\pi_{1ags} = \left(\frac{1}{1 + e^{-\beta_{5gs}}} \right) + \left(1 - \frac{1}{(1 + e^{-\beta_{5gs}})} \right) \left(\frac{\gamma_{0gs} - \gamma_{1gs}}{1 - \gamma_{1gs}} \right) \quad \langle \text{ascending limb} \rangle \quad \text{SS (A.1.32) (12)}$$

$$\pi_{2ags} = 1 + \left[\left(\frac{1}{1 + e^{-\beta_{6gs}}} \right) - 1 \right] \left(\frac{\gamma_{3gs} - 1}{\gamma_{2gs} - 1} \right) \quad \langle \text{descending limb} \rangle \quad \text{SS (A.1.32) (13)}$$

$$\text{where } \gamma_{1gs} = e^{-\left[\frac{(a' - \beta_{1gs})^2}{e^{\beta_{3gs}}} \right]} ; \gamma_{2gs} = e^{-\left[\frac{(a'' - a_{gs}^*)^2}{e^{\beta_{4gs}}} \right]} \quad \text{SS (A.1.33) (14)}$$

$$\text{and } \gamma_{0gs} = e^{-\left[\frac{(a - \beta_{1gs})^2}{e^{\beta_{3gs}}} \right]} ; \gamma_{3gs} = e^{-\left[\frac{(a - a_{gs}^*)^2}{e^{\beta_{4gs}}} \right]} \quad (15)$$

Derived states ($1 \leq t \leq T-1$)

$$L_{ats} = L_{a-1,t-1,s} + (L_{a-1-k,t-1,s} - L_{\infty s}) (e^{-k_s} - 1); \quad a < A \quad \text{SS (A.1.10) (16)}$$

$$L_{Ats} = \frac{N_{A-1,t,s} \bar{L}_{Ats} + N_{Ats} [L_{Ats} - (L_{Ats} + L_{\infty s}) (e^{-k_s} - 1)]}{N_{A-1,t,s} + N_{Ats}} \quad \text{SS (A.1.11) (17)}$$

$$\bar{L}_{ats} = L_{ats} + (L_{ats} - L_{\infty s}) (e^{-0.5k_s} - 1) \quad \text{SS (A.1.12) (18)}$$

$$w_{ls} = \dot{a}_s \dot{L}_l^s \quad \langle \dot{L}_l = \text{mid-size of length bin } l \rangle \quad \text{SS (A.1.15) (19)}$$

$$f_a = \sum_{l=1}^{\Lambda} \varphi_{las} m_l o_l w_{ls} \quad \langle s=1, m=\text{maturity}, o=\text{eggs/kg} \rangle \quad \text{SS (A.1.18) (20)}$$

$$B_t = \sum_{a=0}^A N_{ats} f_a \quad \langle s=1 \text{ (females only)}, f=\text{fecundity} \rangle \quad \text{SS (A.1.19) (21)}$$

$$R_t = \frac{4hR_0B_t}{(1-h)B_0 + (5h-1)B_t} \left(\equiv \frac{B_t}{\alpha + \beta B_t} \right) \quad \text{SS (A.1.7)} \quad (22)$$

$$Z_{ats} = M_{as} + \sum_{g \in 1, \dots, 3} (S_{ags} F_{tg}) \quad \langle F_{tg} = \text{maximum fishing mortality rate} \rangle \quad \text{SS (A.1.21)} \quad (23)$$

$$\mathcal{T}_{1tg} = C_{tg}/(\widehat{D}_{tg} + 0.1C_{tg}) ; \quad \mathcal{J}_{1tg} = 1/[1 + e^{30(\mathcal{T}_{1tg}-0.95)}] \quad \text{SS (A.1.22)} \quad (24)$$

$$\mathcal{T}_{2tg} = \mathcal{J}_{1tg}\mathcal{T}_{1tg} + 0.95(1 - \mathcal{J}_{1tg}) ; \quad F_{1tg} = -\log(1 - \mathcal{T}_{2tg}) \quad \text{SS (A.1.22)} \quad (25)$$

$$C_{ats} = \sum_{g \in 1, \dots, 3} \frac{F_{tg}}{Z'_{ats}} (N_{ats} S_{ags}) \lambda'_{ats} \quad \text{SS (A.1.23)} \quad (26)$$

$$\lambda_{ats} = (1 - e^{-Z_{ats}})/(Z_{ats}) \quad \langle \text{survivorship} \rangle \quad \text{SS (A.1.24)} \quad (27)$$

$$\widehat{C}_t = \sum_{g \in 1, \dots, 3} \sum_{s=1}^2 \sum_{a=0}^A \frac{F_{1tg}}{Z_{ats}} w_{as} N_{ats} S_{ags} \lambda_{ats} ; \quad \text{SS (A.1.25)} \quad (28)$$

$$\text{where } \lambda_{ats} = (1 - e^{-Z_{ats}})/(Z_{ats})$$

$$\overrightarrow{Z}_t = C_t/(\widehat{C}_t + 0.0001) \quad \text{SS (A.1.25)} \quad (29)$$

$$Z'_{ats} = M_{as} + \overrightarrow{Z}_t(Z_{ats} - M_{as}) ; \quad \lambda'_{ats} = (1 - e^{-Z'_{ats}})/(Z'_{ats}) \quad \text{SS (A.1.26)} \quad (30)$$

$$\mathcal{T}_{3tg} = \sum_{s=1}^2 \sum_{a=0}^A w_{as} N_{ats} S_{ags} \lambda'_{ats} \quad \text{SS (A.1.27)} \quad (31)$$

$$F_{2tg} = C_{tg}/(\mathcal{T}_{3tg} + 0.0001) ; \quad \mathcal{J}_{2tg} = 1/[1 + e^{30(F_{2tg}-0.95F_{\max})}] \quad \text{SS (A.1.27)} \quad (32)$$

$$F_{tg} = \mathcal{J}_{2tg}F_{2tg} + (1 - \mathcal{J}_{2tg})F_{\max} \quad \langle \text{updated estimate of } F \text{ using hybrid method} \rangle \quad \text{SS (A.1.28)} \quad (33)$$

$$V_{tg} = \sum_{s=1}^2 \sum_{a=1}^A e^{-M_s(\dot{m}_g/12)} w_{as} N_{ats} S_{ags} ; \quad u_{tg} = C_{tg}/V_{tg}, \quad u_{atgs} = u_{tg}S_{ags} \quad \text{COL (1.2.1)} \quad (34)$$

Ageing error

$$\Phi(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-\mu)/\sigma} e^{-(t^2/2)} dt \quad \langle \text{standard normal cumulative density} \rangle \quad (35)$$

$$\Psi_a = \begin{cases} \Phi\left(\frac{a-\widetilde{a}_a}{\widetilde{a}_a}\right) & ; \quad a = 1 \\ \Phi\left(\frac{a+1-\widetilde{a}_a}{\widetilde{a}_a}\right) - \Phi\left(\frac{a-\widetilde{a}_a}{\widetilde{a}_a}\right) & ; \quad 1 < a < A \\ 1 - \Phi\left(\frac{A-\widetilde{a}_a}{\widetilde{a}_a}\right) & ; \quad a = A \end{cases} \quad \langle \text{expected age w/out ageing error} \rangle \quad \text{SS (A.2.7)} \quad (36)$$

$$\xi_a = \begin{cases} \sigma_1 + (\sigma_A - \sigma_1) \left(\frac{1-e^{-\vartheta(a-1)}}{1-e^{-\vartheta(A-1)}} \right) & ; \quad \vartheta \neq 1 \\ \sigma_1 + (\sigma_A - \sigma_1) \left(\frac{a-1}{A-1} \right) & ; \quad \vartheta = 0 \quad (\text{linear}) \end{cases} \quad \langle \text{ageing error} \rangle \quad \text{AE92 (1.6)} \quad (37)$$

Estimated observations

$$\widehat{I}_{tg} = q_g \sum_{s=1}^2 \sum_{a=1}^A e^{-Z_{ats}(\hat{m}_g/12)} w_{as} S_{ags} N_{ats}; \quad t \in \mathbf{T}_g, g = 1, \dots, 9 \quad \langle \text{abundance} \rangle \quad \text{SS (A.2.2)} \quad (38)$$

$$\widehat{p}_{atgs} = \frac{\Psi_{atgs} \xi_{atgs} + c}{\sum_{s=1}^2 \sum_{a=1}^A \Psi_{atgs} \xi_{atgs} + c}; \quad 1 \leq a \leq A, t \in \mathbf{U}_g, g=1, \dots, 6, s=1, 2 \quad \langle \text{composition} \rangle \quad \text{SS (A.2.8)} \quad (39)$$

3.3. STOCHASTIC COMPONENTS

Table 3. Stochastic (random) components. Calculation of likelihood function $\mathcal{L}(\Theta)$ for stochastic components of the model in Table 2, and resulting objective function $f(\Theta)$ to be minimised. Equation numbers in blue refer to source equations, where SS = Stock Synthesis Appendix A (Methot and Wetzel 2013). Labels on Θ : $\langle \text{typical} \rangle$ = typical FSA using single-area model, $\langle \text{multi-area} \rangle$ = FSA using multi-area model where subareas are allocated coastwide recruitment, $\langle \text{dome-shaped} \rangle$ = selectivity is parameterized to allow dome-shaped fits (descending right-hand limb).

Estimated parameters

$$\Theta = \begin{cases} R_0; M_{1,2}; h; \mu_{1,\dots,6}; \log v_{L1,\dots,6}; \Delta_{1,\dots,6} & \langle \text{typical} \rangle \\ R_0; M_{1,2}; h; \hat{p}_a; \mu_{1,\dots,6}; \log v_{L1,\dots,6}; \Delta_{1,\dots,6} & \langle \text{multi-area} \rangle \\ R_0; M_{1,2}; h; \mu_{1,\dots,6}; \log v_{L1,\dots,6}; \log v_{R1,\dots,6}; \Delta_{1,\dots,6} & \langle \text{dome-shaped} \rangle \end{cases} \quad (40)$$

Recruitment deviations

$$\rho_t = \log R_t - \log B_t + \log(\alpha + \beta B_t) + 0.5 b_t \sigma_R^2 + \epsilon_t \quad \langle \epsilon_t \sim \mathcal{N}(0, \sigma_R^2), 1 \leq t \leq T-1 \rangle \quad \text{SS (A.1.7)} \quad (41)$$

$$\text{where } b_t = \begin{cases} 0 & ; t \leq y_1 \\ b_{\max} [1 - (t - y_1)/(y_2 - y_1)] & ; y_1 < t < y_2 \\ b_{\max} & ; y_2 \leq t \leq y_3 \\ b_{\max} [1 - (y_3 - t)/(y_4 - y_3)] & ; y_3 < t < y_4 \\ 0 & ; y_4 \leq t \end{cases} \quad \langle \rho_t \text{ w/bias adj} \rangle \quad \text{SS (A.1.8)} \quad (42)$$

and:

- y_1 = first year of bias ramp up adjustment period
- y_2 = last year of bias ramp up adjustment period
- y_3 = first year of the bias ramp down adjustment period
- y_4 = last year of the bias ramp down adjustment period
- $b_{\max}$ = maximum bias adjustment applied to recruitment deviations

Log-likelihood components

$$\mathcal{L}_{1g}(\Theta | \{\widehat{I}_{tg}\}) = \sum_{t \in \mathbf{T}_g} \left[\frac{(\log I_{tg} - \log(q_g B_{tg}))^2}{2\kappa_{tg}^2} + \kappa'_{tg} \log \kappa_{tg} \right] \quad \text{SS (A.3.2)} \quad (43)$$

$$\mathcal{L}_{5g}(\Theta|\{a_{tg}\}) = \sum_{t \in \mathbf{U}_g} \sum_{s=1}^2 \sum_{a=1}^A n_{tgs} p_{atgs} \log(p_{atgs}/\widehat{p}_{atgs}) \quad \text{SS (A.3.6)} \quad (44)$$

$$\mathcal{L}_{7g}(\Theta|\{C_{tg}\}) = \sum_{t=1}^T [\log C_{tg} - \log(\widehat{C}_{tg} + 1\mathbf{e-6})]^2 / 2\tau_{tg}^2 \quad \text{SS (A.3.8)} \quad (45)$$

$$\mathcal{L}_R(\Theta|\{R_t\}) = 0.5 \sum_{t=1}^T (\widetilde{R}_t^2 / \sigma_R^2) + b_t \log \sigma_R^2 \quad \text{SS (A.3.10)} \quad (46)$$

$$\mathcal{L}_{\phi_j}(\Theta|\{\phi_j\}) = 0.5 [(\phi_j - \mu_{\phi_j}) / \sigma_{\phi_j}]^2 \quad \langle \text{normal prior distribution} \rangle \quad \text{SS (A.3.11)} \quad (47)$$

$$\mathcal{L}_{\phi_j}(\Theta|\{\phi_j\}) = 0.5 [(\log \phi_j - \mu_{\phi_j}) / \sigma_{\phi_j}]^2 \quad \langle \text{lognormal prior distribution} \rangle \quad \text{SS (A.3.10)} \quad (48)$$

Objective function

$$\mathcal{F}(\Theta) = \sum_{i \in \{1,5,7\}} \sum_{g=1}^G \omega_{ig} \mathcal{L}_{ig} + \omega_R \mathcal{L}_R + \sum_{\phi} \omega_{\phi} \mathcal{L}_{\phi} \quad \langle \omega = \text{weighting : default=1} \rangle \quad \text{SS (A.3.1)} \quad (49)$$

3.4. PRIOR EXPECTATION OF PARAMETERS

Table 4. Example table for model parameter priors (this will vary for each stock assessment). Details include prior distributions used with corresponding means and standard deviations, bounds between which parameters are constrained, and initial values to start the minimisation procedure for the MPD (mode of the posterior density) calculations. In SS3, an analytical solution for q can be calculated when the parameter is allowed to 'float'. Some selectivity parameters adopt estimated ones through SS3's « link » functionality, e.g., $\mu_{linked} \leftarrow \mu_{estimated}$ means parameter μ on the left adopts the estimated value for μ on the right.

Parameter	Phase	Prior distribution	Mean, SD	Bounds	Initial value
Coastwide model					
M_1 (female)	4	normal	0.065, 0.0195	[0.02, 0.2]	0.065
M_2 (male)	4	normal	0.065, 0.0195	[0.02, 0.2]	0.065
h	5	beta	0.67, 0.17	[0.2, 1]	0.67
$\log R_0$	1	normal	10, 10	[1, 16]	10
$\hat{p}_{a=1}$	3	normal	0, 1	[-5, 5]	0
$\hat{p}_{a=2}$	3	normal	0, 1	[-5, 5]	0
$\hat{p}_{a=3}$	-3	fixed	0, 1	[-5, 5]	0
$\log q_{1,...,9}$	1	normal	-3, 6	[-15, 15]	-3
$\mu_{1,...,6}$	3	normal	15, 15	[5, 40]	15
$\mu_7 \leftarrow \mu_5$	—	linked	—	—	—
$\mu_8 \leftarrow \mu_4$	—	linked	—	—	—
$\mu_9 \leftarrow \mu_6$	—	linked	—	—	—
$\log v_{L_1,...,6}$	4	normal	2.5, 2.5	[-15, 15]	2.5
$\log v_{L_7} \leftarrow \log v_{L_5}$	—	linked	—	—	—
$\log v_{L_8} \leftarrow \log v_{L_4}$	—	linked	—	—	—
$\log v_{L_9} \leftarrow \log v_{L_6}$	—	linked	—	—	—
$\Delta_{1,...,6}$	4	normal	0, 10	[-10, 10]	0
$\Delta_7 \leftarrow \Delta_5$	—	linked	—	—	—
$\Delta_8 \leftarrow \Delta_4$	—	linked	—	—	—
$\Delta_9 \leftarrow \Delta_6$	—	linked	—	—	—

4. DESCRIPTION OF MODEL COMPONENTS

Notation (Table 1) and model equations (Table 2) are described below. Acronyms: SS3 = Stock Synthesis 3, AW = Awatea, AF = age frequencies | proportions, RQR = Requested Rockfish.

4.1. AGE CLASSES

Index a represents age classes, going from 1 to the accumulator age class A of 60. Age class $a = 5$, for example, represents fish aged 4-5 years (which is the usual, though not universal, convention, Caswell 2001), and so an age-class 1 fish was born the previous year. Unlike Awatea, SS3 uses an age class 0 that presumably represents fish at birth (new recruits). The variable N_{ats} is the number of age-class a fish of sex s at the start of year t , so the model is run to year T , which corresponds to the year after the last year of data (2025) in the assessment model (i.e., beginning of 2026).

4.2. YEARS

Index t represents model years, going from 1 to $T = 92$, and $t = 0$ represents unfished equilibrium conditions. The actual year corresponding to $t = 1$ is 1935, and so model year $T = 92$ corresponds to 2026. In SS3, all model years represent the beginning of the year. The *interpretation* of year t depends on the model's estimated parameter:

- at start of year t : N_{ats} , B_t
- prior to year t : R_t , F_{tg} (or u_{tg}), $\hat{I}_{tg}$, $\hat{p}_{atgs}$

For example, B_{2026} represents the spawning biomass at the start of 2026 (or at the end of 2025). Meanwhile, harvest rate u_{2026} necessarily means the harvest rate that occurred at some point between the beginning of 2025 and the beginning of 2026. Users can choose to label annual parameter estimates using end-of-year notation ($t - 1$), if they so desire (perhaps more intuitive).

4.3. COMMERCIAL DATA

Commercial rockfish catch is reconstructed back to 1918 for five fisheries – (1) trawl, (2) halibut longline, (3) sablefish trap | longline, (4) dogfish | lingcod | salmon troll, and (5) hook & line rockfish in outside (offshore) and/or inside waters. For shelf and slope rockfish species, inside waters are not considered, i.e., the RCR should exclude PMFC area 4B (waters between Vancouver Island and the mainland from Queen Charlotte Strait in the north, through the Strait of Georgia, and south to Juan de Fuca Strait). In the example base run (Section 2.), one commercial fleet is used – ‘Trawl + Other’ for BC coastwide, where ‘Trawl’ refers to bottom and midwater trawl fisheries, and ‘Other’ refers to negligible non-trawl fisheries. Given the small catches (<1 t/y) in the early years (1918–1934), the model is started in 1935. The time series for catches by fleet are denoted C_{tg} and include retained and discarded catches (either observed or reconstructed). The set $\mathbb{U}_{1,\dots,6}$ (Table 1) gives the years of available ageing data from the commercial fishing fleet and the surveys (see next section). Age frequency data are converted to annual proportions using a stratified weighting scheme (fisheries : catch by

trip, then catch by quarter year; surveys : catch density by stratum, then area by stratum; see Appendix D, Section D.2 in Starr and Haigh 2025) that adjusts for unequal sampling effort across temporal and spatial strata.

4.4. SURVEY DATA

Survey data from six fleets ($g=4, \dots, 9$) are used in the model example. These surveys are indexed using g , with each subscript corresponding to a survey: $g=4$: Queen Charlotte Sound (QCS) Synoptic; $g=5$: West Coast Haida Gwaii (WCHG) Synoptic; $g=6$: West Coast Vancouver Island (WCVI) Synoptic; $g=7$: Hecate Strait (HS) Synoptic; $g=8$: Goose Island Gully (GIG) Historical; $g=9$: US National Marine Fisheries Service (NMFS) Triennial. The years for which data were available for each survey are given in Table 1; T_g corresponds to years for the survey biomass estimates I_{tg} (and corresponding standard deviations κ_{tg}), and U_g corresponds to years for proportion-at-age data $patgs$ (with observed sample sizes n_{tg}). Note that for surveys, sample size refers to the number of tows sampled, where each sample comprises specimens, typically ~ 10 -50 fish/sample.

4.5. SEX

A two-sex model is used, with subscript $s=1$ for females and $s=2$ for males (note that these subscripts are the reverse of the codes used in the GFBioSQL⁷ database). Ageing data are partitioned by sex, as are the weights-at-age inputs. Selectivities and natural mortality are specified by sex.

4.6. WEIGHTS-AT-AGE

The weights-at-age w_{as} are assumed fixed over time and are based on sex-specific allometric (length-weight) and growth (age-length) model parameters derived from the biological data.

4.7. MATURITY OF FEMALES

The proportion of age-class a females that are mature is m_a , and is assumed to be invariant over time. Fecundity-at-age for females f_a is assumed to be proportional to weights-at-age w_a . The default fecundity function: $f_a = w_a(\dot{a} + \dot{b}w_a)$ (option 1, with $\dot{a}=1$ and $\dot{b}=0$) should be used in the SS3 model when assuming that fecundity is proportional to body weight. Fecundity option 2 ($f_a = \dot{a}L^{\dot{b}}$) is approximately equivalent to the option 1 default by setting $\dot{a} = e^{-10.852}$ and $\dot{b} = 2.902$ (assuming allometric parameters from SGR, Starr and Haigh 2026a). Dick et al. (2017) provide length-based fecundity parameters for various *Sebastes* species, which necessitates using fecundity option 2 in SS3; however, not all *Sebastes* species have fecundity parameters specified. Note that using an exponentially increasing function for fecundity inflates the estimates of spawning biomass, and it remains unclear what the units are. The user should also be aware that fecundity parameters from Dick et al. (2017) were estimated by a meta-analysis using 25

⁷GFBioSQL is a DFO SQL Server database that mirrors the GFBio Oracle database, which houses all biological information collected by DFO's Pacific Region Groundfish Section.

data sets from the waters off California (15 collection areas), Washington (five areas), Oregon (three areas), BC (one area), and Greenland (one area). Thus, the parameters estimates may not be representative of fish in BC waters.

4.8. INITIAL CONDITIONS

An unfished equilibrium at the beginning of the reconstruction is assumed because there is no evidence of significant removals prior to 1935. The initial conditions (1) and (2) are obtained by setting $R_t = R_0$ (unfished recruitment), $N_{ats} = N_{a1s}$ (equilibrium condition) and $u_{ats} = 0$ (no fishing). The term « 3A » in (1) and (2) denotes three times the maximum age, and is used to determine the plus group's equilibrium numbers-at-age by calculating the exponential decline in frequency from age A to age 3A, and summing the numbers-at-age in this long tail for the frequency at A (Methot and Wetzel 2013, Appendix A). The unfished spawning biomass B_0 is obtained from (3). The initial lengths are set using the growth equations of Schnute (1981) (4)-(6).

4.9. STATE DYNAMICS

The core of the model is the set of dynamic equations (7) for the estimated number N_{ats} of age-class a fish of sex s at the start of year t . The proportion of female new recruits c in Equation (7) is set to 0.5. Equation (7) calculates the numbers of fish in each age class (and of each sex) that survive to the following year, where Z_{ats} represents the total mortality rate, which in this case comprises the sum of natural mortality M and fishing mortality F . The accumulator age class A retains survivors from this class in following years.

Natural mortality M_s is estimated separately for males and females. This parameter enters the equations in the form e^{-M_s} as the proportion of unfished individuals that survive the year.

4.10. SELECTIVITIES

SS3 offers ~17 « Patterns » for size-based selectivity and ~18 for age-based selectivity. These selectivity functions feature a range of parameters from none to eight; the double-normal (size-based) pattern with six parameters is the most commonly used. The ORP always uses Pattern 20, double-normal (age-based) selectivity, because this mimics how Awatea estimates female selectivity. As in Awatea, SS3's Pattern 20 treats male selectivity as an offset relative to female selectivity, but also allows role reversal (estimate male selectivity and treat females as an offset). Within specific Patterns, « Options » are available on how to treat male selectivity.

- Option = 0 : no male selectivity parameters required; male and female selectivity will be the same;
- Option = 1 : four additional parameters are used to define male *selectivity* relative to that for female *selectivity*;
- Option = 2 : main selectivity parameters are for males, with female *selectivity* as an offset relative to male *selectivity*;

- Option = 3 : male selectivity *parameters* are offsets from female selectivity *parameters*;
- Option = 4 : like Option 3, but female selectivity *parameters* are offsets from male selectivity *parameters*.

Separate selectivities are estimated for fleets with AF data, using SS3's selectivity Pattern 20 for females (Equations 8-14) and selectivity Option 3 for males. (Note that « log » herein refers to natural logarithms.) The female, double-normal selectivity parameters used herein (β_i ; where $i = 1, \dots, 6$) for fleet g are:

1. β_{1g} – age at which selectivity first reaches maximum selectivity:
 - SS3: beginning age (year) for the plateau;
 - AW: age of full selectivity (μ_g) for females;
2. β_{2g} – (SS3 only) used to generate a logistic between peak (β_{1g}) and maximum age (A) that determines width of top plateau ($a_g^* - \beta_{1g}$), where a_g^* is the final age of the top plateau;
3. β_{3g} – used to determine width of the ascending limb of double normal curve:
 - SS3: determines slope of ascending limb by tweaking its variance;
 - AW: log of variance for left limb (v_{Lg}) of selectivity curve;
4. β_{4g} – used to determine width of the descending limb of double normal curve:
 - SS3: determines slope of descending limb by tweaking its variance;
 - AW: log of variance for right limb (v_{Rg}) of selectivity curve;
5. β_{5g} – (SS3 only) determines initial selectivity by generating a logistic between 0 and 1 at first age;
 - where selectivity $S_{a=1,g} = 1/(1 + e^{-\beta_{5g}})$; however,
 - use -999 to ignore initial selectivity algorithm and decay small-fish selectivity using β_{3g} ;
6. β_{6g} – (SS3 only) determines final selectivity by generating a logistic between 0 and 1 at final age bin;
 - where selectivity $S_{Ag} = 1/(1 + e^{-\beta_{6g}})$.

Option 3 for Pattern 20 describes male selectivity parameters as offsets to female selectivity parameters, where Δ_j ($j = 1, \dots, 5$) for fleet g are:

1. Δ_{1g} = male peak offset (Δ_g in AW) added to the first female selectivity parameter, β_{1g} (μ_g in AW);
2. Δ_{2g} = male width offset (log width) added to the third selectivity parameter, β_{3g} (same as female v_{Lg} in AW);
3. Δ_{3g} = male width offset (log width) added to the fourth selectivity parameter, β_{4g} (same as female v_{Rg} in AW);
4. Δ_{4g} = male plateau offset added to the sixth selectivity parameter, β_{6g} (not present in AW);

5. Δ_{5g} = maximum selectivity for males (usually 1 but could be different than that for females; not present in AW).

Generally for males, the same selectivity function is used except that some of the selectivity parameters (β_{ig} for $i \in \{1, 3, 4, 6\}$) may be shifted if male AF data are sufficiently different from female AF data.

Dome selectivity using Pattern 20 occurs under three conditions:

- the width of the top plateau (between β_{1g} and a_g^*) must be less than $A - \beta_{1g}$;
- the steepness of the descending limb (controlled by β_{4g}) must not be too shallow; and
- the final selectivity (controlled by β_{6g}) must be less than peak selectivity (usually 1).

Note that male selectivity can be estimated using the parameters $\beta_{1,...,6g}$, with female selectivity being estimated using the offset parameters $\Delta_{1,...,5g}$. This reversal of order was used to obtain female-only dome-shaped selectivity in various ORP stock assessments by keeping male priors very tight (almost fixed) but estimated so that female offsets would be estimated.

Options 1 & 2: Rather than calculate the dependent male *parameters* as an offset from the independent female parameters (Option 3), or female parameters as an offset from male ones (Option 4), offsets can calculate male (or female) *selectivity* as an offset from female (or male) selectivity. These scenarios comprise Options 1 and 2, respectively (not available in Awatea). Option 1 (male offsets) and Option 2 (female offsets) can be applied to Pattern 20 (double normal selectivity) using four parameters:

1. δ_{1g} = age at which selectivity for dependent sex changes relative to independent sex (set to an integer and *do not* estimate);
2. δ_{2g} = log relative selectivity at minimum age ($a=0$); usually set to zero;
3. δ_{3g} = log relative selectivity at age δ_{1g} ;
4. δ_{4g} = log relative selectivity at maximum age (e.g., $a=60$).

4.11. DERIVED STATES

The spawning biomass (biomass of mature females, in tonnes) B_t at the start of year t is calculated in (21) by multiplying the numbers of females N_{at1} by fecundity f_a (20), which is a function of a length-age matrix φ_{lats} (Eq. A.1.14 in Appendix A of Methot and Wetzel 2013), the maturity ogive (m_l), egg production (o_l), and weights-at-length w_{l1} (19).

The fishing mortality rate F_{tg} (33) is derived through an iterative process to fit observed catches closely rather than removing the catches by subtraction. A mid-season harvest rate is calculated using Pope's approximation (Pope 1972), which is then converted to an instantaneous F using the Baranov equation (Baranov 1918). Each fleet's approximate F is repeated iteratively several times (usually three to four) using the Newton-Raphson procedure until its value yields a close match to the observed catches by the fleet. Details can be found in Methot and Wetzel (2013).

Although SS3 does not report vulnerable biomass *per se* (recent versions do report it for MLE fits), Eq. (34) provides an equation adapted from Awatea (Hilborn et al. 2003) for V_{tg} part way before the start of year t . Assuming that modelled C_{tg} is taken part way before t , the harvest rate is simply C_{tg}/V_{tg} . Further, for year t , the proportion u_{tg} of age-class a and sex s fish that are caught in fishery g can be calculated by multiplying the commercial selectivities S_{ags} and the ratio u_t (34).

4.12. STOCK-RECRUITMENT FUNCTION

SS3 offers several recruitment options including Ricker, Beverton-Holt, and a three-parameter survivorship-based function suitable for low-fecundity species (Taylor et al. 2013). The ORP has used the Beverton-Holt (B-H) recruitment function (22) to date. The B-H is parameterised in terms of steepness, h , which is the proportion of the long-term unfished recruitment obtained when the spawning stock biomass is reduced to 20% of the unfished level (Mace and Doonan 1988; Michielsens and McAllister 2004). The prior on h for ORP FSAs is usually taken from Forrest et al. (2010), where shape parameters for a beta distribution (Hilborn et al. 2003; Michielsens and McAllister 2004) are:

$$\alpha = B_0 \frac{1-h}{4hR_0} \text{ and } \beta = \frac{5h-1}{4hR_0}; \text{ substituting these into (22): } R_t = \frac{B_t}{\alpha + \beta B_t},$$

where R_0 is the unfished recruitment, R_t is the recruitment in year t , B_t is the spawning biomass at the start of year t , and B_0 is the unfished spawning biomass.

4.13. FITTING TO DATA

4.13.1. Abundance indices

Model estimates of the survey biomass indices I_{tg} are denoted $\hat{I}_{tg}$ and are calculated in (38). The estimated numbers N_{ats} are multiplied by the total mortality term $e^{-Z_s(\hat{m}_g/12)}$ (that accounts for $\hat{m}_g/12$ of the annual natural mortality and fishing mortality), weights-at-age w_{as} (to convert to biomass), and selectivity S_{ags} . The sum (over ages and sexes) is then multiplied by the catchability parameter q_g to give the model biomass estimate $\hat{I}_{tg}$.

4.13.2. Age frequencies

The estimated proportions-at-age $\hat{p}_{atgs}$ are calculated in (39). Expected ages without ageing error Ψ_a are multiplied by ageing error (ξ_a), and a small constant c can be added (usually $c=0.0001$)⁸. For each combination of year, fleet, and sex, Ψ_{atgs} gives the expected number of aged fish. Division by the sum over sex and age converts these estimated frequencies to proportions for each year and fleet. The SS3 platform can use either frequencies or proportions, and offers choices on how AF sex data are binned:

⁸Methot et al. (2024) : small constant added to observed and expected proportions at length and age to make log likelihood calculations more robust; the constant should be greater than 0; commonly used values range from 0.00001 to 0.01.

0 = combined male and female, 1 = females only, 2 = males only, and 3 = both sexes (scaled to sum to one over both sexes).

A presentation by Cole Monnahan (Alaska Fisheries Science Center, NOAA) to DFO Science on 4 Apr 2024 prompted DFO Groundfish to adopt one-step ahead (OSA) residuals (Trijoulet et al. 2023) as a replacement for Pearson residuals when assessing fits to composition data. OSA residuals are preferred when using multivariate distributions (e.g., multinomial) to fit AF data because correlations between observations are introduced when enforcing the *sum-to-one* (or *sum-to-almost-one*) requirement, and Pearson residuals⁹ do not correctly compensate for these correlations. OSA residuals decorrelate compositional data so that standardised residuals can be applied correctly. The R package « compResidual », specifically the function « resMulti », generates and plots OSA residuals (Trijoulet and Nielsen 2022).

4.13.3. Ageing error

Ageing error (AE) is applied using SS3's vector-style inputs of bias and precision. The simplest bias vector is 0.5 to 60.5 at increments of 1 year for ages 0 through 60, which in SS3 signifies no age bias. The precision vector for ages 0 through 60 is estimated using NOAA's R package « AgeingError », vignette developed by André E. Punt, Kelli F. Johnson, Ian G. Taylor, and Paul Burch. Equation 1.6 of Richards et al. (1992) appears as equation 3 in Punt et al. (2008); however, it is not clear from the R package vignette which sigma option is used as the default. Using these vectors (bias and precision), SS3 applies a cumulative normal distribution (35) for each age to calculate the frequency of expected age (36) given a mean assigned age and standard deviation (37).

“SS3 never adjusts input data. Rather, it adjusts expected values for data to take into account known factors that influenced the creation of the observations. So, ageing error is applied to a modeled distribution of true ages (after selectivity has taken a subset from the population) to create a new distribution of ages that includes the influence of ageing error.”

– Richard Methot, pers. comm., 21 Sep 2021

5. DESCRIPTION OF STOCHASTIC COMPONENTS

5.1. PARAMETERS

The set Θ gives the parameters that are estimated. The estimation procedure is described in the Bayesian Computations section below.

5.2. RECRUITMENT DEVIATIONS

For recruitment, a log-normal process error is assumed, such that the stochastic version of the deterministic stock-recruitment function (22) is

$$R_t = \frac{B_{t-1}}{\alpha + \beta B_{t-1}} e^{-0.5 b_t \sigma_R^2 + \epsilon_t} \quad (50)$$

⁹(observed - fitted) / standard deviation of observed

where $\epsilon_t \sim \mathcal{N}(0, \sigma_R^2)$, and the bias-correction term $-b_t \sigma_R^2 / 2$ term in (50) ensures that the mean of the recruitment deviations equals 0. This gives the recruitment deviation equation (41) and log-likelihood function (46). The value of σ_R is fixed at 0.9 based on values used in recent BC rockfish stock assessments. Other assessments have used $\sigma_R = 0.6$ following an assessment of Silvergray Rockfish (Starr et al. 2016) in which the authors state that the value was typical for marine ‘redfish’ (Mertz and Myers 1996). An Awatea model of Rock Sole used $\sigma_R = 0.6$ (Holt et al. 2016), citing that it was a commonly used default for finfish assessments (Beddington and Cooke 1983). Recent BC rockfish assessments have adopted $\sigma_R = 0.9$ based on an empirical model fit consistent with the age composition data for 5ABC POP (Edwards et al. 2012b). A study by Thorson et al. (2014) examined 154 fish populations and estimated $\sigma_R = 0.74$ (SD=0.35) across seven taxonomic orders; the marginal value for Scorpaeniformes was $\sigma_R = 0.78$ (SD=0.32) but was only based on 7 stocks.

BC offshore rockfish models in past (using Awatea 2009–2020 and SS3 2021–2023) used a recruitment deviation vector (during the main recruitment period) that summed to zero. However, various issues have been flagged by Ian Taylor (NOAA, 2018) and by Cole Monnahan (NOAA, 2018) when using this option in MCMC runs. For this reason, the « simple deviations » option is sometimes preferred over the « sum-to-zero » option in the control file for SS3. However, there are valid reasons to use the sum-to-zero constraint.

“Without the sum-to-zero constraint, the time-series can do whatever it wants with recruitment and allow for the average estimated recruitment to depart from R_0 (which is the theoretical average recruitment estimated by the model). So, there are two different concepts of R_0 [based on the two constraints]. The sum-to-zero constraint has always been used for internal consistency and for consistent reference points.

The way that I like to think of this is what would happen if you projected the stock far in the future without fishing. What would the projected unfished biomass be? Sampling from the estimated recruitments in the main period, the average of those deviations results in average recruitment being $[X]\%$ greater than R_0 , and I would suspect that projected unfished spawning biomass would be $[X]\%$ greater than B_0 . Therefore, if you were to actually use the projected unfished spawning biomass as the denominator, the status would be lower. Where I really get concerned is if the sum-to-zero is less than zero; you would never be able to bring the stock back to the theoretical B_0 , on average.”

– Allan Hicks, IPHC, pers. comm., Mar 6, 2026

5.3. RECRUITMENT ALLOCATION

If a multi-area model is used in SS3 (e.g, Starr and Haigh 2025), one recruitment is estimated for the coastwide population, which is then allocated amongst the areas (e.g., 5ABC, 3CD, and 5DE). Assuming three areas ($\hat{a} \in \{1, 2, 3\}$), two allocation parameters ($\hat{p}_{\hat{a}=1,2}$) are estimated in natural log space, while the third parameter ($\hat{p}_{\hat{a}=3}$) is fixed at zero ($e^0 = 1$). Note that any one of the three can be fixed to 0 while estimating the other

two. The equation to calculate the proportion allocations by area ($\hat{p}'_{\hat{a}}$) in linear space is:

$$\hat{p}'_{\hat{a}} = e^{\hat{p}_{\hat{a}}} / (e^{\hat{p}_{\hat{a}=1}} + e^{\hat{p}_{\hat{a}=2}} + e^{\hat{p}_{\hat{a}=3}}), \text{ where } \hat{a} \in 1, 2, 3 \quad (51)$$

In order to have varying recruitment by year in each area, a time-varying component can be added to $\hat{p}_{\hat{a}}$ by estimating annual deviance parameters $\nu_{t,\hat{a}}$. The period over which this is applied can vary among areas and runs. For example, in Starr and Haigh (2025), a time-varying component was used from 1935 to 2014 for 5ABC, 1975 to 2014 for 3CD, and 1935 to 2014 for 5DE. These years were selected based on the availability of age frequency data and the number of observations obtained for a cohort. Additionally, a standard error ($\zeta_{\hat{a}}$) for the deviance values can be estimated; however, the POP stock assessment fixed $\zeta_{\hat{a}}$ to 1. Annual time-varying recruitment proportions are calculated by adjusting the estimated proportions in log space:

$$\hat{p}_{t,\hat{a}} = \hat{p}_{\hat{a}} + \nu_{t,\hat{a}} \zeta_{\hat{a}} \quad (52)$$

and then applying (51) to transform these annually adjusted values into linear space, $\hat{p}'_{t,\hat{a}}$.

5.4. LOG-LIKELIHOOD FUNCTIONS

The objective function $\mathcal{F}(\Theta)$ (49) comprises a weighted sum (models used the default weightings) of individual likelihood components that include:

- $\mathcal{L}_{I_g}$ (43) – CPUE or abundance index by fleet
- $\mathcal{L}_{a_g}$ (44) – age composition by fleet
- $\mathcal{L}_{C_g}$ (45) – catch by fleet
- $\mathcal{L}_R$ (46) – recruitment deviations
- $\mathcal{L}_{\phi_j}$ (47) to (48) – parameter priors

See Methot and Wetzel (2013) and Methot et al. (2024) for more likelihood options and details.

6. BAYESIAN COMPUTATIONS

The estimation of model parameters involves minimizing the difference between observed data (such as catch, abundance indices, and proportions-at-age) and the values predicted by the model. Recruitment deviations are estimated as random effects that account for unexplained annual variability in recruitment around a spawning stock-recruitment relationship. Fitting is done by minimizing the objective function $f(\Theta)$, which equation (49) shows is the negative of the sum of the total log-likelihood function comprising the logarithmic components (43)-(48).

The procedure for the Bayesian computations is as follows:

1. minimise the objective function $f(\Theta)$ to give estimates of the mode of the posterior density (MPD) for each parameter: (a) done in phases, and (b) perform a reweighting;
2. generate samples from the joint posterior distributions of the parameters using Monte Carlo Markov Chain (MCMC) procedure, starting the chains from the MPD estimates.

6.1. PHASES

The MPD estimates are obtained by minimising the objective function $f(\Theta)$, from the stochastic (non-Bayesian version) of the model. The resulting estimates are then used to initiate the chains for the MCMC procedure for the full Bayesian model.

Simultaneously estimating all the estimable parameters for complex nonlinear models is ill advised, and so ADMB allows some of the estimable parameters to be kept fixed during the initial part of the optimisation process (ADMB Project 2009). Some parameters are estimated in phase 1, then further ones in phase 2, and so on. The order (if estimated) typically used by ORP is:

- phase 1: unfished recruitment R_0 and survey catchabilities q
(the q fit can also adopt the ‘float’ option, which calculates an analytical solution);
- phase 2: recruitment deviations ϵ_t (held at 0 in phase 1);
- phase 3: natural mortality M_s and age of full selectivity β_{1g} (usually female);
- phase 4: additional selectivity parameters β_{ig} (female) and Δ_{jg} (male)
(male selectivity can be β with female offsets Δ);
- phase 5: steepness h .

6.2. REWEIGHTING

“Sample sizes are used to calculate the variance for a data source and are useful to indicate the relative differences in uncertainty across years within each data source. However, sample size may not represent the relative difference in the variance between different data sources (usually abundance vs. composition). Therefore, the relative weights for each data source in an integrated stock assessment should be adjusted to reflect the information content of each, while retaining the relative differences across years. This can be accomplished by applying adjustment factors to abundance and composition data to weight either data source up or down relative to the other. ”

– Allan Hicks, IPHC, pers. comm., Aug 17, 2021

Previous rockfish stock assessments using the Awatea platform adopted the Francis (2011) reweighting approach (from 2011 on) – adding series-specific process error to abundance index CVs on the first reweight, and iteratively reweighting age frequency (composition data) sample size by mean age on the first and subsequent reweights.

6.2.1. Abundance

For abundance data (survey indices, commercial CPUE indices), Francis (2011) recommends reweighting observed coefficients of variation, c_0 , by adding process error $c_p \sim 0.2$ to give a reweighted coefficient of variation

$$c_1 = \sqrt{c_0^2 + c_p^2}. \quad (53)$$

If survey abundance indices exhibit moderate-to-large relative error, no additional error c_p need be added to these indices. If process error is added, the amount can be tailored to each survey series.

For stock assessments using commercial CPUE, a procedure was developed for estimating process error c_p to add to CPUE indices based on a spline-smoother analysis (Starr and Haigh 2021a). The idea for this analysis came from Francis (2011), citing Clark and Hare (2006), who recommended using a smoothing function to determine the appropriate level of process error to add to CPUE data, with the goal of finding a balance between rigorously fitting the indices while not removing the majority of the signal in the data. In R, the « `spline.smooth` » function determines the « `spar` » argument (smoothing parameter) primarily by minimizing a Generalized Cross-Validation (GCV) score, which balances the goodness of fit with the smoothness of the spline. However, one can also minimise the residual sum of squares (RSS) using the following procedure.

Procedure : An arbitrary sequence of length 50, comprising degrees of freedom (DF, ν_i), where $i = 2, \dots, N$ and N = number of CPUE values I_t from $t = 1996, \dots, 2025$, is used to fit the CPUE data with a spline smoother. At $i = N$, the spline curve fits the data perfectly and the residual sum of squares (ρ_N) is 0. Using spline fits across a range of trial DF ν_i , values of RSS ρ_i form a logistic-type curve with an inflection point at $i = k$ (Figure 1). The difference between point estimates of ρ_i (proxy for the slope δ_i) yield a concave curve with a minimum δ_i , which occurs close to the inflection point k . At the inflection point k , $\nu_k = 7.6$ for SGR in 5DE, corresponding to $\rho_k = 1.415$, which is converted to $c_p = 0.2119$ using:

$$c_p = \sqrt{\frac{\rho_k}{N-2}} \left[\frac{1}{N} \sum_{t=1996}^{2025} I_t \right]^{-1} \quad (54)$$

Sometimes it is not appropriate to use commercial CPUE because the species is targeted by the trawl (or longline or trap) fleet. Trawl fleets in particular often use acoustic signals before catching a species. Regardless, using CPUE for targeted species leads to concerns that the series will be hyperstable.

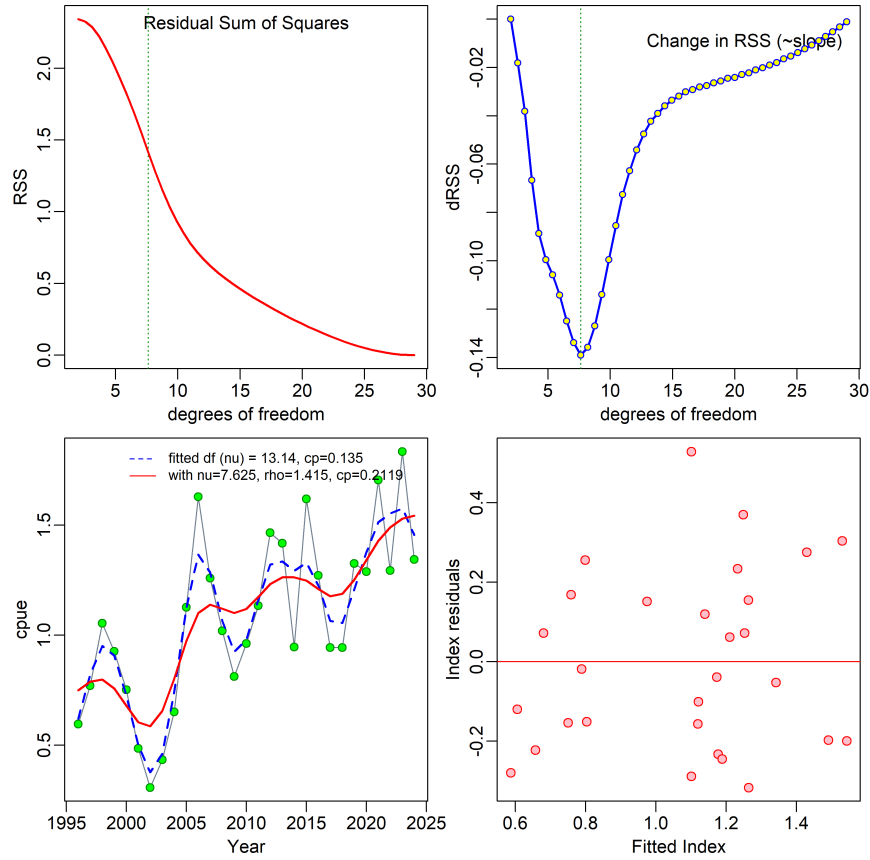


Figure 1. Estimating process error to add to commercial CPUE data using SGR in 5DE as an example: top left – residual sum of squares (RSS) from spline-smoother at various degrees of freedom; top right – slope of RSS ($\sim$ first derivative), vertical dotted line at DF where slope is at a minimum; bottom left – CPUE index data (green dots) with automatic smoothing $\text{DF}=13.1$ (dashed blue curve) and optimised $\text{DF}=7.6$ (solid red curve); bottom right – standardised residual fit.

6.2.2. Composition

The SS3 platform accommodates three options for weighting compositional data: the Francis (2011) mean-age procedure (iterative), the McAllister and Ianelli (1997) harmonic-mean procedure (iterative), and the self-weighting Dirichlet-multinomial (D-M) option (Thorson et al. 2017), which estimates a weighting parameter for each compositional dataset (internal). All three of these procedures have been used in recent ORP stock assessments:

- 2021 Yellowmouth Rockfish (Starr and Haigh 2022c) $\longrightarrow$ harmonic-mean
- 2022 Canary Rockfish (Starr and Haigh 2023) $\longrightarrow$ D-M parameterisation
- 2023 Pacific Ocean Perch (Starr and Haigh 2025) $\longrightarrow$ Francis mean-age

Francis mean age : The method preferred by ORP is the Francis (2011) mean-age procedure (for reasons explained below). The Francis weight w_g given to each fleet's AF

data set g takes into account deviations from the mean age for each year t . The weight is given by equation (TA1.8) of Francis (2011):

$$w_g = \frac{1}{\mathbb{V}\left(\frac{\bar{O}_{tg} - \bar{E}_{tg}}{\sqrt{v_{tg}/n_{tg}}}\right)} ; \quad \text{where finite-sample variance } \mathbb{V}(x_t) = \frac{\sum_t (x_t - \bar{x})^2}{n - 1}, \quad (55)$$

n_{tg} = sample size prior to applying weight w_g , and the observed mean age ($\bar{O}_{tg}$), the expected mean age ($\bar{E}_{tg}$), and the variance of the expected age distribution (v_{tg}) are, respectively:

$$\bar{O}_{tg} = \sum_{a=1}^A \sum_{s=1}^2 a p_{atgs} \quad (56)$$

$$\bar{E}_{tg} = \sum_{a=1}^A \sum_{s=1}^2 a \hat{p}_{atgs} \quad (57)$$

$$v_{tg} = \sum_{a=1}^A \sum_{s=1}^2 a^2 \hat{p}_{atgs} - \bar{E}_{tg}^2 \quad (58)$$

Harmonic mean ratio : A second candidate method for reweighting used by SS3 involves a bootstrapping procedure suggested by Stewart and Hamel (2014) for resampling tows with specimens aged and sexed, and calculating the harmonic mean ratio of McAllister and Ianelli (1997). The Yellowmouth Rockfish and Petrale Sole assessments (Starr and Haigh 2022c; Mazur et al. 2026) could not get models to converge using the Francis reweight, and so the method of Stewart and Hamel (2014) was used. The estimated effective sample size of McAllister and Ianelli (1997) is:

$$\tilde{n}_{tg} = \frac{\sum_{a=1}^A \hat{p}_{atg} (1 - \hat{p}_{atg})}{\sum_{a=1}^A (p_{atg} - \hat{p}_{atg})^2} \quad (59)$$

Dirichlet-multinomial parameterisation : In a previous stock assessment (Starr and Haigh 2023), composition data were reweighted using the Dirichlet-multinomial distribution available in SS3 (Thorson et al. 2017). This approach adds an estimable parameter (θ) which automatically scales the input sample size as part of the likelihood.

“In consultation with Jim Thorson, Ian Taylor proposed a normal $\mathcal{N}(0, 1.813)$ prior for the $\ln(\text{DM_parm})$ parameters to counteract the effect of the logistic transformation between this parameter and the data weighting. The 1.813 value was calculated as the standard deviation of the distribution of $\log(\theta)$ values derived from starting with a uniform distribution on the weights, $\text{weight} = \theta/(1 + \theta) \sim \mathcal{U}(0, 1)$, and solving for $\log(\theta)$.”

– Methot et al. (2023), *Data Weighting*

If the calculated weight $\theta/(1 + \theta)$ ratio is close to 1.0, the model is trying to tune the sample size as high as possible. In this case, Methot et al. (2024) suggest fixing the $\log \text{DM } \theta$ parameter to a high value, like the upper bound of 20, which will result in 100% weight being applied to the input sample sizes. One caveat of using the $\log \text{DM } \theta$ parameter is that it does not allow weights above 100% (by design).

After experimentation with the D-M parameterisation in the 2023 POP stock assessment, the authors found considerable weighting effects on estimated stock productivity based on the sample sizes used. In contrast, the Francis method calculated weightings (by upscaling low sample sizes or downscaling high sample sizes) to yield very similar estimates of stock productivity. The stability of the Francis method obviates the need to rely on alternative methods to reweight sample sizes, and so this is the preferred choice by ORP, assuming its use leads to sensible model convergence.

6.3. PRIOR DISTRIBUTIONS

Descriptions of a typical set of prior distributions for the estimated parameters (without including recruitment deviations) appears in Table 4. A wide normal prior $\mathcal{N}(10, 10)$ is used for $\log R_0$; this provides more stability in the model than using a uniform prior without affecting the estimation process. Steepness is estimated using a beta distribution, with priors generated by Forrest et al. (2010) : $\mathcal{B}(0.67, 0.17)$. Catchability parameters q_g used a wide normal prior $\mathcal{N}(-3, 6)$, but can also be determined analytically using SS3's « float=1 » option.

Natural mortality priors are either based on estimates of M from longevity-based equations (e.g., Hamel and Cope 2022) or taken from priors used by previous stock assessments on the species. If data permit, priors can be diffuse like that for $\log R_0$ (e.g., $\mathcal{N}(0.06, 0.06)$); otherwise, constrained priors with CVs of 20-30% might be needed. Care must be taken on the choice of M priors because they can affect the parameter estimates, and consequently, the estimates of absolute biomass.

Selectivity priors can only be specified for fleets with AF data. As for priors above, diffuse normal priors should be used unless the data result in suspicious estimates like very high age-at-full-selectivity (μ well above maturity) or very low ascending-limb variability ($\log v_L$ leading to knife-edge selectivity). Constrained priors with CVs of 20-30% might be needed. For fleets without AF data (or poor AF data), selectivity parameters can be linked to a fleet with estimated selectivity or simply fixed to reasonable values. Choice of μ (for a prior mean or fixed value) can be based on frequency distribution (histograms or cumulative sums) of female lengths or weights; often one or two surveys catch larger or smaller females than do the majority of surveys.

6.4. MCMC PROPERTIES

While the Metropolis algorithm, which uses a Gaussian distribution to search model space, is feasible (time-wise) for generating MCMC samples in Awatea, its use in SS3 remains largely futile (runs can take days to complete). The No-U-Turn Sampler (NUTS) algorithm uses Hamiltonian dynamics (Duane et al. 1987), which is more efficient than the Metropolis algorithm, especially in high-dimensional spaces. This efficiency necessitates its use in SS3 Bayesian analysis (Monnahan and Kristensen 2018; Monnahan et al. 2019), and was adopted by FSAs for offshore hake, rockfish, and some flatfish.

In ORP assessments, the MCMC procedure using NUTS typically produces 4,000 iterations¹⁰, parsing the workload into eight parallel chains (using the R package « snowfall », Knaus 2015). For each chain, 500 iterations are performed, discarding the first 250 samples as a « burn-in »¹¹, leaving the final 250 samples for use in the MCMC analysis. The eight parallel chains are then merged for a total of 2,000 samples to approximate the posterior distribution. If a higher number of iterations are required¹⁰, the excess of samples are thinned. Code (bash and R) was supplied by Chris Grandin (DFO, pers. comm. 2023) to perform the MCMC simulations on a remote Linux server.

7. REFERENCE POINTS, PROJECTIONS, AND ADVICE TO MANAGERS

Advice to managers is given with respect to a suite of reference points. The first set is based on MSY (maximum sustainable yield) and includes the provisional reference points of the DFO Precautionary Approach (DFO 2006), namely $0.4B_{MSY}$ and $0.8B_{MSY}$ (and also provided are B_{MSY} and u_{MSY} , which denote the estimated equilibrium spawning biomass and harvest rate at MSY, respectively). A second set of reference points, based on the current spawning biomass B_{2026} and harvest rate u_{2025} , is used to show the probability of the stock size increasing from the current female spawning biomass or decreasing from the current harvest rate. A third set of reference points, based on B_0 (the estimated unfished equilibrium spawning biomass) is provided as an alternative to the B_{MSY} reference points.

The probability $P(B_{2026} > 0.4B_{MSY})$ is calculated as the proportion of the 2,000 MCMC samples for which $B_{2026} > 0.4B_{MSY}$ (and similarly for the other biomass-based reference points). For harvest rates, the probability $P(u_{2025} < u_{MSY})$ is calculated so that both B - and u -based stock status indicators (and projections when $t = 2027, \dots, 2036$) state the probability of being in a « good » place.

Projections are typically made for 11 years starting with the biomass for the start of the year after the last data year. All derived values in SS3 are for a start-of-year time period. Therefore, if the end year in the data file is specified as 2025, derived quantities like spawning biomass B_t are estimated to start of year 2026. By default, SS3 will project forward at least one year so that catch in 2025 can be applied and derived quantities will be generated for 2026 (one-year forecast). Therefore, in the file « forecast.ss », a user needs to specify the current year plus any additional forecast years (e.g., a 10-yr forecast would need 11 specified catches from 2026 to 2036). Additionally, if a user needs generational forecasts (e.g, three RQR generations = 75 years), then 76 forecast years need to be specified before any MCMC runs are attempted.

For decision tables, a range of constant catch strategies are used, from 0 to a maximum at various increments, for example. Recruitments are randomly calculated using (22) (i.e., based on lognormal recruitment deviations from the estimated stock-recruitment

¹⁰sometimes 20,000 or 40,000 iterations (thinned every 5 or 10 samples, respectively), depending on the severity of autocorrelation in parameter estimates

¹¹NUTS chains require an initial number of iterations to converge to the posterior before sampling it. This early phase is called the « warm-up » or « burn-in » period, and is discarded before drawing any inferences from the samples.

curve), using randomly generated values of $\epsilon_t \sim \text{Normal}(0, \sigma_R^2)$. Unfortunately, SS3 calculates projected recruitment deviations at the time of the MCMC runs and so the user should be aware that changing the catch policy after the MCMCs have been performed is only possible for the number of projection years previously used (don't expect to project for 30 years if your MCMC run only used 10 years). Each SS3 catch policy requires a separate « -mceval » call. In Awatea, the « -mceval » switch can generate a user-specified time series of $\{\epsilon_t\}$ for each MCMC sample, which means that different catch policies (number of constant catches by number of projection years) can be generated after the MCMC analysis.

8. SS3 INPUTS

Currently, the DFO team that controls departmental web content only allows web links to data sets with digital object identifiers (DOI), and recognises only the European repository called Zenodo, maintained by CERN (Conseil Européen pour la Recherche Nucléaire, which translates to European Council for Nuclear Research). However, SS3 input files¹² for model runs by ORP are hosted on GitHub at :

github.com/pbs-software/pbs-synth/

From there, navigate to

[./tree/master/PBSsynth/inst/input/](#)

for a collection of SS3 model runs for groundfish stocks along the BC coast since 2021 :

- 2021/YMR – Yellowmouth Rockfish in 2021;
- 2022/CAR – Canary Rockfish in 2022;
- 2023/POP – Pacific Ocean Perch using three subareas (5ABC, 3CD, 5DE) in 2023;
- 2024/YTR – Yellowtail Rockfish in 2024;
- 2025/SGR – Silvergray Rockfish in 2025.

A synopsis ([./00ReadMe.txt](#)) for each stock assessment details the model runs, where B = base run, S = sensitivity run, and R00v0 = model run number and its version.

CONTACT INFORMATION

Rowan Haigh

Offshore Rockfish Program, Groundfish Section,
Stock Assessment and Research Division, Science Branch,
Pacific Region, Fisheries and Oceans Canada
Facility : Pacific Biological Station, Nanaimo, BC V9T 6N7
email (until 2026) : rowan.haigh@dfo-mpo.gc.ca
email (from 2027 on) : dana.haggarty@dfo-mpo.gc.ca

¹²File names comprise « starter.ss », « forecast.ss », « data.xx.yy.ss », and « control.xx.yy.ss », where xx = run number and yy = reweight number.

9. REFERENCES – MODEL EQUATIONS

- ADMB Project. 2009. [AD Model Builder: Automatic Differentiation Model Builder](#). Developed by David Fournier and freely available from admb-project.org.
- Allaire, J., Xie, Y., Dervieux, C., McPherson, J., Luraschi, J., Ushey, K., Atkins, A., Wickham, H., Cheng, J., Chang, W. and Iannone, R. 2025. [rmarkdown: Dynamic Documents for R](#). R package version 2.30.
- Anderson, S.C., Grandin, C., Edwards, A.M., Grinnell, M.H., Ricard, D., Haigh, R., Burton, M., Regular, P.M. and Dunic, J.C. 2026. [csasdown: Reproducible CSAS Reports with Bookdown](#). R package version 0.2.2, commit [adb683ae9fc905d53bfb822c5604a7cb4e8f87ba](#).
- Baranov, F.I. 1918. [“On the question of the biological basis of fisheries: on the question of the dynamics of the fishing industry”, \(translated from Russian by W.E. Ricker 1945\)](#). Izvestiya Otdela Rybovodstva I Nauchno-promyslovyykh Issledovaniy 1. 81–128.
- Beamish, R.J. 1979. [New information on the longevity of Pacific ocean perch \(*Sebastes alutus*\)](#). Can. J. Fish. Aquat. Sci. 36(11). 1395–1400.
- Beddington, J.R. and Cooke, J.G. 1983. [The potential yield of fish stocks](#). FAO Fish. Tech. Paper 242. v + 47 p.
- Caswell, H. 2001. Matrix Population Models: Construction, Analysis and Interpretation. Sinauer Associates, Massachusetts.
- Clark, W.G. and Hare, S.R. 2006. [Assessment and management of Pacific halibut: data, methods, and policy](#). Science Report 83, International Pacific Halibut Commission, Seattle, WA.
- DFO. 2006. [A harvest strategy compliant with the precautionary approach](#). DFO Can. Sci. Advis. Sec. Sci. Advis. Rep. 2006/023. 7 p.
- DFO. 2009. [Stock assessment update for British Columbia Canary Rockfish](#). DFO Can. Sci. Advis. Sec. Sci. Resp. 2009/019. 39 p.
- DFO. 2015. [Yellowtail Rockfish \(*Sebastes flavidus*\) stock assessment for the coast of British Columbia, Canada](#). DFO Can. Sci. Advis. Sec. Sci. Advis. Rep. 2015/010. 14 p.
- DFO. 2022. [Update of the 2019 Bocaccio \(*Sebastes paucispinis*\) stock assessment for British Columbia in 2021](#). DFO Can. Sci. Advis. Sec. Sci. Resp. 2022/001. 33 p.
- DFO. 2024. [Update of the 2019 Bocaccio \(*Sebastes paucispinis*\) stock assessment for British Columbia in 2024](#). DFO Can. Sci. Advis. Sec. Sci. Resp. 2024/033.
- Dick, E.J., Beyer, S., Mangel, M. and Ralston, S. 2017. [A meta-analysis of fecundity in rockfishes \(genus *Sebastes*\)](#). Fish. Res. 187. 73–85.
- Dorn, M.W. and Saunders, M.W. 1997. [Pacific hake-offshore stock assessment for 1997 and recommended yield options for 1998](#). DFO PSARC Working Paper G97-1. 84 p.
- Duane, S., Kennedy, A.D., Pendleton, B.J. and Roweth, D. 1987. [Hybrid Monte Carlo](#). Physics Letters B 195(2). 216–222.

- Edwards, A.M., Haigh, R. and Starr, P.J. 2014a. [Pacific Ocean Perch \(*Sebastes alutus*\) stock assessment for the north and west coasts of Haida Gwaii, British Columbia](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2013/092. vi + 126 p.
- Edwards, A.M., Haigh, R. and Starr, P.J. 2014b. [Pacific Ocean Perch \(*Sebastes alutus*\) stock assessment for the west coast of Vancouver Island, British Columbia](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2013/093. vi + 135 p.
- Edwards, A.M. and Auger-Méthé, M. 2019. [Some guidance on using mathematical notation in ecology](#). Methods Ecol. Evol. 10. 92–99.
- Edwards, A.M., Haigh, R. and Starr, P.J. 2012a. [Stock assessment and recovery potential assessment for Yellowmouth Rockfish \(*Sebastes reedi*\) along the Pacific coast of Canada](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2012/095. iv + 188 p.
- Edwards, A.M., Starr, P.J. and Haigh, R. 2012b. [Stock assessment for Pacific ocean perch \(*Sebastes alutus*\) in Queen Charlotte Sound, British Columbia](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2011/111. viii + 172 p.
- Fisch, N., Mazur, M., Tai, T., English, P. and Holt, K. 2026. Assessment of Dover Sole in the Canadian Pacific Exclusive Economic Zone in 2026. DFO Can. Sci. Advis. Sec. Res. Doc. In prep.
- Forrest, R.E., McAllister, M.K., Dorn, M.W., Martell, S.J.D. and Stanley, R.D. 2010. [Hierarchical Bayesian estimation of recruitment parameters and reference points for Pacific rockfishes \(*Sebastes* spp.\) under alternative assumptions about the stock-recruit function](#). Can. J. Fish. Aquat. Sci. 67. 1611–1634.
- Fournier, D.A., Hampton, J. and Sibert, J.R. 1998. [MULTIFAN-CL: a length-based, age-structured model for fisheries stock assessment, with application to South Pacific albacore, *Thunnus alalunga*](#). Can. J. Fish. Aquat. Sci. 55(9). 2105–2116.
- Fournier, D.A., Sibert, J.R., Majkowski, J. and Hampton, J. 1990. [MULTIFAN a likelihood-based method for estimating growth parameters and age composition from multiple length frequency data sets illustrated using data for southern bluefin tuna \(*Thunnus maccoyii*\)](#). Can. J. Fish. Aquat. Sci. 47(2). 301–317.
- Fournier, D. and Archibald, C.P. 1982. [A general theory for analyzing catch at age data](#). DFO Can. J. Fish. Aquat. Sci. 39.
- Francis, R.I.C.C. 2011. [Data weighting in statistical fisheries stock assessment models](#). Can. J. Fish. Aquat. Sci. 68(6). 1124–1138.
- Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. 2004. Bayesian Data Analysis, 2nd edition. Chapman and Hall/CRC, New York.
- Haggarty, D.R., Carruthers, T.R., Walker, L.C., English, P.A. and Huynh, Q.C. 2025. Outside Lingcod (*Ophiodon elongatus*) stock assessment for British Columbia in 2024. DFO Can. Sci. Advis. Sec. Res. Doc. In press.
- Haigh, R., Starr, P.J., Edwards, A.M., King, J.R. and Lecomte, J.B. 2018. [Stock assessment for Pacific Ocean Perch \(*Sebastes alutus*\) in Queen Charlotte Sound, British Columbia in 2017](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2018/038. v + 227 p.

- Hamel, O.S. and Cope, J.M. 2022. [Development and considerations for application of a longevity-based prior for the natural mortality rate](#). Fish. Res. 256. 106,477.
- Hilborn, R., Maunder, M., Parma, A., Ernst, B., Payne, J. and Starr, P. 2003. [Coleraine: A generalized age-structured stock assessment model](#). University of Washington Report SAFS-UW-0116. User's manual version 2.0.
- Holt, K.R., Starr, P.J., Haigh, R. and Krishka, B. 2016. [Stock assessment and harvest advice for Rock Sole \(*Lepidopsetta* spp.\) in British Columbia](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2016/009. ix + 256 p.
- Johnson, K.F., Edwards, A.M., Berger, A.M., Grandin, C.J. and Wetzel, C.R. 2025. [Status of the Pacific Hake \(whiting\) stock in U.S. and Canadian waters in 2025](#). Technical report, Prepared by the Joint Technical Committee of the U.S. and Canada Pacific Hake/Whiting Agreement, National Marine Fisheries Service and Fisheries and Oceans Canada.
- Knaus, J. 2015. [snowfall: Easier cluster computing \(based on snow\)](#). R package version 1.84-6.1.
- Leisch, F. 2002. [Sweave: dynamic generation of statistical reports using literate data analysis](#). In W. Härdle and B. Rönz, eds., Compstat 2002 - Proceedings in Computational Statistics, p. 575–580. Physica Verlag, Heidelberg.
- Mace, P.M. and Doonan, I.J. 1988. [A generalized bioeconomic simulation for fish population dynamics](#). NZ Fish. Assess. Res. Doc. 88/4. 51 p.
- MacLellan, S.E. 1997. [How to age rockfish \(*Sebastes*\) using *S. alutus* as an example – the otolith burnt section technique](#). Can. Tech. Rep. Fish. Aquat. Sci. 2146. 39 p.
- Mazur, M., Holt, K., Fisch, N. and English, P. 2026. Assessment of Petrale Sole in British Columbia in 2024. DFO Can. Sci. Advis. Sec. Res. Doc. In press.
- McAllister, M.K. and Ianelli, J.N. 1997. [Bayesian stock assessment using catch-age data and the sampling – importance resampling algorithm](#). Can. J. Fish. Aquat. Sci. 54(2). 284–300.
- Mertz, G. and Myers, R. 1996. [Influence of fecundity on recruitment variability of marine fish](#). Can. J. Fish. Aquat. Sci. 53(7). 1618–1625.
- Methot, R.D. 1986. [Synthetic estimates of historical abundance and mortality for northern anchovy, *Engraulis mordax*](#). Technical report, U.S. National Marine Fisheries Service. Southwest Fisheries Center. Administrative Report LJ 86–29, La Jolla, California.
- Methot, R.D. 1989. [Synthetic estimates of historical abundance and mortality for northern anchovy](#). In E. F. Edwards and B. A. Megrey, eds., Mathematical analysis of fish stock dynamics. American Fisheries Society Symposium 6, 66–82.
- Methot, R.D., Wetzel, C.R., Taylor, I.G., Doering, K.L., Perl, E.F. and Johnson, K.F. 2024. [Stock Synthesis: User Manual, version 3.30.23](#). Tagged release, NOAA Fisheries, Seattle WA, USA, October 31, 2024.
- Methot, R.D. and Wetzel, C.R. 2013. [Stock Synthesis: A biological and statistical framework for fish stock assessment and fishery management](#). Fish. Res. 142. 86–99.

- Michielsens, C.G.J. and McAllister, M.K. 2004. [A Bayesian hierarchical analysis of stock-recruit data: quantifying structural and parameter uncertainties](#). Can. J. Fish. Aquat. Sci. 61(6). 1032–1047.
- Monnahan, C.C. 2018. [adnuts: No-U-Turn MCMC Sampling for ADMB Models](#). R package ver. 1.1.2.
- Monnahan, C.C., Branch, T.A., Thorson, J.T., Stewart, I.J. and Szuwalski, C.S. 2019. [Overcoming long Bayesian run times in integrated fisheries stock assessments](#). ICES J. Mar. Sci. 76(6). 1477–1488.
- Monnahan, C.C. and Kristensen, K. 2018. [No-U-turn sampling for fast Bayesian inference in ADMB and TMB: Introducing the adnuts and tmbstan R packages](#). PLoS ONE 13(5). e0197,954.
- Pope, J.G. 1972. [An investigation of the accuracy of virtual population analysis using cohort analysis](#). Int. Comm. Northwest Atl. Fish. Res. Bull. 9. 65–74.
- Punt, A.E., Smith, D.C., KrusicGolub, K. and Robertson, S. 2008. [Quantifying age-reading error for use in fisheries stock assessments, with application to species in Australia's southern and eastern scalefish and shark fishery](#). Can. J. Fish. Aquat. Sci. 65. 1991–2005.
- Punt, A.E., Smith, D.C., KrusicGolub, K. and Robertson, S. 2020. [Essential features of the next-generation integrated fisheries stock assessment package: A perspective](#). Fish. Res. 229. 105,617.
- R Core Team. 2025. [R: A Language and Environment for Statistical Computing](#). R Foundation for Statistical Computing, Vienna, Austria.
- Richards, L.J., Schnute, J.T., Kronlund, A.R. and Beamish, R.J. 1992. [Statistical models for the analysis of ageing error](#). Can. J. Fish. Aquat. Sci. 49(9). 1801–1815.
- Schnute, J.T. 1981. [A versatile growth model with statistically stable parameters](#). Can. J. Fish. Aquat. Sci. 38(9). 1128–1140.
- Schnute, J.T., Haigh, R., Krishka, B.A. and Starr, P.J. 2001. [Pacific Ocean Perch assessment for the west coast of Canada in 2001](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2001/138. iv + 90 p.
- Stanley, R.D., Starr, P. and Olsen, N. 2009. [Stock assessment for Canary Rockfish \(*Sebastes pinniger*\) in British Columbia waters](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2009/013. xxii + 198 p.
- Starr, P.J. and Haigh, R. 2021a. [Widow Rockfish \(*Sebastes entomelas*\) stock assessment for British Columbia in 2019](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2021/039. vi + 238 p.
- Starr, P.J. and Haigh, R. 2022a. [Bocaccio \(*Sebastes paucispinis*\) stock assessment for British Columbia in 2019, including guidance for rebuilding plans](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2022/001. vii + 292 p.
- Starr, P.J. and Haigh, R. 2022b. [Rougheye/Blackspotted Rockfish \(*Sebastes aleutianus* / *melanostictus*\) stock assessment for British Columbia in 2020](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2022/20. vii + 385 p.

- Starr, P.J. and Haigh, R. 2021*b*. [Redstripe Rockfish \(*Sebastes proriger*\) stock assessment for British Columbia in 2018](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2021/014. vii + 340 p.
- Starr, P.J. and Haigh, R. 2022*c*. [Yellowmouth Rockfish \(*Sebastes reedi*\) stock assessment for British Columbia in 2021](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2022/010. viii + 288 p.
- Starr, P.J. and Haigh, R. 2023. [Canary Rockfish \(*Sebastes pinniger*\) stock assessment for British Columbia in 2022](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2023/070. vi + 293 p.
- Starr, P.J. and Haigh, R. 2025. [Pacific Ocean Perch \(*Sebastes alutus*\) stock assessment for British Columbia in 2023](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2025/004. vii + 303 p.
- Starr, P.J. and Haigh, R. 2026*a*. Silvergray Rockfish (*Sebastes brevispinis*) stock assessment for British Columbia in 2025. DFO Can. Sci. Advis. Sec. Res. Doc. In prep.
- Starr, P.J. and Haigh, R. 2026*b*. Yellowtail Rockfish (*Sebastes flavidus*) stock assessment for British Columbia in 2024. DFO Can. Sci. Advis. Sec. Res. Doc. In press.
- Starr, P.J., Haigh, R. and Grandin, C. 2016. [Stock assessment for Silvergray Rockfish \(*Sebastes brevispinis*\) along the Pacific coast of Canada](#). DFO Can. Sci. Advis. Sec. Res. Doc. 2016/042. vi + 170 p.
- Stewart, I.J. and Hamel, O.S. 2014. [Bootstrapping of sample sizes for length- or age-composition data used in stock assessments](#). Can. J. Fish. Aquat. Sci. 71(4). 581–588.
- Taylor, I.G., Gertseva, V., Methot, R.D. and Maunder, M.N. 2013. [A stock-recruitment relationship based on pre-recruit survival, illustrated with application to spiny dogfish shark](#). Fish. Res. 142. 15–21.
- Thorson, J.T., Jensen, O.P. and Zipkin, E.F. 2014. [How variable is recruitment for exploited marine fishes? A hierarchical model for testing life history theory](#). Can. J. Fish. Aquat. Sci. 71(7). 973–983.
- Thorson, J.T., Johnson, K.F., Methot, R.D. and Taylor, I.G. 2017. [Model-based estimates of effective sample size in stock assessment models using the Dirichlet-multinomial distribution](#). Fish. Res. 192. 84–93.
- Trijoulet, V., Albertsen, C.M., Kristensen, K., Legault, C.M., Miller, T.J. and Nielsen, A. 2023. [Model validation for compositional data in stock assessment models: calculating residuals with correct properties](#). Fish. Res. 257(106487). 1–10.
- Trijoulet, V. and Nielsen, A. 2022. [compResidual: Residual calculation for compositional observations](#). R package version 0.0.1, commit d4c74845089074d8016454f235044c5d13ded3a5.
- Xie, Y. 2025. [knitr: A General-Purpose Package for Dynamic Report Generation in R](#). R package version 1.51.